Some applications on weakly pseudo-symmetric Riemannian manifolds

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Abstract

This paper deals with weakly pseudo-symmetric Riemannian manifold. In this paper, weakly pseudo-symmetric manifold $(WS)_n$ is introduced and then decomposable weakly pseudo-symmetric manifolds are examined and some theorems about them are proved. In the other part of it, the cyclic Ricci tensor of this manifold is studied.

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1 Introduction

The notions of weakly symmetric and weakly projective symmetric manifolds were introduced by Tamassy and Binh [1]. A non-flat Riemannian manifold V_n ($n > 2$) is called a weakly symmetric manifold if the curvature tensor R_{hijk} satisfies the condition:

$$R_{hijk,l} = a_l R_{hijk} + b_h R_{lijk} + c_i R_{hljk} + d_j R_{hilk} + e_k R_{hijl} \tag{1.1}$$

where a,b,c,d,e are 1-forms (non-zero simultaneously) and the comma ',' denotes covariant differentiation with respect to the metric tensor of the manifold.

It may be mentioned in this connection that although the definition of a $(WS)_n$ is similar to that of a generalized pseudo-symmetric manifold studied by Chaki and Mondal [2], the defining condition of a $(WS)_n$ is weaker than that of a generalized pseudo-symmetric manifold. A reduction in generalized pseudo-symmetric manifolds has been obtained by Chaki and Mondal. But, in [3], the authors investigated and a reduction in $(WS)_n$ is obtained in a simpler form

$$R_{hijk,l} = a_l R_{hijk} + b_h R_{lijk} + b_i R_{hljk} + d_j R_{hilk} + d_k R_{hijl} \tag{1.2}$$

The name weakly-symmetric was chosen, because if in (1.2), a_l, b_l, d_l are taken as zero, then the equation (1.2) takes the form $R_{hijk,l} = 0$ and the manifold reduces to a symmetric manifold in the sense of Cartan.

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In the present paper, some results on a $(PS)_n$ are established. In section III, it is shown that if a $(WS)_n$ is a decomposable manifold $V_r \mathrm{x} V_{n-r}$ $(r > 1, n-r > 1)$, then one of the decomposition manifolds is flat and the other is a weakly symmetric manifold. In section IV, the Ricci-associate of a vector field is defined and some theorems are proved. In addition, for $B \neq 0$ or $D \neq 0$, the decomposable manifold $(WS)_n$ has a zero scalar curvature.

The last Section is concerned with $(WS)_n$ admitting a concurrent or a recurrent vector field, [4].

2 Weakly pseudo-symmetric manifolds $(WS)_n$.

In this section, we shall obtain some formulas which will be required in this study of $(WS)_n$.

Let R_{ij} and R denote the Ricci tensor and the scalar curvature, respectively. Then, from (1.2), we get

$$R_{ij,l} = a_l R_{ij} + b_i R_{lj} + d_j R_{il} + b^k R_{lijk} + d^k R_{kijl} \tag{2.1}$$

Transvecting (2.1) with g^{ij}, we have

$$R_{,l} = a_l R + 2(b^h + d^h) R_{hl} \tag{2.2}$$

Moreover, contraction of (1.2) with $g^{ij} g^{hk}$, we get

$$\frac{R_{,l}}{2} = a^h R_{hl} + d_l R + (b^h - d^h) R_{hl} \tag{2.3}$$

and multiplying (1.2) by $g^{hk} g^{jl}$, we find

$$\frac{R_{,l}}{2} = a^h R_{hl} + b_l R + (d^h - b^h) R_{hl} \tag{2.4}$$

On the other hand, transvecting (1.2) by g^{jl}, we get

$$\frac{R_{,l}}{2} = a^h R_{hl} + b_l R + (b^h - d^h) R_{hl} \tag{2.5}$$

From (2.3) and (2.4), we can easily see that the following expression holds

$$(b_l - d_l) R = 0 \tag{2.6}$$

Similarly, using (2.4) and (2.5), we have

$$b^l R_{hl} = d^l R_{hl} \tag{2.7}$$

From (2.2) and (2.7), we obtain

$$a_l R_{,m} - a_m R_{,l} = 4(a_l b^h R_{hm} - a_m b^h R_{hl}) \tag{2.8}$$

Further, we find

$$R(a_{l,m} - a_{m,l}) + (a_l R_{,m} - a_m R_{,l}) + 4((b^h R_{hl})_{,m} - (b^h R_{hm})_{,l}) = 0 \tag{2.9}$$

3 Decomposable $(WS)_n$.

A Riemannian manifold V_n is said to be decomposable if it can be expressed as a product V_rxV_{n-r} for some r, i.e., if coordinates can be found so that its metric takes the form

$$ds^2 = \sum_{a,b=1}^{r} g_{ab}dx^a dx^b + \sum_{\alpha,\beta=r+1}^{n} g_{\alpha\beta}dx^\alpha dx^\beta \tag{3.1}$$

where g_{ab} are functions of $x^1, x^2, ..., x^r (r < n)$ and $g_{\alpha\beta}$ are functions of $x^{r+1}, x^{r+2}, ..., x^n$ only; a,b,c,... are taken to have range 1 to r and α, β, γ,... are taken to have the range $r+1$ to n. The two parts of (3.1) are metrics of V_r and V_{n-r} which are called the decomposition manifolds.

We now suppose that $(WS)_n$ $(n > 2)$ is decomposable with V_r and V_{n-r} as decomposition manifolds. From [3], we have

$$R_{abcd,\alpha} = A_\alpha R_{abcd} + B_a R_{\alpha bcd} + B_b R_{a\alpha cd} + D_c R_{ab\alpha d} + D_d R_{abc\alpha} \tag{3.2}$$

In view of the fact that the curvature tensor and its covariant derivative are product tensors, the above equation takes the form

$$A_\alpha R_{abcd} = 0 \tag{3.3}$$

Also, from [3], we have

$$R_{\alpha\beta\gamma\delta,a} = A_a R_{\alpha\beta\gamma\delta} + B_\alpha R_{a\beta\gamma\delta} + B_\beta R_{\alpha a\gamma\delta} + D_\gamma R_{\alpha\beta a\delta} + D_\gamma R_{\alpha\beta\gamma a} \tag{3.4}$$

Since the curvature tensor of the space and its covariant derivative are product tensors, the equation (3.4) takes the form

$$A_a R_{\alpha\beta\gamma\delta} = 0 \tag{3.5}$$

Similarly, we get

$$B_\alpha R_{abcd} = 0, \quad B_a R_{\alpha\beta\gamma\delta} = 0 \tag{3.6}$$

$$D_\alpha R_{abcd} = 0, \quad D_a R_{\alpha\beta\gamma\delta} = 0 \tag{3.7}$$

Since A_i, B_i, D_i are non-zero vectors, all its components cannot vanish. Suppose $A_\alpha \neq 0$ for some α. Then, from (3.3), we get $R_{abcd} = 0$ which means that the decomposition manifold is flat. If $A_a \neq 0$ for some a, then by similar argument, we have $R_{\alpha\beta\gamma\delta} = 0$ which means that the decomposition manifold V_{n-r} is flat. Similarly, the conditions $B_\alpha \neq 0, D_\alpha \neq 0$ for some α and $B_a \neq 0, D_a \neq 0$ for some a hold, it is easy to see that the decomposition manifolds V_r and V_{n-r} are flat.

We suppose that $R_{\alpha\beta\gamma\delta=0}$. Then, $R_{abcd} \neq 0$, because, by hypothesis, $(WS)_n$ is not flat. Hence, from (3.3), (3.6) and (3.7), we get $A_\alpha = 0, B_\alpha = 0$ and $D_\alpha = 0$. Since A_i, B_i and D_i are non-zero vectors, all its components can not be vanish. Hence, $A_a \neq 0, B_a \neq 0, D_a \neq 0$ for some a. Therefore, from (1.2), we obtain

$$R_{abcd,e} = A_e R_{abcd} + B_a R_{ebcd} + B_b R_{aecd} + D_c R_{abed} + D_d R_{abce} \tag{3.8}$$

By virtue of (3.8), it follows that the part V_r is a $(WS)_r$.

We can therefore state the following theorem:

Theorem 3.1 *If a $(WS)_n$ is a decomposable manifold $V_r x V_{n-r}$ $(n > 1,\ n - r > 1)$, then one of the decomposition manifolds is flat and the other is a weakly symmetric.*

An n-dimensional Riemannian manifold V^n is said to be decomposable if in some coordinates its metric is given by

$$ds^2 = g_{ij}dx^i dx^j = \sum_{a,b=1}^{r} \bar{g}_{ab}dx^a dx^b + \sum_{a',b'=r+1}^{n} g^*_{a'b'}dx^{a'}dx^{b'} \tag{3.9}$$

where $\bar{g}_{ab}$ are functions of $x^1, x^2, ..., x^r$ $(r < n)$ denoted by $\bar{x}$ and $g^*_{a'b'}$ are functions of $x^{r+1}, x^{r+2}, ..., x^n$ denoted by x^*; $a, b, c, ...$ run from 1 to r and $a', b', c', ...,$ run from $r+1$ to n. The two parts of (3.9) are the metrics of a V^r $(r > 1)$ and a V^{n-r} $(n - r > 1)$ which are called the decomposition manifolds of $V^n = V^r x V^{n-r}$. Throughout this paper each object denoted by a bar is assumed to be from $\bar{g}_{ab}$ and of V^r, and each object denoted by a star is formed from $g^*_{a'b'}$ and of V^{n-r}. From (3.9), we have

$$g_{ab} = \bar{g}_{ab},\ \ g_{a'b'} = g^*_{a'b'},\ \ g^{ab} = \bar{g}^{ab},\ \ g^{a'b'} = g^{*a'b'},\ \ g_{aa'} = g^{aa'} = 0 \tag{3.10}$$

The only non-zero Christoffel symbols of the second kind are as follows: A comma and a dot shall denote covariant differentiation in V^n and V^r, respectively. Hence, we obtain the following relations:

$$R_{abcd} = \bar{R}_{abcd}, \qquad R_{a'b'c'd'} = R^*_{a'b'c'd'} \tag{3.11}$$

We suppose that $V^n = V^r x V^{n-r}$ and $B \neq 0$. By using (1.2), we get

$$R_{a'bcd.a} = A_a R_{a'bcd} + B_{a'} R_{abcd} + B_b R_{a'acd} + D_c R_{a'bad} + D_d R_{a'bca}$$

in this case, we obtain

$$B_{a'} R_{abcd} = 0 \tag{3.12}$$

Similarly, after some calculations, we get

$$B_a R_{a'b'c'd'} = 0 \tag{3.13}$$

Since $B \neq 0$, all its components cannot vanish. Hence, we consider the following two cases:

Case (i): Suppose that $B_{a'} \neq 0$ for a fixed a'. Then from (3.11) and (3.12), we have

$$\bar{R}_{abcd} = 0$$

Transvecting the above equation by $\bar{g}^{ad}$ and $\bar{g}^{bc}$, we get $\bar{R} = 0$. From (3.11), we obtain

$$R = R^* \tag{3.14}$$

(ii): Suppose $B_a \neq 0$ for a fixed a. Then, from (3.13) it follows that $R_{a'b'c'd'} = 0$ for all $a'b'c'd'$. From (3.11), we have

$$R^*_{a'b'c'd'} = 0 \tag{3.15}$$

Multiplying (3.15) with $g^{*a'b'}, g^{*b'c'}$, we get $R^* = 0$. From (3.14), we get

$$R = 0 \tag{3.16}$$

for $D \neq 0$, we obtain the same results.

Theorem 3.2 *For $B \neq 0$ or $D \neq 0$, a decomposable manifold $(WS)_n$ has zero scalar curvature.*

The remaining sections deal with non-decomposable manifolds $(WS)_n$.

4 Ricci-associate of the vector field λ_i.

Let the expression

(4.1) $$v_i = R_{hi}\lambda^h$$

holds. Then the vector field v_i shall be called the Ricci-associate of the vector. Let $\vec{\lambda} = \vec{a}$, then we get

(4.2) $$v_i = R_{hi}a^h \quad (\underset{r}{a}^i \overset{p}{a}_i = \delta^p_r)$$

If R is constant, then from (2.5), (2.7) and (4.2), we get

(4.3) $$v_l = -b_l R$$

If R is zero, then using the same equations, we find $\vec{v} = 0$. Therefore, R is not zero. From (2.6), we say that the vector $\vec{b}$ is not orthogonal to $\vec{d}$. Using (4.3), we have that the vector $\vec{v}$ is not orthogonal to both $\vec{b}$ and $\vec{d}$. If R is a constant, then the vector field v_i is collinear with the vector field b_i.Then we obtain

$$a^i v_i = a^i R_{hi} a^h$$

Thus, if the Ricci form $R_{hi}a^h a^i$ is indefinite, then from the relation $\sum_r \underset{r}{a}^h \underset{r}{a}^i = g^{hi}$, we get $R = 0$. Since $R \neq 0$, the Ricci form $R_{hi}u^h u^i$ is definite, then v_i, a_i are not orthogonal vectors. By the aid of (2.2) and (2.7), we get a_i and b_i are not orthogonal vectors. Therefore, any vector of $(WS)_n$ is not orthogonal to each other.

If R is constant, then the vector field v_i is collinear with the vector field b_i. We consider the manifold $(WS)_n$, $(n > 2)$ of non-constant scalar curvature in which the Ricci-associate of a_i is not necessarily collinear with b_i. Now, for $v_i = R_{hi}a^h$, we get

$$a^i v_i = a^i R_{hi} a^h$$

Thus, if the Ricci form $R_{hi}u^h u^i$ is definite, then $a^i v_i \neq 0$, i.e., a_i and v_i are not orthogonal vectors. Similarly, any vector of $(WS)_n$ is not orthogonal to v_i and the vectors a_i and d_i are not orthogonal.

From (2.6), we get $\vec{b}$ is not orthogonal to $\vec{d}$. If we put $\vec{\lambda} = \vec{b}$ or $\vec{\lambda} = \vec{d}$ in (4.1), we obtain the same results.

This leads to the following theorem:

Theorem 4.1 *In a $(WS)_n$, $(n > 2)$ with constant scalar curvature, if the vector fields a_i form an orthogonal ennuple, then the Ricci-associate of a_i are not orthogonal to the any vector of $(WS)_n$. If in $(WS)_n$, $(n > 2)$ with non-constant scalar curvature, the Ricci form $R_{hi}u^h u^i$ is definite, then the any vector of $(WS)_n$ is not orthogonal to each other.*

Now, we consider the manifold $(WS)_n$ with non-constant scalar curvature. Then R will be another non-zero vector field. A vector field u_i is called closed if $u_{l,m} - u_{m,l} = 0$.

We enquire if the Ricci-associate of a_i and a_i can be both closed. In virtue of $v_i = R_{hi}a^h$, the equations (2.8) and (2.9) can be expressed as follows:

$$a_l R_{,m} - a_m R_{,l} = 4(a_l v_m - a_m v_l) \tag{4.4}$$

and

$$R(a_{l,m} - a_{m,l}) + (a_l R_{,m} - a_m R_{,l}) + 4(v_{l,m} - v_{m,l}) = 0 \tag{4.5}$$

From (4.5), it follows that

$$a_l R_{,m} - a_m R_{,l} = 0 \tag{4.6}$$

and therefore, using (4.4), we get

$$a_l v_m - a_m v_l = 0 \tag{4.7}$$

Now, from (4.6) and (4.7), it follows that the vector field $R_{,i}$ is collinear with both the vector fields a_i and v_i. We can therefore state as follows:

Theorem 4.2 *In the manifold $(WS)_n$ with non-constant scalar curvature, the Ricci-associate of a_i and a_i cannot be both closed, unless the vector field $R_{,i}$ is collinear with both a_i and its Ricci-associate.*

5 The manifold $(WS)_n$ $(n > 3)$ with cyclic Ricci tensor.

A Riemannian manifold is said to be cyclic Ricci tensor if the condition

$$R_{ij,k} + R_{jk,i} + R_{ki,j} = 0 \tag{5.1}$$

holds. According to [5], from (5.1), we have $R = const$. Hence, using (2.5) and (2.7), we get

$$a^h R_{hl} + b_l R = 0, \qquad R = const \neq 0 \tag{5.2}$$

In this section, we suppose that a $(WS)_n(n > 2)$ has cyclic Ricci tensor. Using (2.1) and I. Bianchi Identity, we get

$$R_{ij,k} + R_{jk,i} + R_{ki,j} = (a_i + b_i + d_i)R_{jk} + (a_j + b_j + d_j)R_{ki} + (a_k + b_k + d_k)R_{ij} \tag{5.3}$$

With the help of (2.6), the equation (5.3) can be written as

$$A_i^* R_{jk} + A_j^* R_{ki} + A_k^* R_{ij} = 0 \tag{5.4}$$

where $A_i^* = a_i + 2b_i$. Transvecting (5.4) with A_i^*, we obtain

$$A^{i*} A_i^* R_{jk} + A^{i*} A_j^* R_{ki} + A^{i*} A_k^* R_{ij} = 0 \tag{5.5}$$

where $A_i^* = a^i + 2b^i$. By the aid of the equations (2.2), (2.5), (2.7) and (5.2), we can easily obtain that

$$A^{h*}R_{hl} = -\frac{A_l^* R}{2} \tag{5.6}$$

Then, using (5.5) and (5.6), we have

$$A^{i*}A_i^* R_{jk} = A_j^* A_k^* R \tag{5.7}$$

In this case, transvecting (5.7) with A^{k*} and using the equation (5.6), we get

$$\frac{3}{2}A^{i*}A_i^* A_j^* R = 0 \tag{5.8}$$

i.e.,

$$A_i^* A^{i*} = 0 \quad or \quad A_j^* = 0 \tag{5.9}$$

Suppose that $A_i^* A^{i*} = 0$ and the metric of this manifold is positive definite. In this case, we obtain

$$A_i^* A^{i*} = (\vec{a}_i + 2\vec{b}_i)(\vec{a}^i + 2\vec{b}^i) = (\vec{a} + 2\vec{b})^2 \tag{5.10}$$

Therefore, $A_i^* A^{i*} \neq 0$. From (5.9), we have

$$A_j^* = 0 \tag{5.11}$$

If $A_j^* = 0$, then from (5.7), we have $R_{jk} = 0$. By the aid of (5.2), we get $b_l = 0$. (since $R \neq 0$). Using (2.2) and (2.6), we obtain $a_l = d_l = 0$, i.e., in the manifold $(WS)_n$ with a positive metric, if $A_j^* = 0$, then this manifold is a symmetric manifold. For this reason, the condition $A_j^* \neq 0$ must be hold. Therefore, by the aid of (5.8), we say that $R = 0$. From (5.3), we obtain

$$A_i^{**}A^{i**}R_{jk} + A_j^{**}A^{i**}R_{ki} + A_k^{**}A^{i**}R_{ij} = 0 \tag{5.12}$$

where $A_i^{**} = a^i + b^i + d^i$. In this case, with the help of (2.2), (2.4) and (2.7), we get $A^{h**}R_{hl} = 0$. It is easy to see that the equation (5.12) can be changed into the form $A_i^{**}A^{i**}R_{jk} = 0$. Then, we get

$$A_i^{**}A^{i**} = (a^h + b^h + d^h)(a_h + b_h + d_h) = (\vec{a} + \vec{b} + \vec{d})^2 \tag{5.13}$$

If the metric of the manifold is positive definite, then $A_i^{**}A^{i**} \neq 0$. Therefore, we have R_{jk} which means that the manifold is an Einstein manifold with zero scalar curvature.

It is known [6] that an n-dimensional ($n > 2$) Einstein manifold has constant curvature. Then, an Einstein manifold $(WS)_3$ is locally symmetric. Nevertheless, the condition $R_{hijk,l} \neq 0$ is satisfied.

In view of these results, it follows that an Einstein $(WS)_n (n > 2)$ does not exist, i.e., $R \neq 0$, the manifold $(WS)_n$ can not be an Einstein manifold with a cyclic positive definite Ricci tensor.

Thus, we can state the following theorem:

Theorem 5.1 *If a manifold $(WS)_n$ $(n > 2)$ with positive definite metric has cyclic Ricci tensor, then this manifold is an Einstein manifold with zero scalar curvature.*

6 $(WS)_n$ admitting a concurrent or a recurrent vector field.

This section consists of two parts, the first deals with a $(WS)_n$ admitting a concurrent vector field u^i, [4], given by

(6.1) $$u^i_{,j} = p\delta^i_j$$

where p is a non-zero constant and the second deals with a $(WS)_n$ admitting a recurrent vector field u^i, [4], given by

(6.2) $$u^i_{,j} = \beta_j u^i$$

where β_j is a non-zero covariant vector.

Part I.

According to [5], we have

(6.3) $$u^h R_{hijk} = 0, \quad u^h R_{hk} = 0, \quad pR_{lijk} + u^h R_{hijk,l} = 0$$

Transvecting (1.2) with u^h, we get

$$u^h R_{hijk,l} = a_l u^h R_{hijk} + b_h u^h R_{lijk} + b_i u^h R_{hljk} + d_j u^h R_{hilk} + d_k u^h R_{hijl}$$

In virtue of (6.3), the above equation takes the form

$$R_{lijk}(p + b_h u^h) = 0$$

Since $R_{lijk} \neq 0$, we get

(6.4) $$p + u^h b_h = 0$$

i.e., the condition $u^h b_h = -p \neq 0$ holds.

Multiplying (2.1) by u^k and u^l, respectively, and using the symmetric properties of R_{hijk} and from (6.3), we get

(6.5) $$u^k d_k = -p \neq 0, \qquad u^l R_{hijk,l} = a_l u^l R_{hijk}$$

From (6.5), we have

(6.6) $$u^l R_{,l} = a_l u^l R$$

Using (2.5), (2.7), $(6.3)_2$ and (6.6), we get

(6.7) $$(a_l - 2b_l) u^l R = 0$$

i.e.,

(6.8) $$a_l u^l = 2b_l u^l \qquad or \qquad R = 0$$

We suppose that the condition $R \neq 0$ holds, by the aid of (6.4), (6.5) and (6.8),we say that: If the manifold $(WS)_n$ admits a concurrent vector field u^i given by (6.1), then u_i is not orthogonal to the vectors of $(WS)_n$.

Suppose that $R = 0$, from (2.1) and (6.3), we obtain

$$u^i R_{ij,l} = u^i b_i R_{lj} \tag{6.9}$$

Using (6.1) and $(6.3)_3$, (6.9) takes the form

$$(p - u^i b_i) R_{lj} = 0 \tag{6.10}$$

Then, with the help of (6.4) and (6.10), we find $R_{lj} = 0$, which means that the manifold is an Einstein manifold with zero scalar curvature.

This leads to the following theorem:

Theorem 6.1 *If, in a $(WS)_n$ which admits a concurrent vector field u^i given by (6.1), R is non-zero then u_i is not orthogonal to the vectors of $(WS)_n$. Otherwise, the manifold is an Einstein manifold with zero scalar curvature.*

Part II.

If the recurrent vector field u^i given by (6.2) is non-null, then we have

$$u^h R_{hijk,l} = 0, \qquad u^h R_{hijk} = 0 \tag{6.11}$$

Using the expressions (1.2) and (6.11), we have $u^h b_h R_{lijk} = 0$. Since $R_{lijk} \neq 0$, it is easy to see that

$$u^h b_h = 0 \tag{6.12}$$

By the aid of (1.2) and (6.11) and remembering the symmetric properties of R_{hijk}, we get

$$u^k d_k = 0 \tag{6.13}$$

Using (6.11), we have

$$u^h R_{hk,l} = 0, \qquad u^h R_{hk} = 0 \tag{6.14}$$

By the aid of (2.3), (2.7), (6.13) and (6.14), $u^l R_{,l} = 0$ holds. Then, from (2.2), we have $a_l u^l R = 0$. Let us suppose that $R \neq 0$. Hence, we get

$$a_l u^l = 0 \tag{6.15}$$

Differentiating $(6.11)_2$ covariantly and using (6.2), we get $\beta_l u^l = 0$, i.e., β_l is orthogonal to u^l.

Hence, we can state the theorem as follows:

Theorem 6.2 *If a $(WS)_n$ (with $R \neq 0$) admits a non-null recurrent vector field u^i given by (6.2), then the associated vector field of recurrence β_i is orthogonal to u_i and u_i is orthogonal to the vectors of $(WS)_n$.*

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