ON THE TRIVIALITY OF LIE GROUPOIDS

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Abstract

We give necessary and sufficient conditions ensuring the triviality of a Lie groupoid Ω. These conditions involve the restricted holonomy morphisms of flat connections on Ω, Lie-integrability and appropriate differential equations on the Lie algebroid $L\Omega$. We also introduce the correcting components of flat connections of Lie groupoids, whose existence is equivalent to the triviality of the groupoid.

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0 Introduction and preliminaries

In the theory of Lie groupoids one has often to decide whether a Lie groupoid is isomorphic to a trivial one. The main object of the present paper is the proof of a theorem stating certain criteria for the triviality of Lie groupoids. Holonomy, Lie-integrability and differential equations on Lie algebroids are combined in the formulation of these criteria.

The paper is structured as follows: In Section 1 we define differential equations with total differential on Lie algebroids , which generalize those with total (logarithmic) differential and values in Lie algebras. In Section 2 we study the restricted holonomy morphisms of flat connections on Lie groupoids, which are the analogs of the monodromy homorphisms of flat principal bundles. Based on the results of the previous Sections, we prove in Section 3 the main theorem of the paper, which contains triviality criteria for Lie groupoids. We also introduce the correcting components of

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flat connections of groupoids, a notion whose local version is useful to the study of locally trivial Lie algebroids.

Throughout the present text, all the manifolds are assumed to be smooth (C^∞), Hausdorff, paracompact and finite-dimensional.

An exposition of the geometrical theory of Lie groupoids and algebroids can be found, for example, in [5]. Here we only fix the terminology and the basic notations used throughout.

A groupoid will be denoted as a pair of the form (Ω, B), where Ω is the *total space* and B the *base*. We shall use the symbol α_Ω (resp. β_Ω) for the *source-projection* (resp. the *target-projection*) of (Ω, B) and the symbol $\tilde{x}$ for the *unity* corresponding to $x \in B$. If no danger of confusion arises, we shall simply write Ω, α and β, instead of (Ω, B), α_Ω and β_Ω respectively. The morphism of groupoids $(\beta_\Omega, \alpha_\Omega) : \Omega \to B \times B$ over B will be called the *anchor* of (Ω, B).

The α_Ω*-fiber* (resp. the β_Ω*-fiber*) of Ω over x is the set $\Omega_x := \alpha_\Omega^{-1}(x), \quad x \in B$, (resp. the set $\Omega^x := \beta_\Omega^{-1}(x)$). We also set $\Omega_x^y := \Omega_x \cap \Omega^y$, for every $x, y \in B$. The *left* (resp. *right*) *translation by* $\xi \in \Omega$ is represented by L_ξ (resp. R_ξ).

We shall use the symbol L for the *Lie functor*. Hence, to a differentiable groupoid (Ω, B) corresponds the Lie algebroid $L\Omega$, whereas a morphism φ of differentiable groupoids produces the morphism $L\varphi$ between the corresponding Lie algebroids. The *anchor* of the Lie algebroid $L\Omega$ will be denoted by q^Ω.

We enumerate some differentiable or Lie groupoids which we need in the present paper:

a) The *trivial groupoid* $B \times G \times B$, where B is a manifold and G a Lie group. The cartesian product $B \times B$ and the Lie groups are special cases of trivial groupoids.

b) The *fundamental groupoid* $\wp(B)$ of the manifold B.

c) The *inner subgroupoid* $G\Omega := \bigcup_{x \in B} \Omega_x^x$ of the Lie groupoid (Ω, B).

d) The α_Ω*-fiber product*

$$\Omega \times_\alpha \Omega := \{(\xi, \eta) \in \Omega \times \Omega : \alpha_\Omega(\xi) = \alpha_\Omega(\eta)\}.$$

1 Differential equations on Lie algebroids

We introduce appropriate differential equations with total differential on Lie algebroids. For more details, we refer the reader to [7].

Let (A, M) be a Lie groupoid and $p : M \to M'$ a *surjective submersion with connected fibers*. Then the *quotient groupoid of A by the submersion p* is the pair $(A/p, M)$, with

$$A/p := \{\, \xi \in A : (p \circ \alpha_A)(\xi) = (p \circ \beta_A)(\xi)\}.$$

The quotient groupoid A/p is a differentiable subgroupoid of A, thus the Lie algebroid $L(A/p)$ exists and is a Lie subalgebroid of $L(A)$.

Remark 1.1. A Lie groupoid (Ω, B) is the quotient groupoid of itself by the 1-fibered submersion $p : B \to \{pt\}$. However, an arbitrary quotient groupoid is not, in general, a Lie groupoid.

We define an appropriate total differential for morphisms of Lie groupoids, generalizing the ordinary total (logarithmic) differential for Lie group valued differentiable maps

$$D : C^{\infty}(M, G) \longrightarrow \Lambda^1(M, T_eG) : f \mapsto f^* a_G,$$

where a_G is the Maurer-Cartan 1-form of the Lie group G. As a matter of fact, a morphism of differentiable groupoids $\phi : \Omega_1 \to \Omega_2 \times_\alpha \Omega_2$ induces the morphism of Lie algebroids $L\phi : L\Omega_1 \to T^\alpha\Omega_2 := Ker(T\alpha_{\Omega_2}) \subseteq T\Omega_2$, which, composed with the standard morphism of Lie algebroids

$$\Re : T^\alpha\Omega_2 \longrightarrow L\Omega_2 : u \mapsto T_\xi R_\xi^{-1}(u); \qquad u \in T_\xi\Omega_{2,\alpha(\xi)},$$

yields the morphism of Lie algebroids

$$\Delta\phi := \Re \circ L\phi : L\Omega_1 \longrightarrow L\Omega_2$$

and the desired *(right) total differential*

$$\Delta : \mathrm{Mor}(\Omega_1, \Omega_2 \times_\alpha \Omega_2) \longrightarrow \mathrm{Mor}(L\Omega_1, L\Omega_2) : \phi \mapsto \Delta\phi,$$

where the symbol $\mathrm{Mor}(\Omega_1, \Omega_2 \times_\alpha \Omega_2)$ (resp. $\mathrm{Mor}(L\Omega_1, L\Omega_2)$) is the set of morphisms between the groupoids Ω_1 and $\Omega_2 \times_\alpha \Omega_2$ (resp. the Lie algebroids $L\Omega_1$ and $L\Omega_2$).

Now we fix two Lie groupoids (A, M), (Ω, B), and a submersion $p' : M \to M'$ with connected fibers. We also suppose that an imbedding $i : M' \to B$ exists. The mapping i induces the natural diffeomorphism $i' : M' \to i(M')$ and we set $q := i \circ p'$, $p := i' \circ p'$.

Definition 1.2. If $(\omega, \nu) : (L(A/p), M) \to (L\Omega, B)$ is a morphism of directed vector bundles (see [9]), then an equation of the form

$$\Delta x = \omega \tag{1}$$

is called a *differential equation with total differential* Δ *and coefficient* ω. A *(global) solution* of (1) is a morphism of differentiable groupoids

$$(\psi, f) : (A/p, M) \longrightarrow (\Omega \times_\alpha \Omega, \Omega),$$

which satisfies the following conditions:

$$\begin{gathered} \alpha_\Omega \circ f = q, \qquad \beta_\Omega \circ f = \nu, \\ \Delta\psi = \omega. \end{gathered} \tag{2}$$

The definition of local solutions (over an open $U \subseteq M$) is similar; it suffices to replace Ω by $\Omega_U^U := \alpha_\Omega^{-1}(U) \cap \beta_\Omega^{-1}(U)$.

The equations with total differential Δ generalize the corresponding equations with logarithmic differential D; the latter equations arise if $A = M \times M$, p is the 1-fibered submersion and Ω is a Lie group (see [7, Proposition 3.4]).

In the present paper, we shall use only equations of the form $\Delta x = \gamma$, where $\gamma : TB \to L\Omega$ is a *connection* of the Lie groupoid (Ω, B).

2 Restricted holonomy morphisms

Let γ be a *flat* connection on the Lie groupoid Ω. Then, working as in [5, Theorem III.7.3], we can prove the following

Proposition 2.1. *Let $c : [0,1] \mapsto B$ be a differentiable curve in the manifold B. Then, for every $\xi \in \Omega^{c(0)}_{c(0)}$, there exists a unique curve $\hat{c}_\xi$ of Ω, satisfying the conditions*

$$\begin{aligned} \hat{c}_\xi(0) &= \xi, \\ \hat{c}_\xi(t) &\in \Omega^{c(t)}_{c(0)}, \end{aligned} \tag{3}$$

$$TR^{-1}_{\hat{c}_\xi(t)}\left(\left.\frac{d\hat{c}_\xi}{dt}\right|_t\right) = (\gamma \circ \dot{c})(t), \tag{4}$$

for every $t \in [0,1]$.

Definition 2.2. The curve $\hat{c}_\xi$ is called *the lifting of c with initial condition ξ.*

On the other hand, if we set $\beta_{c(0)} := \beta|\Omega_{c(0)}$, the connection γ induces the mapping

$$\bar{\gamma} \equiv \bar{\gamma}_{c(0)} : \beta^*_{c(0)}(TB) \longrightarrow T(\Omega_{c(0)}) : (\eta, v) \mapsto TR_\eta(\gamma(v)), \tag{5}$$

which is flat infinitesimal connection on the principal vertex bundle $\Omega_{c(0)}$. Equality (4) implies that

$$\left.\frac{d\hat{c}_{\widetilde{c(0)}}}{dt}\right|_t = \bar{\gamma}(\hat{c}_{\widetilde{c(0)}}(t), \dot{c}(t)) \in Im(\bar{\gamma}),$$

for every $t \in [0,1]$; hence, $\hat{c}_{\widetilde{c(0)}}$ is the horizontal lifting of the curve c, with respect to the infinitesimal connection $\bar{\gamma}$ and initial condition $\widetilde{c(0)}$. If c is a loop, then $\hat{c}_{\widetilde{c(0)}}(1) = h_{\bar{\gamma}}([c])$, where $h_{\bar{\gamma}}$ is the monodromy homomorphism of the flat principal bundle $(\Omega_{c(0)}, \bar{\gamma})$ (see [2], [6]). Therefore we define the mapping

$$\mu_\gamma : G(\wp(B)) \longrightarrow G\Omega : [c] \mapsto \hat{c}_{\tilde{x}}(1), \qquad c(0) = x.$$

It is directly proved that μ_γ is morphism of differentiable groupoids.

Definition 2.3. The mapping μ_γ is called *the restricted holonomy morphism* of the flat connection γ.

Since in the next section we shall use the restricted holonomy morphisms of trivial Lie groupoids, we prove some preparatory results about them. First observe that a flat connection on the (trivial) groupoid $B \times G \times B$ is a mapping of the form

$$\gamma : TB \longrightarrow TB \oplus (B \times T_eG) : v \mapsto v \oplus (x, a_x(v)), \tag{6}$$

where $v \in T_xB$ and $a \in \Lambda^1(B, T_eG)$. Since the connection γ is flat, the differential form a is Maurer-Cartan ([5, Example III.2.3]) and the differential equation

$$Df = a; \qquad f \in C^\infty(B, G),$$

is integrable ([4, Chapter 1]). The lifting of this differential equation on the α-fibers $\hat{B}_x \equiv \wp(B)_x,\ x \in B$, of the fundamental groupoid $\wp(B)$, which are (mutually diffeomorphic) universal covering spaces of the manifold B, leads to the following differential equations with unitary initial conditions

$$DF = \pi_x^* a; \qquad F([c_x]) = e,$$

where $[c_x]$ is the homotopy class of the constant loop at x and $\pi_x : \hat{B}_x \to B$ are the corresponding covering maps. The latter equations are globally solvable (ibid, Section 1.6) and, by using their solutions $F_{a,x} : \hat{B}_x \to G$, we define the differentiable mapping

$$F_a : \wp(B) \longrightarrow B \times G \times B : [c] \mapsto (\pi_x([c]), F_{a,x}([c]), x), \qquad [c] \in \hat{B}_x. \tag{7}$$

Proposition 2.4. *The restricted holonomy morphism of the flat connection (6) of the trivial groupoid $B \times G \times B$ is given by the equality*

$$\mu_\gamma = F_a \mid G(\wp(B)). \tag{8}$$

Proof. We consider a smooth loop c of the manifold B and set $x := c(0) = c(1)$. The lifting $\hat{c}_{\tilde{x}}$ is a curve of the form $\hat{c}_{\tilde{x}}(t) = (c_1(t), c_2(t), c_3(t)),\ \ t \in [0, 1]$, where c_1 and c_3 (resp. c_2) are smooth curves of the manifold B (resp. of the Lie group G). We shall determine the values $c_i(1),\ \ i = 1, 2, 3$.

In fact, relation (3) implies that $c_3(1) = c(0) = x$ and $c_1(1) = (\beta \circ \hat{c}_{\tilde{x}})(1) = c(1) = x$. On the other hand, equality (4) and the condition $\hat{c}_{\tilde{x}}(0) = \tilde{x}$ lead to the differential equation

$$Dc_2 = c^* a; \qquad c_2(0) = e,$$

from which we get $c_2(1) = F_{\alpha,x}([c])$.

Therefore, the definition of μ_γ and (7) imply that

$$\mu_\gamma([c]) = \hat{c}_{\tilde{x}}(1) = (x, F_{a,x}([c]), x) = (F_a \mid G(\wp(B)))([c]),$$

which completes the proof. □

Since the α-fibers of the groupoid $\wp(B)$ are connected and simply connected, the Lie theory of differentiable groupoids ([5, Proposition III.6.4 and Theorem III.6.5]) ensures the existence of a unique Lie-integral of the composition $\gamma \circ q^{\wp(B)}$, that is, of a morphism of Lie groupoids (often called *the holonomy morphism* of the flat connection γ) $h^\gamma \equiv h : \wp(B) \to \Omega$, such that $Lh = \gamma \circ q^{\wp(B)}$.

Proposition 2.5. *The mapping F_a coincides to the holonomy morphism of the flat connection (6).*

Proof. Since $F_a \mid \hat{B}_x = (\pi_x, F_{a,x}, \sigma_x)$, where σ_x is the constant mapping $\hat{B}_x \to \{x\}$, we see that

$$LF_a \mid T_{\tilde{x}}\hat{B}_x = (T_{\tilde{x}}\pi_x, T_{\tilde{x}}F_{a,x}, \bar{\sigma}_x),$$

where $\tilde{x}$ is the unity of the groupoid $\wp(B)$ at x and $\bar{\sigma}_x$ the (constant) mapping $T_{\tilde{x}}\hat{B}_x \to \{x\}$. The above equality allows to compute the morphism of Lie algebroids LF_α. Indeed, taking into account the equalities

$$T_{\tilde{x}}\pi_x = T_{\tilde{x}}(\beta_{\wp(B)} \mid \hat{B}_x) = q^{\wp(B)} \mid T_{\tilde{x}}\hat{B}_x,$$
$$T_{\tilde{x}}F_{a,x} = TR_e^{-1} \circ T_{\tilde{x}}F_{a,x} = DF_{a,x} = \pi_x^* a,$$

and the isomorphism of vector bundles $TB \oplus (B \times T_eG) \cong TB \oplus (T_eG \times B)$, we obtain the relation

$$LF_a(v) = q^{\wp(B)}(v) \oplus (x, a_x(q^{\wp(B)}(v))); \qquad v \in L(\wp(B))|_x := T_{\tilde{x}}\hat{B}_x,$$

which leads directly to the result. □

Corollary 2.6. *If γ is a connection on the trivial groupoid $B \times G \times B$, then μ_γ is the restriction of h^γ to the inner subgroupoid $G(\wp(B))$.*

3 Triviality criteria for Lie groupoids

We state the following criterion for the triviality of flat principal bundles ([10, Theorem A]) needed in the sequel.

Proposition 3.1. *Let $P \equiv (P, G, B, \pi)$ be a principal bundle, which is equipped with a flat connection (form) ω. The bundle P is trivial if, and only if, the monodromy homomorphism $h_\omega : \pi_1(B, b) \to G$ of (P, ω), where $b \in B$ fixed, extends to a smooth mapping $\hat{h} : \hat{B}_b = \alpha^{-1}_{\wp(B)}(b) \to G$, such that*

$$\hat{h}([\hat{x}] \cdot [c]) = \hat{h}([\hat{x}]) \cdot h_\omega([c]),$$

for every $[\hat{x}] \in \hat{B}_b$ and $[c] \in \pi_1(B, b)$.

We are now in a position to prove the main result of the present paper.

Theorem 3.2. *For every Lie groupoid (Ω, B) the following conditions are equivalent:*
(i) Ω is a trivial groupoid.
(ii) The restricted holonomy morphism μ_γ of any flat connection γ on Ω extends to a differentiable mapping $H : \wp(B) \to G\Omega$ satisfying conditions

$$\alpha_{G\Omega} \circ H = \alpha_{\wp(B)}, \tag{9}$$
$$H(\xi \cdot \eta) = H(\xi) \cdot \mu_\gamma(\eta), \tag{10}$$

for every $(\xi, \eta) \in \wp(B) \times G(\wp(B))$ *with* $\alpha_{\wp(B)}(\xi) = \beta_{\wp(B)}(\eta)$.

(iii) Ω *can be equipped with a Lie-integrable connection.*

(iv) There exists a flat connection $\hat{\gamma}$ *on* Ω*, so that the differential equation* $\Delta x = \hat{\gamma}$ *has a global solution.*

Proof. *(i)* $\Rightarrow$ *(ii).* If Ω is the trivial differentiable groupoid $B \times G \times B$ and γ the connection (6) on it, then the morphism μ_γ extends to the differentiable mapping $F_a : \wp(B) \to B \times G \times B$, as a result of (8). Obviously, the smooth mapping

$$H := (\alpha_{\wp(B)}, p_2 \circ F_a, \alpha_{\wp(B)}),$$

satisfies equality (9). The validity of (10) is implied by the properties of the fundamental solutions $F_{\alpha,x}$ (see [4, Proposition 1.6.4]).

(ii) $\Rightarrow$ *(i).* We fix a point $b \in B$ and consider the principal vertex bundle $(\Omega_b, \Omega_b^b, B, \beta_\Omega|\Omega_b)$ endowed with the flat infinitesimal connection $\bar{\gamma}_b$ (see (5)). By the comments preceding Definition 2.3, the mapping $\mu_\gamma|\pi_1(B, b)$ coincides with the monodromy homomorphism $h_{\bar{\omega}}$ of the principal bundle $(\Omega_b, \bar{\omega})$, where $\bar{\omega}$ is the connection form which corresponds to the splitting $\bar{\gamma}_b$. Since μ_γ extends to the smooth mapping H, we conclude that $h_{\bar{\omega}}$ extends to the smooth mapping $H_b \equiv H|\hat{B}_b : \hat{B}_b \to \Omega_b^b$. Also, equality (10) implies that

$$H_b([c_1] \cdot [c_2]) = H_b([c_1]) \cdot h_{\bar{\omega}}([c_2]), \qquad [c_1] \in \hat{B}_b, \;\; [c_2] \in \pi_1(B, b).$$

Consequently (see Proposition 3.1) the vertex bundle Ω_b is trivial, a fact implying the triviality of the groupoid Ω.

(i) $\Rightarrow$ *(iii).* If $\Omega = B \times G \times B$, we consider the flat connection

$$\gamma^0 : TB \longrightarrow TB \oplus (B \times T_eG) : v \mapsto v \oplus (x, 0); \quad v \in T_xB,$$

and also the morphism of Lie groupoids

$$\varphi^0 : B \times B \longrightarrow B \times G \times B : (y, x) \mapsto (y, e, x).$$

It is directly checked that $L(\varphi^0) = \gamma^0$; hence, the connection γ^0 is Lie-integrable.

(iii) $\Rightarrow$ *(iv).* If $\hat{\gamma} := L\hat{\varphi}$ is a Lie-integrable connection , where $\hat{\varphi} : B \times B \to \Omega$ is a morphism of Lie groupoids, we fix a point $b \in B$ and define the differentiable (partial) mapping

$$l \equiv \hat{\varphi}_b : B \longrightarrow \Omega_b : x \mapsto l(x) := \hat{\varphi}(x, b).$$

The latter produces the morphism of differentiable groupoids

$$(l \times l, l) : (B \times B, B) \longrightarrow (\Omega \times_\alpha \Omega, \Omega),$$

which is global solution of the differential equation $\Delta x = \hat{\gamma}$, hence condition (iv) holds true.

(iv) $\Rightarrow$ *(i).* Finally, we assume that the morphism of C^∞-groupoids

$$(f, f_0) : (B \times B, B) \longrightarrow (\Omega \times_\alpha \Omega, \Omega)$$

is a solution of the differential equation $\Delta x = \hat{\gamma}$. Taking into account (2), we conclude that the composition $\alpha_\Omega \circ f_0$ is a constant mapping and, furthermore, that the composition $\beta_\Omega \circ f_0$ coincides with the base map id_B of the connection $\hat{\gamma}$. Therefore the mapping f_0 is global C^∞-section of the Lie groupoid Ω, a fact implying the triviality of the latter. □

Corollary 3.3. *Let (Ω, B) be a Lie groupoid equipped with a flat connection γ. If the base B is simply connected, then Ω is trivial.*

Proof. In this case, $G(\wp(B))$ coincides with the set of the unit elements of the fundamental groupoid $\wp(B)$, thus the restricted holonomy morphism of every flat connection γ of Ω is the mapping $\mu : G(\wp(B)) \to G\Omega : \tilde{x} \mapsto \tilde{x}$. As a consequence, it suffices to take the mapping

$$H : \wp(B) \longrightarrow G\Omega : [c] \mapsto \widetilde{c(0)},$$

and apply Theorem 3.2. □

Definition 3.4. Let γ be a flat connection on the Lie groupoid Ω . A smooth mapping $H : \wp(B) \to G\Omega$, satisfying equality $H|G(\wp(B)) = \mu_\gamma$ and conditions (9), (10), is called *correcting component* of the flat connection γ.

Corollary 3.5. *A Lie groupoid (Ω, B) is trivial if, and only if, any flat connection of Ω admits correcting component.*

Scholium. If a Lie algebroid is only *locally* trivial (for example, if it is transitive), then it is equipped with a system of local correcting components. Also, condition (iv) of Theorem 3.2 implies the existence of a system of local differential equations $\{\Delta x = \gamma_i | \ \ i \in I\}$, where γ_i are connections on trivial groupoids (compare with [1], [3], [8] and [10], dealing with vector/principal bundles and corresponding equations with total differential). As a conclusion, the local counterparts of the results of the present paper can be used in the structural study of general classes of Lie algebroids.

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