Some new N-topological spaces by generalized open sets

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Abstract. This paper introduces $N\tau\tilde{g}$-closure operator and characterize its properties. Moreover, some N-topological generalized continuous functions are defined and their properties are investigated. Furthermore, $N\tau\tilde{g}$-regular and $N\tau\tilde{g}$-normal spaces are introduced.

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1 Introduction

Abd E. Monsef et al. [1] introduced the concept of β-open sets and its continous functions. Maki et al. [9] defined and investigated some properties of generalized α-continuous functions. Reilly.I.L [11] characterized some bitopological separation axioms. Dorrset.C [2] defined semi-normal space. The concept of N-topological spaces and its open sets were initiated by Lellis Thivagar et al. [5]. He [6] 6] also established the properties of some weak forms of N-topological open sets and generalized closed sets. This paper focus on the introduction of $N\tau\tilde{g}$-closure operator and some generalized continuous functions. Further, we establish the properties of $N\tau\tilde{g}$-regular and $N\tau\tilde{g}$-normal spaces.

2 Preliminaries

In this section we recall some known results of N-topological spaces which are used in the following sections. By a space $(X, N\tau)$, we mean an N-topological space with N-topology $N\tau$ on X on which no separation axioms are assumed unless explicitly stated.

Definition 2.1. [5] Let X be a non empty set, $\tau_1, \tau_2, \ldots, \tau_N$ be N-arbitrary topologies defined on X. Then the collection $N\tau = \{S \subseteq X : S = (\bigcup_{i=1}^{N} A_i) \cup (\bigcap_{i=1}^{N} B_i), A_i, B_i \in \tau_i\}$, is said to be N-topology if it satisfying the following axioms:

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(i) $X, \varnothing \in N\tau$.

(ii) $\bigcup_{i=1}^{\infty} S_i \in N\tau$ for all $\{S_i\}_{i=1}^{\infty} \in N\tau$.

(iii) $\bigcap_{i=1}^{n} S_i \in N\tau$ for all $\{S_i\}_{i=1}^{n} \in N\tau$.

Then the pair $(X, N\tau)$ is called an N-topological space on X and the elements of the collection $N\tau$ are known as $N\tau$-open sets on X. A subset A of X is said to be $N\tau$-closed on X if the complement of A is $N\tau$-open on X. The set of all $N\tau$-open sets on X and the set of all $N\tau$-closed sets on X are respectively denoted by $N\tau O(X)$ and $N\tau C(X)$.

Definition 2.2. [5] Let $(X, N\tau)$ be an N-topological space and S be a subset of X. Then

(i) the $N\tau$-interior of S is defined as $N\tau\text{-}int(S) = \cup\{G : G \subseteq S$ and G is $N\tau$-open$\}$.

(ii) the $N\tau$-closure of S is defined as $N\tau\text{-}cl(S) = \cap\{F : S \subseteq F$ and F is $N\tau$-closed$\}$.

Definition 2.3. [6] A subset A of N-topological space $(X, N\tau)$ is said to be

(i) $N\tau\alpha$-open if $A \subseteq N\tau\text{-}int(N\tau\text{-}cl(N\tau\text{-}int(A)))$.

(ii) $N\tau$ semi-open if $A \subseteq N\tau\text{-}cl(N\tau\text{-}int(A))$.

(iii) $N\tau$ pre-open if $A \subseteq N\tau\text{-}int(N\tau\text{-}cl(A))$.

(iv) $N\tau\beta$-open if $A \subseteq N\tau\text{-}cl(N\tau\text{-}int(N\tau\text{-}cl(A)))$.

Definition 2.4. [7] A subset A of N-topological space $(X, N\tau)$ is said to be

(i) $N\tau$ generalized-closed (briefly $N\tau g$-closed) if $N\tau\text{-}cl(A) \subseteq U$ whenever $A \subseteq U$and U is $N\tau$-open in $(X, N\tau)$.

(ii) $N\tau\alpha$ generalized-closed (briefly $N\tau\alpha g$-closed) if $N\tau\text{-}\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$-open in $(X, N\tau)$.

(iii) $N\tau$ generalized α-closed (briefly $N\tau g\alpha$-closed) if $N\tau\text{-}\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau\alpha$-open in $(X, N\tau)$.

(iv) $N\tau$ generalized semi-closed (briefly $N\tau gs$-closed) if $N\tau\text{-}scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$-open in $(X, N\tau)$.

(v) $N\tau$ semi generalized-closed (briefly $N\tau sg$-closed) if $N\tau\text{-}scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$ semi-open in $(X, N\tau)$.

(vi) $N\tau\hat{g}$-closed if $N\tau\text{-}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$ semi-open in $(X, N\tau)$.

(vii) $N\tau^*g$-closed if $N\tau\text{-}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau\hat{g}$-open in $(X, N\tau)$.

(viii) $N\tau^{\#}g$-semi closed (briefly $N\tau^{\#}gs$-closed) if $N\tau\text{-}scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau^*g$-open in $(X, N\tau)$.

(ix) $N\tau\tilde{g}$-closed if $N\tau\text{-}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau^{\#}gs$-open in $(X, N\tau)$.

The complement of above N-topological generalized closed set is called respective generalized open sets. The set of all $N\tau g$-closed (resp. $N\tau\alpha g$-closed, $N\tau g\alpha$-closed, $N\tau gs$-closed, $N\tau sg$-closed, $N\tau\hat{g}$-closed, $N\tau^{*}g$-closed, $N\tau^{\#}gs$-closed, $N\tau\tilde{g}$-closed) sets of $(X, N\tau)$ is denoted by $N\tau GC(X)$ (resp. $N\tau\alpha GC(X)$, $N\tau G\alpha C(X)$, $N\tau GSC(X)$, $N\tau SGC(X)$, $N\tau\hat{G}C(X)$, $N\tau^{*}GC(X)$, $N\tau^{\#}GSC(X)$, $N\tau\tilde{G}C(X)$).

Definition 2.5. [7] An N-topological space $(X, N\tau)$ is called a

(i) $N\tau T_{1/2}$-space, if every $N\tau g$-closed set is $N\tau$-closed.

(ii) $N\tau_{gs}T^{\#}_{1/2}$-space, if every $N\tau^{\#}gs$-closed set is $N\tau$-closed.

(iii) $N\tau\alpha$-space, if every $N\tau\alpha$-closed set is $N\tau$-closed.

(iv) $N\tau_{\alpha}T_{b}$-space, if every $N\tau\alpha g$-closed set is $N\tau$-closed.

(v) $N\tau_{\alpha}T_{d}$-space, if every $N\tau\alpha g$-closed set is $N\tau g$-closed.

(vi) $N\tau T_{b}$-space, if every $N\tau gs$-closed set is $N\tau$-closed.

(vii) $N\tau^{*}T_{1/2}$-space, if every $N\tau g$-closed set is $N\tau^{*}g$-closed.

(viii) $N\tau T_{\hat{g}}$-space, if every $N\tau\hat{g}$-closed set is $N\tau$-closed.

(ix) $N\tau_{\alpha}T_{\tilde{g}}$-space, if every $N\tau\alpha g$-closed set is $N\tau\tilde{g}$-closed.

(x) $N\tau T_{\tilde{g}}$-space, if every $N\tau\tilde{g}$-closed set is $N\tau$-closed.

(xi) $N\tau_{g}T_{\tilde{g}}$-space, if every $N\tau g$-closed set is $N\tau\tilde{g}$-closed.

3 $N\tau$-$\tilde{g}$ closure operator in N-topological space

In this section we introduce the properties of $N\tau$-$\tilde{g}$ closure operators in N-topological space.

Definition 3.1. For any subset A of an N-topological space $(X, N\tau)$, $N\tau$-$\tilde{g}$ closure of A is denoted by $N\tau$-$\tilde{g}cl(A)$ and defined by intersection of all $N\tau\tilde{g}$-closed sets containing A. That is, $N\tau\text{-}\tilde{g}cl(A) = \cap\{F : A \subseteq F, F \in N\tau\tilde{G}C(X)\}$.

Definition 3.2. Let A be a subset of N-topological space $(X, N\tau)$ and $x \in X$. Then A is said to be a $N\tau$-$\tilde{g}$ neighbourhood of x in X if there exists a $N\tau\tilde{g}$-open set U in X such that $x \in U \subseteq A$.

Definition 3.3. A point $x \in X$ is said to be a $N\tau$-$\tilde{g}$ interior point of A if there exists a $N\tau\tilde{g}$-open set U containing x such that $U \subseteq A$. The set of all $N\tau$-$\tilde{g}$ interior points of A is known as the $N\tau\tilde{g}$-interior of A and denoted by $N\tau$-$\tilde{g}int(A)$.

Theorem 3.1. *Let $(X, N\tau)$ be an N-topological space on X and let $A, B \subseteq X$. Then*

(i) $N\tau$-$\tilde{g}cl(A)$ is the smallest $N\tau\tilde{g}$-closed set which containing A.

(ii) A is $N\tau\tilde{g}$-closed if and only if $N\tau\text{-}\tilde{g}cl(A) = A$.
In particular, $N\tau\text{-}\tilde{g}cl(\varnothing) = \varnothing$ and $N\tau\text{-}\tilde{g}cl(X) = X$.

(iii) $A \subseteq B \Rightarrow N\tau\text{-}\tilde{g}cl(A) \subseteq N\tau\text{-}\tilde{g}cl(B)$.

(iv) $N\tau\text{-}\tilde{g}cl(A \cup B) = N\tau\text{-}\tilde{g}cl(A) \cup N\tau\text{-}\tilde{g}cl(B)$.

(v) $N\tau\text{-}\tilde{g}cl(A \cap B) \subseteq N\tau\text{-}\tilde{g}cl(A) \cap N\tau\text{-}\tilde{g}cl(B)$.

(vi) $N\tau\text{-}\tilde{g}cl(N\tau\text{-}\tilde{g}cl(A)) = N\tau\text{-}\tilde{g}cl(A)$.

Proof. *Here we shall prove parts (ii) only and similarly we can prove the remaining parts. Assume A is $N\tau\tilde{g}$-closed, then A is the only smallest $N\tau\tilde{g}$-closed set which containing itself and therefore, $N\tau\text{-}\tilde{g}cl(A) = A$. Conversely, assume $N\tau\text{-}\tilde{g}cl(A) = A$. Then A is the smallest $N\tau\tilde{g}$-closed set containing itself. Therefore, A is $N\tau\tilde{g}$-closed. Particularly, since $\varnothing$ and X are $N\tau\tilde{g}$-closed sets, then $N\tau\text{-}\tilde{g}cl(\varnothing) = \varnothing$ and $N\tau\text{-}\tilde{g}cl(X) = X$.*

Example 3.4. Let $X = \{a, b, c, d\}$. For $N = 4$, consider $\tau_1 O(X) = \{X, \varnothing, \{a\}\}$, $\tau_2 O(X) = \{\varnothing, X, \{c, d\}\}$, $\tau_3 O(X) = \varnothing, X, \{a, c, d\}\}$ and $\tau_4 O(X) = \{X, \varnothing, \{b, c, d\}\}$, then $4\tau O(X) = \{X, \varnothing, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ and also $4\tau\text{-}\tilde{G}C(X) = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. Let $A = \{c\}$ and $B = \{d\}$, then $4\tau\text{-}\tilde{g}cl(A) = \{b, c\}$, $4\tau\text{-}\tilde{g}cl(B) = \{b, d\}$ and $4\tau\text{-}\tilde{g}cl(A \cap B) = \varnothing$. Therefore, $4\tau\text{-}\tilde{g}cl(A \cap B) \neq 4\tau\text{-}\tilde{g}cl(A) \cap 4\tau\text{-}\tilde{g}cl(B)$. That is, equality does not hold in (v) of theorem 3.1.

Theorem 3.2. *In an N-topological space $(X, N\tau)$, the $N\tau\text{-}\tilde{g}$ closure operator is a Kuratowski closure operator.*

Proof. The proof follows from (i), (ii), (iv) and (vi) of Theorem 3.1. □

Theorem 3.3. *For any subset Aof an N-topological space X, $x \in N\tau\text{-}\tilde{g}cl(A)$ if and only if for any $N\tau\tilde{g}$-open set U containing x such that $A \cap U \neq \varnothing$.*

Proof. Suppose that $x \in N\tau\text{-}\tilde{g}cl(A)$ and U is any $N\tau\tilde{g}$-open set containing x such that $A \cap U = \varnothing$. Then $X - U$ is a $N\tau\tilde{g}$-closed set in X containing A. Therefore, $N\tau\text{-}\tilde{g}cl(A) \subseteq X - U$. Since $x \notin X - U$, $x \notin N\tau\text{-}\tilde{g}cl(A)$, which is a contradiction. On the other hand, assume that every $N\tau\tilde{g}$-closed set is containing x intersects A. If $x \notin N\tau\text{-}\tilde{g}cl(A)$, then there exists a $N\tau\tilde{g}$-closed set F such that $A \subseteq F$ and $x \notin F$. Then $X - F$ is a $N\tau\tilde{g}$-open set in X containing x such that $A \cap (X - F) = \varnothing$, which is a contradiction. Therefore, $x \in N\tau\text{-}\tilde{g}cl(A)$. □

Theorem 3.4. *For any subset A of an N-topological space $(X, N\tau)$,*

(i) $X - N\tau\text{-}\tilde{g}int(A) = N\tau\text{-}\tilde{g}cl(X - A)$

(ii) $X - N\tau\text{-}\tilde{g}cl(A) = N\tau\text{-}\tilde{g}int(X - A)$

Proof. *The proof is directly follows from theorem 3.1.*

4 Generalized functions in N-topological spaces

This section introduce some N-topological generalized continuous functions and investigate the relationship between them.

Definition 4.1. Let $(X, N\tau)$ and $(Y, N\sigma)$ be two N-topological spaces, then a function $f : (X, N\tau) \to (Y, N\sigma)$ is said to be

(i) a N^*-g continuous if the inverse image of every $N\sigma$-closed set in $(Y, N\sigma)$ is $N\tau g$-closed in $(X, N\tau)$.

(ii) a N^*-αg continuous if the inverse image of every $N\sigma$-closed set in $(Y, N\sigma)$ is $N\tau\alpha g$-closed in $(X, N\tau)$.

(iii) a N^*-$g\alpha$ continuous if the inverse image of every $N\sigma$-closed set in $(Y, N\sigma)$ is $N\tau g\alpha$-closed in $(X, N\tau)$.

(iv) a N^*-gs continuous if the inverse image of every $N\sigma$-closed set in $(Y, N\sigma)$ is $N\tau gs$-closed in $(X, N\tau)$.

(v) a N^*-sg continuous if the inverse image of every $N\sigma$-closed set in $(Y, N\sigma)$ is $N\tau sg$-closed in $(X, N\tau)$.

(vi) a N^*-$\hat{g}$ continuous if the inverse image of every $N\sigma$-closed set in $(Y, N\sigma)$ is $N\tau\hat{g}$-closed in $(X, N\tau)$.

(vii) a N^*-$^*g$ continuous if the inverse image of every $N\sigma$-closed set in $(Y, N\sigma)$ is $N\tau^*g$-closed in $(X, N\tau)$.

(viii) a N^*-$^{\#}gs$ continuous if the inverse image of every $N\sigma$-closed set in $(Y, N\sigma)$ is $N\tau^{\#}gs$-closed in $(X, N\tau)$.

(ix) a N^*-$\tilde{g}$ continuous if the inverse image of every $N\sigma$-closed set in $(Y, N\sigma)$ is $N\tau\tilde{g}$-closed in $(X, N\tau)$.

Theorem 4.1. *Let $(X, N\tau)$ and $(Y, N\sigma)$ be two N-topological spaces, then*

(i) every N^*-continuous function is N^*-g continuous.

(ii) every N^*-semi continous function is N^*-$^{\#}gs$ continuous.

(iii) every N^*-α continuous function is N^*-$^{\#}gs$ continuous.

(iv) every N^*-g continuous function is N^*-αg continuous.

(v) every N^*-g continuous function is N^*-gs continuous.

(vi) every N^*-sg continuous function is N^*-β continuous.

(vii) every N^*-$g\alpha$ continuous function is N^*-pre continuous.

(viii) every N^*-continuous function is N^*-$\tilde{g}$ continuous.

(ix) every N^*-$\tilde{g}$ continuous function is N^*-$\hat{g}$ continuous.

(x) *every N^*-$\tilde{g}$ continuous function is N^*-g continuous.*

(xi) *every N^*-$\tilde{g}$ continuous function is N^*-αg continuous.*

(xii) *every N^*-$\tilde{g}$ continuous function is N^*-sg continuous.*

(xiii) *every N^*-$\tilde{g}$ continuous function is N^*-β continuous.*

(xiv) *every N^*-$\tilde{g}$ continuous function is N^*-$g\alpha$ continuous.*

(xv) *every N^*-$\tilde{g}$ continuous function is N^*-pre continuous.*

(xvi) *every N^*-$\tilde{g}$ continuous function is N^*-gs continuous.*

Proof. The proof follows from proposition 3.3 and 3.7 of [7]. □

Remark 4.2. The following examples shows that the converse of theorem 4.1 need not be true.

Example 4.3. Let $X = Y = \{a, b, c, d\}$, $N = 3$, consider $\tau_1 O(X) = \{X, \varnothing\}$, $\tau_2 O(X) = \{\varnothing, X, \{a, b\}\}$, $\tau_3 O(X) = \{\varnothing, X, \{a, b\}\}$, then $3\tau O(X) = \{X, \varnothing, \{a, b\}\}$ and $\sigma_1 O(Y) = \{Y, \varnothing, \{a\}\}$, $\sigma_2 O(Y) = \{\varnothing, Y, \{b, c\}\}$, $\sigma_3 O(Y) = \{\varnothing, Y, \{a, b, c\}\}$, then $3\sigma O(Y) = \{Y, \varnothing, \{a\}, \{b, c\}, \{a, b, c\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b, f(b) = a$, $f(c) = c$ and $f(d) = d$. Then f is 3^*-pre continuous, 3^*-β contunuous, 3^*-$^{\#}gs$ continuous, 3^*-$\tilde{g}$ continuous, and 3^*-g continuous, but not 3^*-continuous, not 3^*-semi continuous and not 3^*-α continuous.

Example 4.4. Let $X = Y = \{a, b, c, d\}$, $N = 3$, consider $\tau_1 O(X) = \{X, \varnothing, \{a\}\}$, $\tau_2 O(X) = \{\varnothing, X, \{b\}\}$, $\tau_3 O(X) = \{\varnothing, X, \{a, b\}\}$, then $3\tau O(X) = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma_1 O(Y) = \{Y, \varnothing, \{a\}\}$, $\sigma_2 O(Y) = \{\varnothing, Y, \{c, d\}\}$, $\sigma_3 O(Y) = \{\varnothing, Y, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, then $3\sigma O(Y) = \{Y, \varnothing, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c, f(b) = d$, $f(c) = a$ and $f(d) = b$. Then f is 3^*-α continuous, 3^*-semi contunuous, 3^*-pre continuous, 3^*-β continuous, 3^*-g continuous, 3^*-$g\alpha$ continuous, 3^*-αg continuous, 3^*-sg continuous and 3^*-gs continuous but not 3^*-continuous, not 3^*-$\tilde{g}$ continuous, and not 3^*-$\hat{g}$ continuous.

Example 4.5. Let $X = Y = \{a, b, c, d\}$, $N = 3$, consider $\tau_1 O(X) = \{X, \varnothing, \{a, b\}\}$, $\tau_2 O(X) = \{\varnothing, X, \{a, b, c\}\}$, $\tau_3 O(X) = \{\varnothing, X, \{a, b\}, \{a, b, c\}\}$, then $3\tau O(X) = \{X, \varnothing, \{a, b\}, \{a, b, c\}\}$ and $\sigma_1 O(Y) = \{Y, \varnothing, \{b\}, \{b, c\}\}$, $\sigma_2 O(Y) = \{\varnothing, Y, \{b\}, \{b, d\}\}$,$\sigma_3 O(Y) = \{\varnothing, Y, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$, then $3\sigma O(Y) = \{Y, \varnothing, \{b\}, \{b, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a, f(b) = b$, $f(c) = c$ and $f(d) = d$. Then f is 3^*-pre continuous, 3^*-β continuous but not 3^*-continuous, not 3^*-$g\alpha$ continuous, not 3^*-αg continuous, not 3^*-sg continuous, not 3^*-gs continuous and not 3^*-g continuous.

Example 4.6. Let $X = Y = \{a, b, c, d\}$, $N = 3$, consider $\tau_1 O(X) = \{X, \varnothing, \{a, b, c\}\}$, $\tau_2 O(X) = \{\varnothing, X, \{a, b, c\}\}$, $\tau_3 O(X) = \{\varnothing, X\}$, then $3\tau O(X) = \{X, \varnothing, \{a, b, c\}\}$ and $\sigma_1 O(Y) = \{Y, \varnothing, \{b\}, \{b, c\}\}$, $\sigma_2 O(Y) = \{\varnothing, Y, \{b\}, \{b, d\}\}$, $\sigma_3 O(Y) = \{\varnothing, Y, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$, then $3\sigma O(Y) = \{Y, \varnothing, \{b\}, \{b, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a, f(b) = b$, $f(c) = c$ and $f(d) = d$. Then

f is 3^*-α continuous, 3^*-semi continuous, 3^*-pre continuous, 3^*-β continuous, 3^*-$g\alpha$ continuous, 3^*-αg continuous, 3^*-sg continuous, 3^*-gs continuous, 3^*-$^{\#}gs$ continuous, 3^*-$\hat{g}$ continuous, 3^*-$^*g$ continuous and 3^*-$\tilde{g}$ continuous but not 3^*-g continuous and not 3^*-continuous.

Theorem 4.2. *Let $(X, N\tau)$ and $(Y, N\sigma)$ be two N-topological spaces, then*

(i) in $N\tau T_{1/2}$-space, every N^*-g continuous function is N^*-continuous.

(ii) in $N\tau_{gs}T^{\#}_{1/2}$-space, every N^*-$^{\#}gs$ continous function is N^*-semi continuous.

(iii) in $N\tau_{gs}T^{\#}_{1/2}$-space, every N^*-$^{\#}gs$ continuous function is N^*-α continuous.

(iv) in $N\tau_\alpha T_d$-space, every N^*-αg continuous function is N^*-g continuous.

(v) in $N\tau T_b$-space, every N^*-gs continuous function is N^*-g continuous.

(vi) in $N\tau T_{\tilde{g}}$-space, every N^*-$\tilde{g}$ continuous function is N^*-continuous.

(vii) in $N\tau T_{\hat{g}}$-space, every N^*-$\hat{g}$ continuous function is N^*-$\tilde{g}$ continuous.

(viii) in $N\tau_g T_{\tilde{g}}$-space, every N^*-g continuous function is N^*-$\tilde{g}$ continuous.

(ix) in $N\tau_\alpha T_{\tilde{g}}$-space, every N^*-αg continuous function is N^*-$\tilde{g}$ continuous.

(x) in $N\tau T_b$-space, every N^*-gs continuous function is N^*-$\tilde{g}$ continuous.

Proof. The proof trivially follows from definitions and propositions of [7]. □

Theorem 4.3. *Let $(X, N\tau)$ and $(Y, N\sigma)$ be two N-topological spaces, then the following are equivalent for a function $f : X \to Y$.*

(i) f is N^*-$\tilde{g}$ continuous.

(ii) For every $N\sigma$-open set V of Y, $f^{-1}(V)$ is $N\tau$-$\tilde{g}$ open.

(iii) For each $x \in X$ and each $N\sigma$-open set V of Y containing $f(x)$, there exists a $N\tau$-$\tilde{g}$ open set U containing x such that $f(U) \subseteq V$.

(iv) $f(N\tau\text{-}\tilde{g}cl(A)) \subseteq N\sigma\text{-}cl(f(A))$ for any subset A of X.

(v) $N\tau\text{-}\tilde{g}cl(f^{-1}(B)) \subseteq f^{-1}(N\sigma\text{-}cl(B))$ for any subset B of Y.

(vi) $f^{-1}(N\sigma\text{-}\tilde{g}int(B)) \subseteq N\tau\text{-}\tilde{g}int(f^{-1}(B))$ for any subset B of Y.

Proof.$(i) \Rightarrow (ii)$: It follows from $f^{-1}(Y - V) = X - f^{-1}(V)$, for any subset V of Y and the definition of $N\tau$-$\tilde{g}$ open set.
$(ii) \Rightarrow (iii)$: Let $x \in X$ and V be any $N\sigma$-open set in Y containing $f(x)$. Since f is N^*-$\tilde{g}$ continuous, $f^{-1}(V)$ is $N\tau\tilde{g}$-open in X and $x \in f^{-1}(V)$. If we take $U = f^{-1}(V)$, then U is $N\tau\tilde{g}$-open set in X containing x and $f(U) \subseteq V$.
$(iii) \Rightarrow (iv)$: Let A be any subset of X and $x \in N\tau\text{-}\tilde{g}cl(A)$ and V be any $N\sigma$-open set in Y containing $f(x)$. By hypothesis, there exists a $N\tau\tilde{g}$-open set U in X containing x such that $f(U) \subseteq V$. Since $x \in N\tau\text{-}\tilde{g}cl(A)$ and by theorem 3.3, $U \cap A \neq \emptyset$ and

hence $\varnothing \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$. Therefore, $f(x) \in N\sigma\text{-}cl(f(A))$. Thus $f(N\tau\tilde{g}cl(A)) \subseteq N\sigma\text{-}cl(f(A))$.
$(iv) \Rightarrow (v)$: Let B be any subset of Y. By hypothesis, $f(N\tau\text{-}\tilde{g}cl(f^{-1}(B))) \subseteq N\sigma\text{-}cl(f(f^{-1}(B))) \subseteq N\sigma\text{-}cl(B)$ and hence $N\tau\text{-}\tilde{g}cl(f^{-1}(B)) \subseteq f^{-1}(N\sigma\text{-}cl(B))$.
$(v) \Rightarrow (vi)$: Let B be any subset in Y. By hypothesis, $N\tau\text{-}\tilde{g}cl(f^{-1}(Y-B)) \subseteq f^{-1}(N\sigma\text{-}cl(Y-B))$. Then $X - N\tau\text{-}\tilde{g}int(f^{-1}(B)) \subseteq X - f^{-1}(N\sigma\text{-}int(B))$. Therefore, $f^{-1}(N\sigma\text{-}int(B)) \subseteq N\tau\text{-}\tilde{g}int(f^{-1}(B))$.
$(vi) \Rightarrow (i)$: Let F be any $N\sigma$-closed set in Y. By hypothesis, $f^{-1}(Y - F) = f^{-1}(N\sigma\text{-}int(Y-F)) \subseteq N\tau\text{-}\tilde{g}int(f^{-1}(Y-F)) = X - N\tau\text{-}\tilde{g}cl(f^{-1}(F))$. Then $N\tau\text{-}\tilde{g}cl(f^{-1}(F)) \subseteq f^{-1}(F)$. Therefore, $N\tau\text{-}\tilde{g}cl(f^{-1}(F)) = f^{-1}(F)$. By theorem 3.3, $f^{-1}(F)$ is $N\tau\tilde{g}$-closed in X and hence f is $N^*\text{-}\tilde{g}$ continuous. □

5 N-topological regular and normal spaces

In this section we investigate the properties of $N\tau\tilde{g}$-regular and $N\tau\tilde{g}$-normal spaces in N-topological spaces.

Definition 5.1. An N-topological space $(X, N\tau)$ is said to be $N\tau\tilde{g}$ regular if for each $N\tau\tilde{g}$-closed set F of $(X, N\tau)$ and a point $x \notin F$, there exists disjoint $N\tau$-open sets U and V such that $F \subseteq U$ and $x \in V$.

Example 5.2. Let $X = \{a, b, c, d\}$. For $N = 2$, consider $\tau_1 O(X) = \{X, \varnothing, \{a, b\}\}$ and $\tau_2 O(X) = \{\varnothing, X, \{c, d\}\}$ then $2\tau O(X) = \{X, \varnothing, \{a, b\}, \{c, d\}\}$. Also $2\tau\text{-}\tilde{G}C(X) = \{X, \varnothing, \{a, b\}, \{c, d\}\}$. Here $(X, 2\tau)$ is $2\tau\tilde{g}$ regular-space.

Theorem 5.1. *Let $(X, N\tau)$ be an N-topological space, then the following are equivalent:*

(i) $(X, N\tau)$ is $N\tau\tilde{g}$ regular.

(ii) For every point x of $(X, N\tau)$ and each $N\tau\tilde{g}$-neighbourhood W of x, there exists a $N\tau$-open set V of $(X, N\tau)$ such that $x \in V \subseteq N\tau\text{-}cl(V) \subseteq W$.

(iii) For every point x of $(X, N\tau)$ and each $N\tau\tilde{g}$-closed set F not containing x, there exists a $N\tau$-open set V of $(X, N\tau)$ containing x such that $N\tau\text{-}cl(V) \cap F = \varnothing$.

Proof: $(i) \Rightarrow (ii)$: Let W be any $N\tau\tilde{g}$-neighbourhood of $x \in X$. Then there exists a $N\tau\tilde{g}$-open set G such that $x \in G \subseteq W$. Since $X - G$ is a $N\tau\tilde{g}$-closed set in $(X, N\tau)$ such that $x \notin X - G$. By hypothesis, there exist disjoint $N\tau$-open sets U and V such that and $X - G \subseteq U$ and $x \in V$. Now $V \subseteq X - U$ implies $N\tau\text{-}cl(V) \subset N\tau\text{-}cl(X - U) = X - U$. Also $X - U \subseteq G \subseteq W$. Hence V is a $N\tau$-open set such that $x \in V \subseteq N\tau\text{-}cl(V) \subseteq W$.
$(ii) \Rightarrow (i)$: Let F be a $N\tau\tilde{g}$-closed set of $(X, N\tau)$ such that $x \notin F$. Then $X - F$ is a $N\tau\tilde{g}$-open set containing x and hence $X - F$ is a $N\tau\tilde{g}$-neighbourhood of x. By hypothesis, there exists a $N\tau$-open set V of $(X, N\tau)$ such that $x \in V \subseteq N\tau\text{-}cl(V) \subset X - F$ which implies $F \subseteq X - N\tau\text{-}cl(V)$. Now $X - N\tau\text{-}cl(V)$ is a $N\tau$-open set containing F such that $V \cap (X - N\tau\text{-}cl(V)) = \varnothing$. Hence $(X, N\tau)$ is $N\tau\tilde{g}$ regular.
$(ii) \Rightarrow (iii)$: Let $x \in X$ and F be a $N\tau\tilde{g}$-closed set not containing x. Then $X - F$ is a $N\tau\tilde{g}$-neighbourhood of x. Hence by hypothesis, there exists a $N\tau$-open set V of

$(X, N\tau)$ containing x such that $N\tau\text{-}cl(V) \subseteq X - F$ which implies $N\tau\text{-}cl(V) \cap F = \varnothing$.
$(iii) \Rightarrow (ii)$: Let $x \in X$ and W be any $N\tau\tilde{g}$-neighbourhood of x. Then there exists a $N\tau\tilde{g}$-open set G such that $x \in G \subseteq W$. Since $X - G$ is a $N\tau\tilde{g}$-closed set in $(X, N\tau)$ such that $x \notin X - G$ and by hypothesis, there exists a $N\tau$-open set V of $(X, N\tau)$ containing x such that $N\tau\text{-}cl(V) \cap (X - G) = \varnothing$ which implies $N\tau\text{-}cl(V) \subseteq G \subseteq W$. Hence V is a $N\tau$-open set of $(X, N\tau)$ such that $x \in V \subseteq N\tau\text{-}cl(V) \subseteq W$. □

Definition 5.3. An N-topological space $(X, N\tau)$ is said to be $N\tau\tilde{g}$ normal if for any pair of disjoint $N\tau\tilde{g}$-closed sets A and B of $(X, N\tau)$, there exists disjoint $N\tau$-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Example 5.4. Let $X = \{a, b, c, d\}$. For $N = 5$, consider $\tau_1 O(X) = \{X, \varnothing, \{a\}\}$, $\tau_2 O(X) = \{\varnothing, X, \{c\}\}$, $\tau_3 O(X) = \{\varnothing, X, \{a, c\}\}$, $\tau_4 O(X) = \{\varnothing, X, \{a, b, d\}\}$ and $\tau_5 O(X) = \{\varnothing, X, \{a\}, \{a, c\}\}$. Then $5\tau O(X) = \{X, \varnothing, \{a\}, \{c\}, \{a, c\}, \{a, b, d\}\}$ and $5\tau\text{-}\tilde{G}C(X) = \{X, \varnothing, \{c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$. Here $(X, 5\tau)$ is $5\tau\tilde{g}$ normal-space.

Theorem 5.2. *Let $(X, N\tau)$ be an N-topological space, then the following are equivalent:*

(i) $(X, N\tau)$ *is $N\tau\tilde{g}$ normal.*

(ii) For every $N\tau\tilde{g}$-closed set A and every $N\tau\tilde{g}$-open set B containing A, there exists a $N\tau$-open set U of $(X, N\tau)$ such that $A \subseteq U \subseteq N\tau\text{-}cl(U) \subseteq B$.

Proof. $(i) \Rightarrow (ii)$: Let A be a $N\tau\tilde{g}$-closed set and B be a $N\tau\tilde{g}$-open set containing A. Since A and $X - B$ are disjoint $N\tau\tilde{g}$-closed sets in $(X, N\tau)$ and by hypothesis, there exists disjoint $N\tau$-open sets U and V such that $A \subseteq U$ and $X - B \subseteq V$. Also $U \cap V = \varnothing$ implies $N\tau\text{-}cl(U) \cap V = \varnothing$ and so $N\tau\text{-}cl(U) \subseteq X - V$. Hence $A \subseteq U \subseteq N\tau\text{-}cl(U) \subseteq X - V \subseteq B$ which is the required result.
$(ii) \Rightarrow (i)$: Let A and B be disjoint $N\tau\tilde{g}$-closed sets of $(X, N\tau)$. Then $A \subseteq X - B$ where $X - B$ is $N\tau\tilde{g}$-open. By hypothesis, there exists a $N\tau$-open set U of $(X, N\tau)$ such that $A \subseteq U \subseteq N\tau\text{-}cl(U) \subseteq X - B$. Let $V = X - N\tau\text{-}cl(U)$. Then U and V are disjoint $N\tau$-open sets containing A and B respectively which implies required result. □

Theorem 5.3. *In an N-topological space $(X, N\tau)$, the followings are equivalent:*

(i) $(X, N\tau)$ *is $N\tau\tilde{g}$ normal.*

(ii) For each pair of disjoint $N\tau\tilde{g}$-closed sets A and B of $(X, N\tau)$, there exists a $N\tau$-open set U of $(X, N\tau)$ containing A such that $N\tau\text{-}cl(U) \cap B = \varnothing$.

(iii) For each pair of disjoint $N\tau\tilde{g}$-closed sets A and B of $(X, N\tau)$, there exists a $N\tau$-open set U of $(X, N\tau)$ containing A and a $N\tau$-open set V containing B such that $N\tau\text{-}cl(U) \cap N\tau\text{-}cl(V) = \varnothing$.

Proof. $(i) \Rightarrow (ii)$: Let A and B be disjoint $N\tau\tilde{g}$-closed sets of $(X, N\tau)$. Then the $N\tau\tilde{g}$-closed set A is contained in the $N\tau\tilde{g}$-open set $X - B$. By theorem 5.2, there exists a $N\tau$-open set U such that $A \subseteq U \subseteq N\tau\text{-}cl(U) \subseteq X - B$. Thus U is the

required $N\tau$-open set containing A such that $N\tau$-$cl(U) \cap B = \varnothing$ which proves (ii).
$(ii) \Rightarrow (iii)$: Let A and B be disjoint $N\tau\tilde{g}$-closed sets of $(X, N\tau)$. By hypothesis, there exists a $N\tau$-open set U containing A such that $N\tau$-$cl(U) \cap B = \varnothing$. Now $N\tau$-$cl(U)$ and B are disjoint $N\tau\tilde{g}$-closed sets of $(X, N\tau)$. Again by(ii), there exists a $N\tau$-open set V of $(X, N\tau)$ containing B such that $N\tau$-$cl(U) \cap N\tau$-$cl(V) = \varnothing$ which proves (iii).
$(iii) \Rightarrow (i)$: Let A and B be disjoint $N\tau\tilde{g}$-closed sets of $(X, N\tau)$. By hypothesis, there exists a $N\tau$-open set U containing A and a $N\tau$-open set V containing B such that $N\tau$-$cl(U) \cap N\tau$-$cl(V) = \varnothing$. Hence U and V are $N\tau$-open sets containing A and B respectively such that $U \cap V = \varnothing$ which proves (i). □

6 Conclusions

In topology and related fields of mathematics, there are several restrictions that one often makes on the kinds of topological spaces that one wishes to consider. The separation axioms are about the use of topological means to distinguish disjoint sets and distinct points. In this paper, we have introduced and investigated some properties of $N\tau\tilde{g}$-regular and $N\tau\tilde{g}$-normal spaces. We can extend most of the results to other research area of general topology such as nano-topology, fuzzy topology, ideal topology, supra topology and so on.

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