A note on chaos synchronization between two differential three-dimensional systems

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Abstract

Chaos synchronization with feedback control between two different systems is presented by using techniques from active control theory. The results are tested by numerical simulations.

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Key words: Lorenz system, chaos synchronization, active control theory.

1 Introduction

Chaos synchronization, i.e., making two or more systems oscillate in the same manner, is a relative challenging new problem and of current interest. In [1] is presented a method to synchronize two identical chaotic systems with different initial conditions. There was obtained chaos synchronization between a lot of systems, not only between systems belonging to the same family [11, 13, 14, 15], as, for example, between two Lorenz systems, but also between two different systems [12, 16] - as between the Chen system the Lorenz systems. Very recently in [16], the author has presented chaos synchronization between the pairs provided by the Lorentz, and *Lü* and Chen systems. A Rössler system and a Lorenz system are synchronized in [12]. Chaos and in particular chaos synchronization is widely explored in a variety of fields ranging from physics and chemistry to ecology and biology [2, 3]. Two laser systems have been studied for controlling chaos in [4]. Each laser system has been modelled by means of the Lorenz system. Using the Melnikov method, in [6] it is studied the Duffing modified oscillator, and are determined the conditions of transition to chaos. Different methods have been successfully applied to chaos synchronization. We mention here: active control [5], nonlinear control [7] and adaptive control [8]. In this paper we synchronize two different systems applying active control theory following a method presented in [16]. The two systems are the system (2.2) and the Lorenz system. The system (2.2) is introduced in [9]. The Lorenz system is considered as a paradigm in the field of chaotic systems, since it captures many of the features of chaotic dynamics. Originally it was used as a model of convection.

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2 Description of the systems

At its origin, the Lorenz system was a model of Earth‘s atmospheric convection flow heated from bellow and cooled from above [18]. The system has a bounded , zero volume, globally attracting set. This means that the trajectories are always bounded and continuously differentiable. The Lorenz system is described by the equations

$$\text{(2.1)} \qquad \begin{cases} x' = \alpha\,(y - x) \\ y' = \gamma x - xz - y \\ z' = xy - \beta z \end{cases}$$

The Lorenz chaotic attractor is displayed bellow in Fig.1.

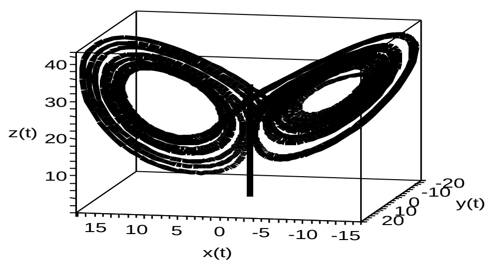

Figure 1: The Lorenz attractor. The phase portrait is obtained for the parametric vector $(\alpha, \beta, \gamma) = (10, 8/3, 28)$.

The system (2.2) is described by the following three nonlinear differential equations [9]:

$$\text{(2.2)} \qquad \begin{cases} x' = a\,(y - x) \\ y' = (c - a)x - axz \\ z' = xy - bz \end{cases}$$

where a, b, c with $a \neq 0$ are the real parameters of the system. The flow of the system in three particular cases is drawn in Fig. 2 a)-c). In some conditions, $a(c - a) < 0$, this system is a dual system to the Lorenz system, Fig 2 b). The duality is defined in the sense of Vanecek and Celikovsky [16]. The system (2.2) is conservative if and only if $a + b = 0$, Fig. 2 a).

3 Chaos synchronization

In order to synchronize the two systems we consider that system (2.2) drives the Lorenz system (2.1). Hence, the drive system is

$$\text{(3.3)} \qquad \begin{cases} x' = a\,(y - x) \\ y' = (c - a)x - axz \\ z' = xy - bz \end{cases}$$

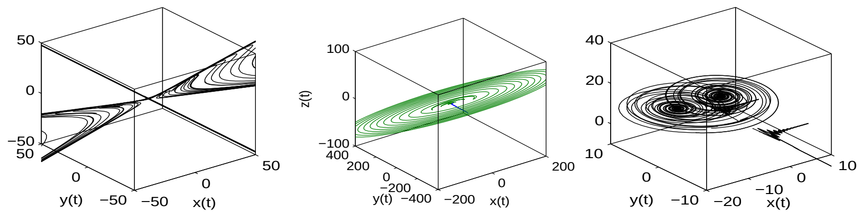

Figure 2: The flow of the system (2). The phase portraits are obtained for the parametric vector: a) (a,b,c)=(-8,8,10), b) (a,b,c)=(4,5,3), c) (a,b,c)=(2,3,20)

and the response system is:

$$\left\{ \begin{array}{l} X' = \alpha\,(Y - X) + C_1(t) \\ Y' = \gamma X - XZ - Y + C_2(t) \\ Z' = XY - \beta Z + C_3(t) \end{array} \right. \tag{3.4}$$

where $C_1(t), C_2(t)$ and $C_3(t)$ are three control functions.

Our aim is to ascertain the three control functions $C_1(t), C_2(t)$ and $C_3(t)$. Define the error system as the difference between the response system and the drive system. Substracting the system (3.3) from (3.4) and using the notation for the state errors between the response system and the drive system:

$e_1 = X - x, e_2 = Y - y$ and $e_3 = Z - z$

we have that:

$$\left\{ \begin{array}{l} e_1' = \alpha(e_2 - e_1) + (\alpha - a)(y - x) + C_1(t) \\ e_2' = \gamma e_1 - e_2 + (\gamma - c + a)x - (1 - a)xz - e_1e_3 - e_1z - e_3x - y + C_2(t) \\ e_3' = -\beta e_3 + e_1y + e_2x + e_1e_2 - z(\beta - b) + C_3(t) \end{array} \right. \tag{3.5}$$

Defining the active control functions $C_1(t), C_2(t)$ and $C_3(t)$ by:

$$\left\{ \begin{array}{l} C_1(t) = I_1(t) - (\alpha - a)(y - x) \\ C_2(t) = I_2(t) - (\gamma - c + a)x + (1 - a)xz + e_1e_3 + e_1z + e_3x + y \\ C_3(t) = I_3(t) - e_1y - e_2x - e_1e_2 + z(\beta - b) \end{array} \right. \tag{3.6}$$

we obtain that the error system (3.5) becomes:

$$\left\{ \begin{array}{l} e_1' = \alpha(e_2 - e_1) + I_1(t) \\ e_2' = \gamma e_1 - e_2 + I_2(t) \\ e_3' = -\beta e_3 + I_3(t) \end{array} \right. \tag{3.7}$$

with the input control functions $I_1(t), I_2(t)$ and $I_3(t)$. Consequently, the problem of synchronization of the two systems is to define a controller vector $(I_1(t), I_2(t), I_3(t))$ such that $\lim_{t\to\infty}(\| e(t) \|) = 0$. This means that system (2.2) and Lorenz system are synchronized with feedback control. As we can see the choice of the feedback is not unique. We choose here:

$$\begin{pmatrix} I_1(t) \\ I_2(t) \\ I_3(t) \end{pmatrix} = A \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}, \tag{3.8}$$

where A is a 3×3 constant matrix, such that the error system (3.6) has all eigenvalues with negative real parts. We choose the matrix A as follows:

$$A = \begin{pmatrix} \alpha-1 & -\alpha & 0 \\ -\gamma & -1 & 0 \\ 0 & 0 & \beta-3 \end{pmatrix}. \tag{3.9}$$

With this choice, the feedback error system (3.6) has the eigenvalues $-1, -2$ and -3. Therefore, the error vector (e_1, e_2, e_3) converges to zero as the time t goes to infinity. This implies that the system (2.2) and the Lorenz system are synchronized with feedback control.

4 Numerical simulations

In order to solve the two differential systems, the numerical simulations are carried out by using MATLAB 7.0.1. For the Lorenz system, the selected parameters are $(\alpha, \beta, \gamma) = (10, 8/3, 28)$. We use this choice in order to achieve a chaotic behavior of the system. For the second system (2.2), we choose $(a, b, c) = (2, 3, 20)$. The initial states of the drive system and of the response system are $x(0) = y(0) = 1, z(0) = 0$ and, respectively, $x(0) = 0.2, y(0) = 0.3, z(0) = 0.1$. The initial values of the error system are $e_1 = -0.8, e_2 = -0.7$ and $e_3 = 0.1$. From the Figures 3) and 4) we can observe how different oscillate the two systems before activating the control, while from the Figures 3) and 5) we see that the systems oscillate in the same manner after using the control functions. The last figure displays the behavior of the error system. As we can see the error functions tend to zero some seconds after start.

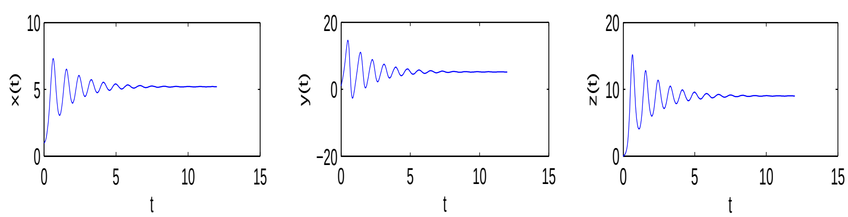

Figure 3: The solution of the system (2.2); the signals x, y and z.

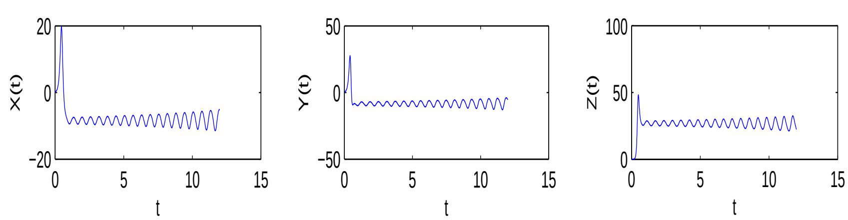

Figure 4: The solution of the Lorenz system with the active control deactivated; the signals X, Y and Z.

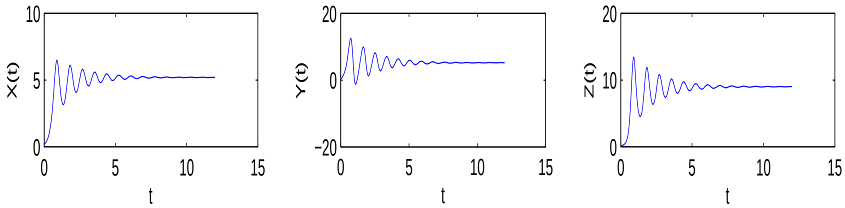

Figure 5: The solution of the Lorenz system with the active control activated; the signals X, Y and Z.

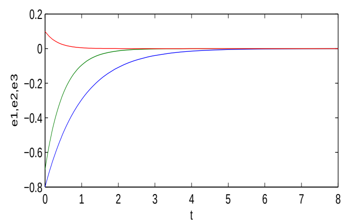

Figure 6: The dynamics of the error system e_1, e_2, e_3.

Conclusions. In this paper we studied a matter of chaos synchronization between two different systems. By using active control theory we have obtained that the two systems are synchronized with feedback control. The figures displayed show the manner of oscillating before and after using the control functions. These simulations are carried out by numerical computations by using MATLAB. In forthcoming papers we intend to study the chaos synchronization between the system (2.2), on one hand, and the Rossler system and Duffing system, on the other hand.

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References

[1] Pecora L.M., Carroll T.L., *Synchronization in chaotic systems*, Phys Rev Lett 64 (1990), 8, 821.

[2] Lakshmanan M, Murali K., *Chaos in Nonlinear Oscillators: Controlling and Sinchronization*, World Scientific, Singapore, 1996.

[3] Han S.K., Kerrer C., Kuramoto Y., *Dephasing and bursting in coupled neural oscillators*, Phys. Rev. Lett. 75 (1995), 3190-3193.

[4] Liu Y., Barbosa L.C., *Periodic locking and coupled Lorenz system*, Phys. Lett. A 197 (1995), 13-18.

[5] Ho M.C., Hung Y.C. *Synchronization of two different systems by using generalized active control*, Phys. Lett. A 301 (2002), 424-428.

[6] Tigan Gh., *Transition to chaos of the Duffing oscillator modified*, Mathematiki Bilten, Skopje, 2003.

[7] Huang L., Feng R., Wang M., *Synchronization of chaotic systems via nonlinear control*, Phys. Lett. A 320 (2004), 271-275.

[8] Chen S.H., Lü J., *Synchronization of an uncertain unified system via adaptive control*, Chaos, Solitons and Fractals 14 (2002), 643-647.

[9] Tigan Gh., *Hopf bifurcation on a dual system to Lorenz system*, International Journal of Nonlinear Sciences and Numerical Simulations, China, to appear.

[10] Hu G., Xiao J., Zheng Z. *Chaos Control*, Shanghai Scientific and Technological Education Publishing House, China, 2000.

[11] Hendrik R., *Controlling the Lorenz system: combining global and local schemes*, Chaos, Solitons and Fractals, 12 (2001), 2375-2380.

[12] John R.T., Gregory D.V., *Chaotic communication using generalized synchronization*, Chaos, Solitons and Fractals, 12 (2001), 145-152.

[13] Shyi-Kae Y., Chen C., Yau H., *Control of chaos in Lorenz systems*, Chaos, Solitons and Fractals 13 (2002), 767-780.

[14] Shihua L., Yu-Ping T., *Finite time synchronization of chaotic systems*, Chaos, Solitons and Fractals, 15 (2003), 303-310.

[15] Er-Wei B., Karl E.L., *Synchronization of two Lorenz systems using active control*, Chaos, Solitons and Fractals, 8 (1997), 51-58.

[16] Yassen M.T., *Chaos synchronization between two chaotic systems using active control*, Chaos, Solitons and Fractals, 23 (2005), 130-140.

[17] Vanecek A., Celikovsky S., *Control Systems: from Linear Analysis to Synthesis of Chaos*, London, Prentice-Hall, 1996.

[18] Sparrow C., *The Lorenz Equations: Bifurcations, Chaos and Strange Attractors*, Springer-Verlag, New York, 1982.

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