EINSTEIN-SCHWARZSCHILD FORCE (SPEED OF LIGHT AND g OF DEUTERON)

Masataka Mizushima

Abstract

Galileo Galilei asserted that everything on Earth falls downward with the same acceleration $g_\circ$. Einstein proposed that light, a mass zero object, should be included among Galileo's "everything." Thus, a light traveling vertically (down) cannot be accelerated any further, so g for such light must be zero. By using Einstein's general relativistic equation of motion with the Schwarzschild metric, we see that the accerelation of a test particle toward a gravitational source depends on the component of the velocity of the test particle along that direction, as expected.

The speed of light is expected to show an anisotropy due to an existing gravitational source within our experimental accuracy. In a deuteron, a proton and a neutron are orbiting around each other. The effective g would be less than $g_\circ$, according to the above-stated Einstein-Schwarzschild effect on the Newtonian theory of g.

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1. Prologue

Einstein's theory of general relativity [1] is based on a general metric

$$(ds)^2 = g_{00}(cdt)^2 - g_{11}(dx^1)^2 - g_{22}(dx^2)^2 - g_{33}(dx^3)^2. \tag{1-1}$$

If a test particle is moving along the direction of x^1, then this metric gives

$$\left(\frac{dx^1}{dt}\right)^2 = \frac{g_{00}}{g_{11}}c^2 - \left(\frac{ds}{dt}\right)^2 \leq \frac{g_{00}}{g_{11}}c^2, \tag{1-2}$$

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which implies that the speed of a test particle cannot exceed $c\sqrt{g_{00}/g_{11}}$, and this maximum possible speed is reserved for light, which is defined by $ds = 0$.

Einstein [1] derived an equation of motion from this general metric as

$$\frac{d^2x^i}{ds^2} = -\Gamma^i_{k\ell}\frac{dx^k}{ds}\frac{dx^\ell}{ds}, \tag{1-3}$$

where $\Gamma^i_{k\ell}$ is a component of the Christoffel symbol:

$$\Gamma^i_{k\ell} = \frac{1}{2}g^{im}\left(\frac{\partial g_{mk}}{\partial x^\ell} + \frac{\partial g_{m\ell}}{\partial x^k} - \frac{\partial g_{k\ell}}{\partial x^m}\right). \tag{1-4}$$

In the nonrelativistic limit, where the spatial displacements dx^α are small, we see from eq. (1-1) that $ds \simeq cdt$ by eq. (1-1). Therefore, the left-hand side of eq. (1-3) is essentially the Newtonian acceleration, if we take i for a space coordinate. If we interpret the right hand side of eq. (1-3) as the i-th component of a force per inertial mass (of a test particle) times c^2, eq. (1-3) reduces to Newton's equation of motion for a test particle under gravity, assuming Galileo's equivalence principle. An important point is that Einstein assumed [2] that eq. (1-3) was applicable to light (a mass zero particle) as well as to any test particle.

2. Schwarzschild Metric

Schwrzschild showed [3] that Einstein's wave equation for an empty space can be satisfied by

$$(ds)^2 = (1-2a)(cdt)^2 - \frac{1}{1-2a}(dr)^2 - r^2((d\theta)^2 + (\sin\theta d\varphi)^2), \tag{2-1}$$

where

$$a = \frac{GM}{c^2r} \tag{2-2}$$

and M is the mass of a stationary spherical point gravitation source placed at a distance r from the observation point. By using the Schwarzschild metric, eq. (2-1), in Einstein's equation of motion, eq. (1-6), we obtain

$$\frac{dt}{ds} = \frac{1}{c(1-2a)}, \tag{2-3}$$

and

$$c^2(1-2a)\frac{d^2r}{ds^2} = -\left(1 - \frac{1}{(c(1-2a))^2}\left(\frac{dr}{dt}\right)^2\right)\frac{c^2a}{r} + r\left(\left(\frac{d\theta}{dt}\right)^2 + \sin^2\theta\left(\frac{d\phi}{dt}\right)^2\right) \tag{2-4}$$

for the radial component [4]. The right-hand side of eq. (2-4) may be called the radial component of the Einstein-Schwarzschild force per unit mass. The second line of eq.

(2-4) gives the centrifugal correction terms, which equal zero when the test particle is moving in the radial direction.

For a test particle on the surface of Earth, the gravitational effect given by a is almost negligible compared to 1, in which case $ds = cdt$ to a good approximation. Thus, eq. (2-4) reduces to

$$\frac{d^2z}{dt^2} = g, \tag{2-5}$$

where the effective gravitational constant g is given by

$$g = \left(1 - \left(\frac{dz}{cdt}\right)^2\right) g_\circ, \tag{2-6}$$

where

$$g_\circ = \frac{GM_E}{r_E^2} = 9.8 \ \ \text{m/s}^2, \tag{2-7}$$

taking M_E and r_E as the mass and radius of Earth, respectively. We took the vertical distance z from the surface of Earth for r. Eq. (2-5) is the Newtonian gravitational equation of motion with Galileo's equivalence principle, except for the relativistic correction term given in eq. (2-6).

3. Deuterium under Gravity

A deuteron is made of one proton and one neutron bound by the Yukawa force, and has an angular momentum of magnitude $\hbar$ in its ground state. The ground state is mostly a ^3S state, where two nucleonic intrinsic spins are aligned and no orbital angular momentum exists. The nucleonic intrinsic spins do not contribute to the anisotropy term of the gravitational force in the nucleons, because the spin is 1/2. Due to the tensor component of the Yukawa force, however, the deuteron ground state is a mixture of the ^3S and ^3D states. In the ^3D state, the nucleons, with mass M_P each, are orbiting around the center of mass with a total angular momentum of $2\hbar$. The speed V of each nucleon in this orbital motion is $V = \sqrt{6}\hbar/(M_P R)$, where R is the distance between the nucleons. If we take a horizontal axis as the quantization axis, then the $(dz/dt)^2$ in eq. (2-6), when averaged, is

$$\left(\frac{dz}{dt}\right)^2 = m^2 \left(\frac{\hbar f(^3D)}{M_P R}\right)^2, \tag{3-1}$$

where m is the orbital magnetic quantum number of the deuteron along the z-axis, and $f(^3D)$ is the fraction of the ^3D state in the ground state of the deuteron. We see that for the deuteron,

$$g(D) = \left(1 - m^2 \left(\frac{\hbar f(^3D)}{cM_P R}\right)^2\right) g_\circ. \tag{3-2}$$

We assume that the structure of the deuteron is determined by the Yukawa force, or is rigid against the gravitational deformation.

If we take $f(^3D) = 0.07$ and $R = 10^{-15}$ m, then the correction factor in eq. (3-2) is

$$g(D) = (1 - m^2 10^{-4}) g_\circ, \tag{3-3}$$

or the effective g for the $m = \pm 1$ states is less than that for the $m = 0$ state by the factor $10^{-4} g_\circ$.

The deuteron usually exists as a heavy water, HDO or D_2O. The oxygen nucleus is bound with the proton or deuterons by a chemical bond, and they fall down together. The gravitational constant for the compound D_2O is

$$g(D_2O) = (2m(D)g(D) + m(O)g_\circ)(2m(D) + m(O)) = (1 - 10^{-5})g_\circ, \tag{3-4}$$

where $m(D)$ and $m(O)$ are the masses of the deuteron and the oxygen nuclei, respectively. We assumed that the average of m^2 is $2/3$, and the gravitational constant of the oxygen nucleus is $g_\circ$.

We propose to measure the gravitational acceleration, g, of D_2O [5].

4. Anisotropy in the Speed of Light

The Einstein-Schwarzschild equation of motion (2-4) is applicable to light as well as to any test particle. It shows that light, which is moving with an ultimate speed given by eq. (1-2), cannot be accelerated any further. Thus the speed of light propagating in the radial direction is

$$\frac{dr}{dt} |_{ds=0} = c(1 - 2a). \tag{4-1}$$

On the other hand, if the light is propagating in the tangential direction, eq. (1-2) gives

$$\frac{rd\theta}{dt} |_{ds=0} = c\sqrt{1 - 2a} \simeq c(1 - a), \tag{4-2}$$

which is different from eq. (4-1).

When we take the mass of Earth for M and the radius of Earth for r in eq. (2-2) we obtain

$$a_E = g_\circ r_E / c^2 = 0.7 \times 10^{-10}. \tag{4-3a}$$

When we take the mass of the sun for M and the distance from Earth to the sun for r in eq. (2-2) we obtain

$$a_s = 0.99 \times 10^{-8}. \tag{4-3b}$$

We are in a circular orbit around the center of the Milky Way galaxy moving at $V_M \simeq$ 250 km/s. Thus, the value of a due to the center of the Milky Way galaxy is

$$a_M = (V_M/c)^2 \simeq 0.5 \times 10^{-6}, \tag{4-3c}$$

if the orbital motion is circular under the Newtonian gravitational force, neglecting the relativistic effect in eq. (2-4).

Schamir and Fox [6] set up a Michelson interferometer to measure the anisotropy due to the sun with an accuracy of 3×10^{-11}, but could not detect any anisotropy. They used fused quartz tubes as spacers. Brillet and Hall [7] placed a rigid cavity on a rotating plate, but could not detect any anisotropy. Although light can penetrate through these transparant materials, their index of refraction, n, is about 1.5, or the wavelength of light is dictated by the atomic structure, according to the Maxwell theory. The atoms are bound together by chemical bonds, and the deformation of these solid materials under gravity is negligible. The anisotropy due to the gravitational factor a, which is of the order of 10^{-8}, is concealed by the chemical bond effect shown by $n-1$. For the air at a pressure of one atmosphere, we know that $n-1 = 2 \times 10^{-4}$. Therefore, to detect the general relativistic anisotropy with a cavity we have to reduce the pressure inside the cavity to at least 10^{-3} torr.

We suggested [4] that the Hills-Hall experiment [8] should be tried again by placing the Fabry-Perot cavity in the East-West direction under a pressure less than 10^{-3} torr. Such an experiment is now under way at the University of Colorado.

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Author's address:

Masataka Mizushima
Department of Physics,
University of Colorado,
Boulder, Colorado 80309 U.S.A.