Some characterizations of slant submanifolds of trans-Sasakian manifolds

A. Sarkar and P. Bhakta

Abstract. The object of the present paper is to study slant submanifolds of trans-Sasakian manifolds with second fundamental form satisfying certain conditions. Totally umbilical slant submanifolds of trans-Sasakian manifolds have been considered. Nature of slant submanifolds of trans-Sasakian manifolds as Ricci soliton has also been analyzed.

M.S.C. 2010: 53C40; 53C25.
Key words: trans-Sasakian manifolds; second fundamental form; recurrent; slant submanifold; totally geodesic; totally umbilical.

1 Introduction

As a unification of cosymplectic, Sasakian and Kenmotsu structure, the notion of trans-Sasakian structure arose from the works of Blair and Oubina [1], [2], [19]. Trans-Sasakian structures have further been analyzed by Marrero [18]. Marrero proved that a trans-Sasakian structure of dimension ≥ 5 is either α-Sasakian or β-Kenmotsu. Three-dimensional trans-Sasakian manifolds have been studied in the paper [8].

The theory of slant submanifolds was introduced by Chen [6] as a generalization of holomorphic and totally real submersions in Kaehlerian geometry. The almost contact analogue of such properties can be found in the work of Lotta [17]. Slant submanifolds of Sasakian manifolds have been characterized by Cabrerizo et al [3]. Slant submanifolds of trans-Sasakian manifolds have been studied by R. S. Gupta [14]. Invariant submanifolds of trans-Sasakian manifolds have been characterized in [10], [24], [26]. Pseudo slant submanifolds of trans-Sasakian manifolds have been studied in [9].Umbilical submanifolds of Kenmotsu manifolds have been considered in [28], [31]. In [30], invariant submanifolds of Kenmotsu manifolds have been studied. The Kenmotsu case has been generalized to trans-Sasakian case in the papers [10], [24]. In [24], some necessary and sufficient conditions for invariant submanifolds of trans-Sasakian manifolds to be totally geodesic have been obtained. Since totally geodesic submanifolds are the simplest submanifolds, a natural tendency of research

BSG Proceedings 26. The International Conference "Differential Geometry - Dynamical Systems" DGDS-2018, 30 August - 2 September 2018, Mangalia-Romania, pp. 102-111.

in submanifold theory is to establish relation between totally geodesic submanifolds and other submanifolds. In [16], necessary and sufficient conditions for invariant submanifolds of Sasakian manifolds to be totally geodesic have been deduced. In [11], necessary and sufficient conditions for submanifolds of Sasakian manifolds to be totally geodesic have been obtained. Invariant submanifolds of trans-Sasakian manifolds have been studied in [7]. The anti invariant case is treated in [27].

Considering the above works, in the present paper we would like to study slant submanifolds of trans-Sasakian manifolds whose second fundamental form satisfy certain conditions. The present paper is organized as follows:

After the introduction and preliminaries, slant submanifolds of trans-Sasakian manifolds with second fundamental form satisfying some parallel and recurrent conditions have been studied in Section 3. Totally umbilical slant submanifolds of trans-Sasakian manifolds have been considered in Section 4. Nature of slant submanifolds of trans-Sasakian manifolds as Ricci soliton has been analyzed in the last section.

2 Preliminaries

Let $\tilde{M}$ be an almost contact metric manifold of dimension $2n+1$, that is, a $(2n+1)$-dimensional differentiable manifold endowed with an almost contact metric structure (ϕ, ξ, η, g). By definition ϕ, ξ, η are tensor fields of type $(1,1)$, $(1,0)$, $(0,1)$, respectively, and g is a Riemannian metric such that [1]

$$\phi^2 X = -X + \eta(X)\xi, \;\; \eta(\xi) = 1,$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$$

for all differentiable vector fields X, Y on $\tilde{M}$. Then also

$$\phi\xi = 0, \;\; \eta(\phi X) = 0, \;\; \eta(X) = g(X, \xi).$$

Let Φ be the fundamental 2-form defined by $\Phi(X,Y) = g(X, \phi Y)$, for all differentiable vector fields X, Y on $\tilde{M}$. An almost contact metric structure (ϕ, ξ, η, g) on $\tilde{M}$ is called trans-Sasakian structure [2], [18] [19] if $(\tilde{M} \otimes \mathbb{R}, J, G)$ belongs to the class W_4 [13] where J is the almost complex structure on $\tilde{M} \otimes \mathbb{R}$ defined by

$$J(X, fd/df) = (\phi X - f\xi, \eta(X)d/dt),$$

for a vector field X on $\tilde{M}$, a smooth functions f on $\tilde{M} \otimes \mathbb{R}$ and the product metric G on $\tilde{M} \otimes \mathbb{R}$. This may be expressed by the condition [2]

$$(\tilde{\nabla}_X \phi)Y = \alpha(g(X,Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X) \tag{2.1}$$

for smooth functions α and β on $\tilde{M}$, where $\tilde{\nabla}$ is the Levi-Civita connection on $\tilde{M}$. In such case, we say that the trans-Sasakian structure is of type (α, β). From (2.1) it follows that

$$\tilde{\nabla}_X \xi = -\alpha\phi X + \beta(X - \eta(X)\xi), \tag{2.2}$$

$$(\tilde{\nabla}_X \eta)Y = -\alpha g(\phi X, Y) + \beta g(\phi X, \phi Y).$$

For a $(2n+1)$-dimensional trans-Sasakian manifold of type (α,β) we know [21] the following:

$$\begin{aligned}\tilde{R}(X,Y)\xi = \;& (\alpha^2-\beta^2)(\eta(Y)X-\eta(X)Y)+2\alpha\beta(\eta(Y)\phi X-\eta(X)\phi Y)\\ & -(X\alpha)\phi Y+(Y\alpha)\phi X-(X\beta)\phi^2 Y+(Y\beta)\phi^2 X.\end{aligned}$$

$$\tilde{S}(X,\xi)=2n(\alpha^2-\beta^2)\eta(X)-(2n-1)X\beta-\eta(X)\xi\beta-(\phi X)\alpha.$$

We also know [18]

$$2\alpha\beta+\xi\alpha=0. \tag{2.3}$$

Let $f:(M,g)\to(\tilde{M},g)$ be an isometric immersion from an n-dimensional Riemannian manifold M to a $(2n+1)$ dimensional trans-Sasakian manifold $\tilde{M}$. Then we have [4]

$$\begin{aligned}\tilde{\nabla}_X Y &= \nabla_X Y+h(X,Y),\\ \tilde{\nabla}_X N &= -A_N X+\tilde{\nabla}^{\perp}_X N,\end{aligned} \tag{2.4}$$

for all vector fields X,Y tangent to M and normal vector field N on M, where ∇ is the Riemannian connection on M defined by the induced metric g and $\nabla^{\perp}$ is the normal connection on $T^{\perp}M$ of M. h is the second fundamental form of the immersion.

We consider the Codazzi equation

$$R^N(X,Y)Z=(\tilde{\nabla}_X h)(Y,Z)-(\tilde{\nabla}_Y h)(X,Z).$$

A submanifold M of $\tilde{M}$ is said to be invariant if the structure vector field ξ is tangent to M at every point of M and ϕX is tangent to M for every vector X tangent to M at every point on M. A submanifold M of $\tilde{M}$ is said to be anti invariant if the structure vector field ξ is tangent to M at every point of M and ϕX is normal to M for every vector X tangent to M at every point on M. The submanifold is called totally geodesic if its second fundamental form vanishes identically on it [16]. In view of (2.2) and (2.4), we get

$$\nabla_X\xi+h(X,\xi)=-\alpha\phi X+\beta(X-\eta(X)\xi).$$

Assume that ξ is tangent to M. For each non-zero vector fields X tangent to M at a point p such that X is not proportional to ξ_p, we denote by $\theta(X)$ the Weingarten angle of X, that is the angle between ϕX and T_pM. The submanifold is called slant if the Weingarten angle $\theta(X)$ is constant which is independent of the choice of $p\in M$. The angle θ is called slant angle. If $\theta=\frac{\pi}{2}$, the submanifold is anti invariant and for $\theta=0$ it is invariant.

Let M be a slant submanifold of a $(2n+1)$ dimensional manifold $\tilde{M}$ with induced metric g. For any X tangent to M, we write

$$\phi X=PX+FX$$

and for N normal to M we write

$$\phi N=tN+fN,$$

where PX and FX, respectively, denote the tangential and normal part of ϕX; tN and fN, respectively, denote the tangential and normal part of ϕN.

Suppose the submanifold is slant. Then we get the following:

$$\nabla_X \xi = -\alpha PX + \beta\big(X - \eta(X)\xi\big). \tag{2.5}$$

$$h(X,\xi) = -\alpha FX. \tag{2.6}$$

$$\begin{aligned} R^T(X,Y)\xi &= (\alpha^2-\beta^2)(\eta(Y)X - \eta(X)Y) \\ &+ 2\alpha\beta(\eta(Y)PX - \eta(X)PY) \\ &- (X\alpha)PY + (Y\alpha)PX \\ &- (X\beta)P^2Y + (Y\beta)P^2X. \end{aligned} \tag{2.7}$$

$$\begin{aligned} R^N(X,Y)\xi &= 2\alpha\beta(\eta(Y)FX - \eta(X)FY) \\ &- (X\alpha)FY + (Y\alpha)FX \\ &- (X\beta)F^2Y + (Y\beta)F^2X. \end{aligned} \tag{2.8}$$

$$\begin{aligned} S^T(X,\xi) &= 2n(\alpha^2-\beta^2)\eta(X) - (2n-1)X\beta \\ &- \eta(X)\xi\beta - (PX)\alpha. \end{aligned}$$

$$S^N(X,\xi) = -(FX)\alpha.$$

Here R^T and R^N denote respectively the tangential and normal part of $\tilde{R}$, S^T and S^N denote respectively the tangential and normal part of the Ricci curvature $\tilde{S}$.

For a slant submanifold of trans-Sasakian manifold with slant angle θ we get [14]

$$P^2X = \cos^2\theta(X - \eta(X)\xi).$$

$$F^2X = \sin^2\theta(X - \eta(X)\xi). \tag{2.9}$$

Example 2.1. Let us consider $\mathbb{R}^5$ with

$$\eta = \frac{1}{2}(dz - \sum_{i=1}^{2} y^i dx^i), \quad \xi = 2\frac{\partial}{\partial z},$$

$$g = \eta\otimes\eta + \frac{1}{4}\big(\sum_{i=1}^{2}(dx^i\otimes dx^i + dy^i\otimes dy^i)\big),$$

and

$$\phi(X_1,X_2,Y_1,Y_2,Z) = (-X_2,-X_1,Y_2,-Y_1,y^2X_1 - y^1X_2).$$

Then $(\mathbb{R}^5,\phi,\xi,\eta,g)$ is a trans-Sasakian manifold. The map $f:\mathbb{R}^3\to\mathbb{R}^5$ defined by

$$f(u,v,t) = 2(u\cos\theta, v, u\sin\theta, 0, t)$$

is a slant submersion. For details, we refer to [3].

3 Slant submanifolds of trans-Sasakian manifolds with the second fundamental form satisfying some conditions

Definition 3.1. The second fundamental form h of a submanifold is called parallel if it satisfies

$$(\tilde{\nabla}_W h)(X, Y) = 0 \tag{3.1}$$

for the vector fields X, Y, W tangent to the submanifold.

Theorem 3.1. *If the second fundamental form of a slant submanifold of a proper trans-Sasakian manifold is parallel, then it is a 1-dimensional manifold with Ker$\{F\}$ as the section of its tangent bundle.*

Proof. Consider a slant submanifold of a trans-Sasakian manifold with parallel second fundamental form. From (3.1), we have

$$\nabla^{\perp}_W h(X, Y) - h(\nabla_W X, Y) - h(X, \nabla_W Y) = 0.$$

Putting $X = Y = \xi$ and using (2.5) and (2.6) in the above equation, we get

$$\alpha\beta FW = 0.$$

If α and β both are non-zero, then $W \in \text{Ker}\{F\}$. $\text{Ker}\{\phi\}$ is 1-dimensional, $\text{Ker}\{F\}$ is so. □

From the above theorem, we immediately obtain the following:

Corollary 3.1. *A hypersurface with parallel second fundamental form of a proper trans-Sasakian manifold is not slant.*

Definition 3.2. The second fundamental form of a submanifold is called recurrent [22] if

$$(\tilde{\nabla}_X h)(Y, Z) = \omega(X) h(Y, Z), \tag{3.2}$$

where ω is an 1-form on M.

Theorem 3.2. *If the second fundamental form of a slant submanifold of a proper trans-Sasakian manifold is recurrent, then it is a 1-dimensional manifold with Ker$\{F\}$ as the section of its tangent bundle.*

Proof. Consider a slant submanifold of a trans-Sasakian manifold with recurrent second fundamental form. From (3.2), we have

$$\nabla^{\perp}_W h(X, Y) - h(\nabla_W X, Y) - h(X, \nabla_W Y) = \omega(X) h(Y, Z). \tag{3.3}$$

Putting $X = Y = \xi$ and using (2.5) and (2.6) in the above equation, we get

$$\alpha\beta FW = 0.$$

If α and β both are non-zero, then $W \in \text{Ker}\{F\}$. Thus, as before, the claim follows. □

The above theorem also gives

Corollary 3.2. *A hypersurface with recurrent second fundamental form of a proper trans-Sasakian manifold is not slant.*

Definition 3.3. The second fundamental form of a submanifold is called semi parallel if

$$\tilde{R}(X,Y)h(U,V) = 0 \tag{3.4}$$

for the vector fields X, Y, U, V tangent to the submanifold.

Theorem 3.3. *If the second fundamental form of a slant submanifold of a trans-Sasakian manifold of type (α, β) is semi parallel, then the submanifold is a 1-dimensional manifold with Ker$\{F\}$ as the section of its tangent bundle, provided $\alpha \neq 0$ and $\alpha^2 \neq \beta^2$.*

Proof. Consider a slant submanifold of trans-Sasakian manifold with semi parallel second fundamental form. Then, expanding (3.4), we get

$$R^N(X,Y)h(U,V) - h(R^T(X,Y)U,V) - h(U,R^T(X,Y)V) = 0. \tag{3.5}$$

In (3.5) putting $Y = U = V = \xi$ and using (2.5), (2.6) and (2.7), we get after simplification

$$\alpha(\alpha^2 - \beta^2)FX = 0.$$

The above equation concludes the proof. □

From the above theorem, we immediately get the following:

Corollary 3.3. *A hypersurface with semi parallel second fundamental form of a trans-Sasakian manifold is not slant, provided provided $\alpha \neq 0$ and $\alpha^2 \neq \beta^2$.*

Definition 3.4. The second fundamental form of a submanifold is called pseudo parallel if

$$\tilde{R}(X,Y)h(U,V) = fQ(g,h)(X,Y,U,V) \tag{3.6}$$

for the vector fields X, Y, U, V tangent to the submanifold. Here f is a smooth function on the submanifold and Q is the pseudo symmetry operator defined in the sense of Deszcz [12].

Theorem 3.4. *If the second fundamental form of a slant submanifold of a trans-Sasakian manifold of type (α, β) is pseudo parallel, then the submanifold is a 1-dimensional manifold with Ker$\{F\}$ as the section of its tangent bundle, provided $\alpha \neq 0$ and $\alpha^2 \neq \beta^2 + 2f$.*

Proof. Consider an anti invariant submanifold of trans-Sasakian manifold with pseudo parallel second fundamental form. Then expanding (3.6), we get

$$\begin{aligned} R^N(X,Y)h(U,V) - h(R^T(X,Y)U,V) &- h(U,R^T(X,Y)V) \\ &= -f(g(Y,h(U,V))X - g(X,h(U,V))Y \\ &+ h(g(Y,U)X - g(X,U)Y,V) \\ &+ h(U,g(Y,V)X - g(X,V)Y)). \end{aligned} \tag{3.7}$$

In (3.7) putting $Y = U = V = \xi$ and using (2.5), (2.6) and (2.7), we get after simplification

$$\alpha(\alpha^2 - \beta^2 - 2f)FX = 0,$$

which concludes the proof. □

An immediate consequence of the above theorem is

Corollary 3.4. *A hypersurface with pseudo parallel second fundamental form of a trans-Sasakian manifold is not slant, provided $\alpha \neq 0$ and $\alpha^2 \neq \beta^2 + 2f$.*

4 Totally umbilical slant submanifolds of trans-Sasakian manifolds

Definition 4.1. A submanifold of a trans-Sasakian manifold is called totally umbilical [5] if its second fundamental form satisfies

$$h(X, Y) = g(X, Y)H$$

for the tangent vector fields X, Y of the submanifold. Here H is the mean curvature vector.

Theorem 4.1. *A totally umbilical slant submanifold of a trans-Sasakian manifold with $\xi\beta \neq 0$ is a 1-dimensional manifold and it is integral curve of the Reeb vector field ξ.*

Proof. Let M be a totally umbilical proper slant submanifold of a trans-Sasakian manifold. Then from Codazzi equation, we get

$$R^N(X, Y)\xi = \eta(Y)\nabla^{\perp}_X H - \eta(X)\nabla^{\perp}_Y H.$$

Putting $Y = \xi$ and using (2.3) and (2.8), in the above equation, we obtain

$$\nabla^{\perp}_X H = \xi\beta F^2 X.$$

But from the umbilical condition

$$g(U, V)\nabla^{\perp}_X H = (\tilde{\nabla}_X h)(U, V).$$

Putting $U = V = \xi$ in above

$$\nabla^{\perp}_X H = 0. \tag{4.1}$$

In view of the above equations, we see that

$$F^2 X = 0.$$

Using (2.9), we get for proper slant submanifold with $\xi\beta \neq 0$

$$X = \eta(X)\xi.$$

Hence $X \in \{\xi\}$ and we conclude the proof. □

Remark 4.1. The above theorem is also true for Kenmotsu case. In Kenmotsu case the above theorem has been proved in the paper [31] in another way. Our process is simpler than that of the paper [31].

5 Ricci soliton on slant submanifolds of trans-Sasakian manifolds

The notion of Ricci flow [15] was introduced by Hamilton and was later successfully used by Perelman [20] to prove the well known Poincare conjecture. A self similar solution of Ricci flow equation is known as Ricci soliton. Ricci soliton in the context of almost contact geometry was studied by Sharma [29].

Definition 5.1. A submanifold of a $(2n+1)$-dimensional trans-Sasakian manifold is said to be Ricci soliton if it satisfies the following:

$$\pounds_V g(X,Y) + 2S^T(X,Y) + \lambda g(X,Y) = 0. \tag{5.1}$$

Here λ is a constant called the soliton constant. The soliton is called shrinking if $\lambda < 0$, expanding if $\lambda > 0$ and steady if $\lambda = 0$.

Theorem 5.1. *A slant submanifold of a trans-Sasakian manifold as a Ricci soliton is expanding, steady or shrinking according as* $\xi\beta > (\alpha^2 - \beta^2)$, $\xi\beta = (\alpha^2 - \beta^2)$ *or* $\xi\beta < (\alpha^2 - \beta^2)$.

Proof. Let a slant submanifold of a trans-Sasakian manifold admits a Ricci soliton. Then putting $X = Y = V = \xi$ in (5.1), we get after simplification

$$\lambda = 4n(\xi\beta - \alpha^2 + \beta^2).$$

Hence by definition, we obtain the above theorem. □

As immediate consequences of the above theorem, we obtain the following result:

Corollary 5.1.

a) *A slant submanifold of a Sasakian manifold as a Ricci soliton is shrinking.*
b) *A slant submanifold of a Kenmotsu manifold as a Ricci soliton is expanding.*
c) *A slant submanifold of a cosymplectic manifold as a Ricci soliton is steady.*

References

[1] D. E. Blair, *Geometry of contact and symplectic manifolds*, Birkhäuser, Boston-Basel-Berlin, 2002.

[2] D. E. Blair and J. A. Oubina, *Conformal and related changes of metric on the product of two almost contact metric manifolds*, Publications Mathematiques, 34 (1990), 199-207.

[3] J. L. Cabrerizo, A. Carriazo, and L. M. Fernandez, *Submanifolds in Sasakian manifolds*, Glasgow Math. J., 42 (2000), 125-138.

[4] B. Y. Chen, *Geometry of submanifolds, Marcel Dekker*, New York, 1973.

[5] B. Y. Chen, *Totally umbilical submanifolds*, Soochow J. Math., 5 (1979), 9-36.

[6] B. Y. Chen, *Slant immersions*, Bull. Australian Math. Soc., 41 (1990), 135-147.

[7] D. Chinea, and P. S. Prestelo, *Invariant submanifolds of a trans-Sasakian manifold*, Publ. Math. Debreceen, 38 (1991), 103-109.

[8] U. C. De, and A. Sarkar, *On three-dimensional trans-Sasakian manifolds*, Extracta Mathematicae, 23 (2008), 265-277.

[9] U. C. De, and A. Sarkar, *On pseudoslant submanifolds of trans-Sasakian manifolds*, Proceedings of Estonian Academy of Science, 60 (2011), 1-15.

[10] A. De, *Totally geodesic submanifolds of a trans-Sasakian manifold*, Proc. Estonian Acad. Sci., 62 (2013), 249-257.

[11] S. Deshmukh, and M. A. Al-Gwaiz, *Totally umbilical submanifolds of a Sasakian manifold*, New Zealand J. Math., 22 (1993), 43-47.

[12] R. Deszcz, *On pseudo symmetric spaces*, Bull. Soc. Math. Belg. Ser. A., 44 (1992), 1-34.

[13] A. Gray, and L. M. Hervella, *The sixteen classes of almost Harmite manifolds and their linear invariants*, Ann. Mat. Pura Appl., 123 (4) (1980), 35-58.

[14] R. S. Gupta, S. M. K. Haider and A. Sharifuddin, *Slant submanifolds of a trans-Sasakian manifold*, Bull. Math. Soc. Roumanie, 47 (2004), 45-47.

[15] R. S. Hamilton, *The Ricci flow on surfaces*, Mathematics and general Relativity, Contemp. Math., 71, A. M. S., 1988.

[16] M. Kon, *Invariant submanifolds of normal contact metric manifolds*, Kodai Math Sem Rep., 27 (1973), 330-336.

[17] A. Lotta, *Slant submanifolds in contact geometry*, Bull. Math. Soc. Roumanie, 39 (1996), 183-198.

[18] J. C. Marrero, *The local structure of trans-Sasakian manifolds*, Ann. Mat. Pura Appl., 162 (1992), 77-86.

[19] J. A. Oubina, *New classes of almost contact metric structures*, Publ. Math. Debrecen, 32 (1985), 187-193.

[20] G. Perelman, *The entropy formula for the Ricci flow and its geometric applications*, arxiv math DG, 1021111599.

[21] R. Prasad, *Quasi-conformal curvature on trans-Sasakian manifolds*, Proc. Indian Nat. Sci. Acad., 76 (2010), 7-35.

[22] W. Roter, *On conformally recurrent, Ricci recurrent manifolds*, Colloq. Math., 46 (1982), 45-57.

[23] A. Sarkar, and M. Sen, *On invariant submanifolds of trans-Sasakian manifolds*, Proc. Estonian Acad. Sci., 61 (2012), 29-37.

[24] A. Sarkar, and M. Sen, *Invariant submanifolds of LP-Sasakian manifolds*, Extracta Mathematicae, 27 (2012), 145-154.

[25] A. Sarkar, A. Sil, and A. Paul, *Some characterizations of η-Ricci solitons and Kagan sub projective spaces*, to appear in Ukrainian Math. J., 2020.

[26] A. Sarkar and N. Biswas, *Some conditions for totally umbilical submanifolds of trans-Sasakian manifolds to be totally geodesic*, submitted.

[27] A. Sarkar and P. Bhakta, *Some characterizations of anti invariant submanifolds of trans-Sasakian manifolds*, submitted.

[28] J. A. Schauten, *Ricci's Calculus*, Springer Verlag, Berlin, 1954.

[29] R. Sharma, *Certain results on K-Contact and (κ, μ)-contact manifolds*, J. Geom., 89 (2008), 138-247.

[30] S. Sular and C. Özgur, *On some submanifolds of Kenmotsu manifolds*, Chaos, Solitons and Fractals, 42 (2009), 1990-1995.

[31] S. Uddin, Z. Ahsan and A. H. Yakub, *Classification of totally umbilical slant submanifolds of Kenmotsu manifolds*, arxiv math D.G., 1404.3791v2.

Authors' address:

Avijit Sarkar and Pradip Bhakta
Department of Mathematics, University of Kalyani,
Kalyani 741235, West Bengal, India.
E-mail: avjaj@yahoo.co.in , pradip020791@gmail.com