Models for the reliability of the manufacturing systems

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Abstract. The paper describes some models with application in the manufacturing systems. In the first part is proposed a generalization of the Taguchi loss function in the multidimensional case with interactions for $2, 3, \ldots, n$ order. Using the graph theory, reliability of systems and conditional probabilities, further is analyzed a practical application for a lathe gearbox. In the last part of the work is conceived an extension of Adamyan-He model based on the Nakada-Yoneyama method.

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1 Introduction

Several methods can be applied to the system reliability analysis, such as reliability graph, fault tree analysis, Markov chain and Monte Carlo simulation [7]. System reliability models are based on the reliability of its components, which frequently are assumed independent. For many systems there might be dependencies between the components of the system. Section two presents different forms of Taguchi loss function in the one-dimensional and multidimensional cases; for the multidimensional case it exists many models for Taguchi loss functions: the authors analyze the dependencies, with the order greater as two, with this type of loss functions.

In the past, it was necessary to resort to Markov chains in order to model some intricate dynamic interactions among failure models [14;15]. As a substitute, the Petri nets are widely used as a tool for analyzing system safety and reliability of the complex systems. In our days the new non-Markov technique, Stochastic Petri Nets, can be as a preprocessor for creating larger Markov models in an automatic fashion [1;3].

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2 Taguchi's Loss Functions for Reliability

Many problems of robust design and forms of quality loss functions have been proposed in the literature [4;9;11;12]. The loss function parabolic type and its variants in the one-dimensional case is well known and has different applications. Another representation of the loss function it is proposed for the reliability case:

$$QLF(x) = k(x^{-\alpha} - 1),\ \alpha \in \mathcal{R}_+\ ,\ x \in [0,1]$$

where x denoted as variable and k is a cost coefficient:

Parameter α depends on the costs of the reliability. If the cost of the loss is smaller ($\alpha_1 < \alpha_2$) the loss function is changing in the same way; the bigger values for α correspond to critical systems with very high reliability. If x approaches to zero, the losses will be very high.

Feng and Kusiak extended the notion of quality loss function to multidimensional chains in the independence case. The loss function in this case is given by the sum of loss functions for every variable [6;13]: $L(y) = y^T Dy$, where D is the diagonal matrix,

$$D = \begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0; \\ 0 & 0 & d_{33} \end{pmatrix}$$

it result $L(y) = \sum_i d_{ii} y_i^2$.

In many practical applications, it is also interested in the analysis of interaction between two factors, since such of potential interactions can cause a fraction of quality loss. Considering also the influence of pairs of factors, statistical inferences become more conclusive.

For multidimensional variables it exists many possibilities for extended Taguchi quadratic loss function. Pignatiello [10] and Feng and Kusiak [6] proposed for multivariate variables the quality loss function as a quadratic form positive defined:

$$L(y) = y^T Cy$$

where C is a n order symmetrical quadratic matrix positive defined In the three-dimensional case it will be obtained:

$$L(y) = (y_1; y_2, y_3) \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \sum_{i=1}^{3} C_{ii} y_i^2 + 2 \sum_i \sum_j C_{ij} y_i y_j,\ i < j.$$

This take into account also the losses due to the every variable and pair of variables imprecision, but not appear the losses dues to the order $3, 4, \ldots, n$ interactions.

We therefore extended the Taguchi quality loss function to the multidimensional case with interactions for $2, 3, \ldots, n$ order[9]. In the three-dimensional case it will be proposed:

$$L(y) = \sum_{i=1}^{3} C_{ii} y_i^2 + 2 \sum_{i,j=1}^{3} C_{ij} y_i y_j, + C_{123} y_1 y_2 y_3,\ i < j.$$

Obviously the model can be naturally generalized to the n-dimensional case.

3 Reliability graphs of the machine-tools subsystems

Example. Reliability of a lathe gearbox (fig. 1) is studied on the basis of the connections graph of its elements [5;16]. The connections graph of constituent elements of the gearbox is visualized in fig.2; the functional flow of the gearbox is marked with orientated transmission arcs and symbolized with arrows.

Fig. 1.

Fig. 2.

The condition to achieve a transmission of the moment from the shaft a_1 to the shaft a_2, through the arc $P_{2-3-4} = P_2 \cdot P_3 \cdot P_4$, seems to be conditioned by the working of both bearings L_1 and L_2, so it exists the temptation to consider that the probability P_{2-3-4} is a probability conditioned by the probabilities P_8 and P'_8, which will leads to the expression:

$$P'_{(2-3-4/L_4 \cap L_2)} = \frac{P[2-3-4 \cap (L_1 \cap L_2)]}{P(L_1 \cap L_2)}.$$

This expression is valid only when the two events - here the transmission of the movement from a_1 to Z'_2 and then to Z''_2 and a_2 and respectively the working of both the bearing L_1, and the bearing L_2 - would be dependant.

Since the achievement of each of the arcs P_2, P_3, P_4 does not depend on the achievement of the arcs P_8 and P_8', namely the couplings $a_1 - Z_2'$, $Z_2' - Z_2''$ or $Z_2'' - a_2$ are not conditioned by the working of the bearings L_1 and L_2, the events $\{2-3-4\}$, $\{L_1 \cap L_2\}$ might be considered independent, which determines that the expression written before, becomes:

$$P'_{(2-3-4/L_1 \cap L_2)} = P(2-3-4).$$

Consequently the probability of transmitting the movement from a_1 to a_2 is not conditioned by the working of L_1 and L_2. Nevertheless the working of L_1 and L_2 influences the good running of the whole gearbox, in the same way in which any of the partial probabilities P_2, P_3, P_4 etc. are influenced, which leads to the conclusion that the probabilities P_{20} and P_{22} would be considered "in series", in the product of the constituent probabilities of the other elements of the gearbox [5].

Determining the reliability P_{CV} supposes, consequently, the study of each of the constituent probabilities of the gearbox elements.

The total reliability of the gearbox may be calculated based on the given graph, if the values of the components reliability are known; these values may be either taken directly from the existing specialised literature, or may be estimated based on experiments.

4 A Petri net extension with the time dependent failure rate

The authors follow the developments of Adamyan, He, Nakada, Yoneyama from [2;8], but it will proposed a model with variable failure rate. Instead of memory less exponential distribution, which has a constant failure rate and a constant hazard rate, it is proposed a Weibull distribution, with the time dependent hazard rate. A set of transitions is given as $T = \{t_1, t_2, \ldots, t_n\}$; each transition represents an event or an action and can be fired with firing rate $\lambda_i(t_i)$, corresponding to transition t_i, $i = 1, 2, \ldots, n$; $M = \{M_1, M_2, \ldots, M_n\}$ is a set of markings in the reach ability tree.

Fig. 3.

The reach ability tree (fig.3) [2;8] describes the dynamic behavior of the system, shows all possible markings and possible fairings at each marking. First one it is analyzed the case without branches. Because the time for firing the transition t_l is less or equal to τ, the transitional probability that marking M_i changes to M_l is the failure function of the Weibull distribution. It results that the transitional probability, denoted as $Q_{M_i,M_l}(\tau)$, is:

$$Q_{M_i,M_l}(\tau) = F_l(\tau) = \int_0^\tau \frac{\beta}{\eta_l^\beta} x^{\beta-1} \exp\left[-\frac{x^\beta}{\eta_l^\beta}\right] dx.$$

If exists only one branch, it is estimated the probability that the transition t_1 is fired given the alternative transition t_k is not fired up to time τ. It results the transitional probability:

$$Q_{M_i,M_l}(\tau) = \int_0^\tau \frac{\beta}{\eta_l^\beta} x^{\beta-1} \exp\left[-\left(\frac{1}{\eta_l^\beta} + \frac{1}{\eta_k^\beta}\right) x^\beta\right] dx.$$

In the general case there are more than one branch; it is estimated the probability that the transition t_l will be fired given that the alternative transition $t_l, t_k, \ldots, t_n$ are not fired up to time τ:

$$Q_{M_i,M_l}(\tau) = \int_0^\tau \frac{\beta}{\eta_l^\beta} x^{\beta-1} \exp\left[-\left(\frac{1}{\eta_l^\beta} + \frac{1}{\eta_k^\beta} + \ldots\right) x^\beta\right] dx.$$

Next is possible to develop the model for different Weibull distributions with 3, 4 or 5 parameters, but the calculus become very complicated.

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