The Barbosu-Constantin potential

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Abstract. In this paper, we study a post-Newtonian (the Barbosu-Constantin) potential. This is similar with the Mücket-Treder potential, obtained mainly through replacement of the logarithm-term with a square-term. In the frame of the two-body problem, we investigate some mathematical aspects associated to the field generated by this potential. We emerge from the motion equations in an analytical frame and the first integrals written in polar coordinates. Then, applying McGehee type transformations, we remove the singularity from the associated array-field. We also address the study at the origin of the system, for the proposed McGehee transformations.

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1 Introduction

In 1977, Mücket and Treder [2] introduced a post-Newtonian potential in order to explain Mercury's perihelion advance and the motion of Moon's perigee.

In the present paper, in the frame of two-body problem, we propose the Barbosu-Constantin potential (BC-potential) which is similarly with the Mücket-Treder potential (MT-potential). We obtain BC-potential from MT-potential by replacing the logarithm-term with a square-term, and considering $\alpha = 1$ [4].

The Barbosu-Constantin (BC) potential is the pseudo Mücket-Treder potential, which has the following form:

$$V(r) = \frac{\beta + \sqrt{r}}{\gamma \cdot r}, \tag{1.1}$$

where $\beta = 2$, $\gamma = \frac{1}{\mu}$, $\mu = GM$ (G being the Newtonian constant of gravitation), M is the mass of field generated by body, and $r = |\mathbf{r}|$ is the distance between two point-like masses.

In Sections 2 and 3, we further initiate a brief approach in the problem associated to the BC-potential. We focus on several mathematical aspects such as McGehee

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type transformations for this potential, and we begin a study at the origin of the system, corresponding to these transformations.

2 McGehee type transformations

Definition 2.1. In an analytical frame, $\mathbf{q} = (q_1, q_2) \in \mathbb{R}^2 \setminus \{(0,0)\}$ and $\mathbf{p} = (p_1, p_2) \in \mathbb{R}^2$ are the position vector and the momentum vector of a particle with respect to the field centre, respectively.

Definition 2.2. We introduce the polar coordinates (r,θ):

$$r = |\mathbf{q}|, \tag{2.1}$$

$$\theta = \arctan\left(\frac{q_2}{q_1}\right). \tag{2.2}$$

Theorem 2.1. *The Hamilton-Jacobi system associated of two-body problem in the BC-potential is [4]:*

$$\xi = \dot{r} = \frac{q_1 p_1 + q_2 p_2}{|\mathbf{q}|}, \tag{2.3}$$

$$\eta = r\dot{\theta} = \frac{q_1 p_2 - q_2 p_1}{|\mathbf{q}|}, \tag{2.4}$$

$$\dot{\xi} = \frac{\eta^2}{|\mathbf{q}|} + V_{,|\mathbf{q}|}, \dot{\eta} = -\frac{\eta \cdot \xi}{r}. \tag{2.5}$$

Proof. We calculate: $V_{,|\mathbf{q}|} = -\frac{(\beta\prime + \sqrt{|\mathbf{q}|})}{\gamma\prime \cdot |\mathbf{q}|^2}$, $\beta\prime = \frac{2}{3} \cdot \beta, \gamma\prime = \frac{2}{3} \cdot \gamma$. and, after some computations, we obtain the motion equations. □

Remark 2.3. We observe in Definition 1 and Theorem 2.1 that the potential and the motion equations have an isolated singularity at the origin, $\mathbf{q} = (0,0)$.

In this paper, we consider the McGehee type transformations [3]:

Definition 2.4.

$$z = r^{1/2} \cdot \xi, \tag{2.6}$$

$$w = r^{1/2} \cdot \eta, \tag{2.7}$$

$$d\rho = r^{-3/2} \cdot dt. \tag{2.8}$$

To remove the singularity at the origin $\mathbf{q} = (0,0)$, we use these McGehee transformations.

Theorem 2.2. *The associated array field for the BC-potential is:*

$$r_{,\rho} = r \cdot \xi\sqrt{r} = r \cdot z, \tag{2.9}$$

$$\theta_{,\rho} = w, \tag{2.10}$$

$$z_{,\rho} = \frac{z^2}{2} + w^2 + (-\mu)\left(2 + \frac{\sqrt{r}}{2}\right), \tag{2.11}$$

$$w_{,\rho} = -\frac{\xi \cdot \eta}{2} \cdot r. \tag{2.12}$$

and also the first integrals are:

$$z^2 + w^2 = 2h \cdot r + 2\mu \cdot \sqrt{r} + 4\mu, \tag{2.13}$$

$$C = w \cdot \sqrt{r}. \tag{2.14}$$

3 The study at the origin

By taking $r = 0$ in Theorem 2.2., we infer

Corollary 3.1. *The array field from Theorem 2.2 becomes:*

$$r_{,\rho} = 0, \tag{3.1}$$

$$\theta_{,\rho} = w, \tag{3.2}$$

$$z_{,\rho} = \frac{z^2}{2} + w^2 - 2\mu, \tag{3.3}$$

$$w_{,\rho} = 0. \tag{3.4}$$

We note that the first integrals from Theorem 2.2 may be written as $C = 0$, and:

$$z^2 + w^2 = 4\mu. \tag{3.5}$$

By considering the equation (3.5) and the parametrization:

$$\begin{cases} z = 2\sqrt{\mu} \cdot \sin \alpha, \\ w = 2\sqrt{\mu} \cdot \cos \alpha, \end{cases}$$

we infer the corresponding derivation [1]:

$$\frac{d\alpha}{d\theta} = \frac{1}{2}.$$

4 Conclusions

By applying the McGehee transformations, we have removed the isolated singularity at the origin $\mathbf{q} = (0, 0)$ both for the motion equations and the integral of energy associated to the particle in the field generated by Barbosu-Constantin potential. This brief study at the origin will be subject of our further research.

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