Detection of gravitational waves II

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Abstract. In the theory of general relativity, gravity is assumed to propagate with the speed of light. The Newtonian gravitational force is obtained at a nonrelativistic limit, where the propagation speed is taken to be infinity. The gravito-radiative force appears as a relativistic correction to the Newtonian theory. The emission of gravitational energy is predicted in terms of the time derivative of the gravito-tensor potentials and confirmed to establish the gravitational wave theory. The detection of the gravitational wave on Earth is discussed in terms of the gravito-radiative force, which is due to the time derivative of the gravito-tensor potential. The gravito-radiative force transmits energy and angular momentum to Earth and masses on Earth. Because Earth's crust is rigid by means of nongravitational forces, the gravitational wave can be detected by observing the extra energy and angular momentum transmitted by gravitational waves (pulses) relative to the center of Earth. We installed verticity meters, stationary pendulums with devices to measure and record the displacements of bobs, in Boulder, Colorado. Some impulses of about 10^{-8} m/s were observed during 1999, and are attributed to gravitational pulses emitted by evaporating stars at the galactic center. If incoming gravitational pulses have this magnitude with a circular polarization, the induced change in the rotational speed of Earth would be measurable as leap seconds in a few decades later.

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1 Gravito-radiative forces

Einstein assumed that

$$\left(\frac{dx}{dt}\right)^2 = c^2 - \left(\frac{ds}{dt}\right)^2 \leq c^2 \tag{1.1}$$

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for a particle moving in the x-direction, and obtained the well-known expression of its energy as $E = mc^2/\sqrt{1-(dx/cdt)^2}$. He [1] generalized this assummption into a metric

$$ds^2 = g_{00}(ct)^2 - g_{xx}(dx)^2 - g_{yy}(dy)^2 - g_{zz}(dz)^2, \tag{1.2}$$

and proposed that gravity could be described in terms of the space-time dependence of the metric coefficients, g_{ij}. Thus, in this theory, the general relativity theory, gravity, as everything else, follows Einstein's principle,

$$\left(\frac{dx}{dt}\right)^2 \leq \frac{g_{00}}{g_{11}} c^2; \tag{1.3}$$

that is, everything propagates with a finite speed.

In Einstein's theory, equations of motion under gravitational fields can be obtained from the variational principle $\delta \int ds = 0$, as

$$\frac{d^2x^i}{ds^2} = -\Gamma^i_{k\ell} \frac{dx^k}{ds}\frac{dx^\ell}{ds} \equiv F^i_E/(mc^2), \tag{1.4}$$

where the $\Gamma^i_{k\ell}$ are called Christoffel symbols. Because the space component of the left-hand side of eq. (1.4) reduces to the acceleration in c^2, we call the space component of the right-hand side of eq. (1.4) the Einstein force F_E in mc^2. The wave equations of Einstein [1], when linearized (i. e., when the higher-order terms of the deviation of the metric components from 1 are neglected) show [2,3] that these metric components can be expressed in terms of gravitational scalar, vector, and tensor potentials,

$$\phi_{00}(p,t) = \frac{4GM}{c^2 r'}, \quad \boldsymbol{\phi}(p,t) = -\frac{4GMv}{c^3 r'}, \quad \text{and} \quad \phi_{\alpha\beta} = \frac{4GMv_\alpha v_\beta}{c^4 r}, \tag{1.5}$$

respectively, where $\alpha, \beta = 1$, 2, and 3. We neglected the retardation effects, which are not important in our cases. In the linear approximation, equation (1.4) can be approximated as [4]

$$\frac{d^2r}{dt^2} = -\frac{c^2}{4}\nabla\phi_{00} - c\frac{\partial\boldsymbol{\phi}}{\partial t} + cV \times (\nabla \times \boldsymbol{\phi}) + \frac{\partial[v(V\cdot\boldsymbol{\phi})]}{c^2\partial t} \equiv \frac{F_E}{m} \tag{1.6}$$

where v and V are the velocities of a source particle (of mass M) and a test particle (of mass m), respectively. In expression (1-7), we do not differentiate the velocity of the test particle, V, by time t.

In eq. (1.6), we see that the Newtonian term appears as the first approximation, as expected, but there are other correction terms due to Einstein's principle. We notice that there are two terms in eq. (1.6) that are proportional to $1/r$ instead of $1/r^2$ as the Newtonian term is. The Newtonian term does give the orbit of a test particle to the first approximation, but the two terms proportional to $1/r$ transmit energy and momentum through space [3], as the Poynting theorem shows in the theory of electricity and magnetism. These two terms are

$$F_{rad1}/m = \frac{4GM\dot{v}}{c^2 r}, \tag{1.7}$$

and

$$F_{rad2}/m = -\frac{4GM\,[\dot{v}(V \cdot v) + v(V \cdot \dot{v}]}{c^4 r}. \tag{1.8}$$

Equations (1.7) and (1.8) are called the gravito-radiative forces of order 1 and 2, respectively. F_{rad1} comes from the vector potentials of eq. (1.5). F_{rad2} actually comes from the gravitational tensor potentials produced by colliding massive stars or black holes at the center.

When the two massive particles are located near the origin, Newton's third law $\Sigma_i M_i \dot{v}_i = 0$, is applied to null the total contribution of $\Sigma_i F_{rad1}/m = 0$. Therefore, we will consider the gravito-radiative force, F_{rad2}, only in the following discussions.

2 Transmission of energy and angular momentum

The total energy of a test particle of mass m and speed V, under the gravitational field due to mass M at the origin is

$$\mathcal{E} = mc^2 + \frac{1}{2}mV^2 - \frac{GMm}{r}, \tag{2.1}$$

to the first approximation in $(V/c)^2$ of static general relativity, and it agrees with the Newtonian theory. In the Newtonian theory we know that $d\mathcal{E}/dt = 0$, but in the general relativity theory, assuming that $dm/dt = 0$, we see that

$$\frac{d\mathcal{E}}{dt} = m\frac{V \cdot dV}{dt} - GMm\frac{d(1/r)}{dt}$$

$$= V \cdot F_E - GMm\frac{d(1/r)}{dt} = -\frac{8GMm(V \cdot v)(V \cdot \dot{v})}{c^4 r}, \tag{2.2}$$

taking F_E for mdV/dt from eq. (1.6).

The orbital angular momentum, $\mathcal{L} = R \times V$, is conserved under the Newtonian gravitational force. But with the general relativistic force, given by eq. (1.6), we obtain

$$\frac{d\mathcal{L}}{dt} = R \times F_E = T_{GM} + T_{GR}, \tag{2.3}$$

where

$$T_{GM} = -4GMm\frac{(R \times v)(V \cdot r)}{c^2 r^3}, \tag{2.4}$$

and

$$T_{GR} = -4GMm\frac{R \times [v(V \cdot \dot{v}) + \dot{v}(V \cdot v)]}{c^4 r}. \tag{2.5}$$

The extra torque produced by the gravito-magnetic force, T_{GM}, is proportional to $1/r^2$ and dissipates before propagating through the space, but that produced by the gravito-radiative force, T_{GR}, can reach stars at great distances.

It is known that, when the gravity source (mass M) has an accelerated mass quadrupole moment, a gravitational wave is emitted and the source particle loses its energy [2], and that theory is confirmed by observing an energy loss of a binary star [5]. Our eq. (2.2) shows that the gravitational energy is transmitted to a moving test particle (mass m and velocity V) through the gravito-radiative force F_{rad2}. Thus, our theory is the corresponding theory of detecting gravitation waves. In fact, the rate of energy loss, $d\varepsilon/dt$, from the gravitation wave source (mass M) is given as

$$\frac{d\varepsilon}{dt} = \frac{G}{45c^5}(Mv\dot{v})^2. \tag{2.6}$$

with the assumption that $v \cdot \dot{v} = 0$ [2]. Thus, to estimate the order of magnitude of the gravito-radiative force, we can use

$$F_{rad2}/m \simeq \frac{V}{r}\sqrt{\frac{45G}{c^3}\frac{d\varepsilon}{dt}}. \tag{2.7}$$

Extra factors would appear in this relation, depending on the relative angles between V, v, and $\dot{v}$.

3 Detection of gravitational waves by verticity meters

A test particle fixed to the surface of Earth at latitude Θ is moving toward East with speed

$$V = 464\cos\Theta \quad \mathrm{m/s} \tag{3.1}$$

with respect to the center of Earth. The gravito-radiative force, F_{rad2}, exerted on a test particle fixed on the surface of Earth, is obtained by eq. (1.8), using eq. (3.1) for V. Take the z-axis along the North direction and x- and y-axes in the East-West plane, which contains an Earth-bound observation point. Let the spherical angles that v and $\dot{v}$ make in this xyz-system be θ, φ and θ', φ', respectively. If the longitude of the observation point is Φ, then the East component of F_{rad2} is

$$F(E)_{rad2}/m = \frac{2GV}{c^4 r}\Sigma Mv\dot{v}\sin\theta\sin\theta'[(\cos(2\Phi - \varphi - \varphi') - \cos(\varphi - \varphi')]. \tag{3.2}$$

We see that the first term of $F(E)_{rad2}/m$, proportional to $\cos(2\Phi - \varphi - \varphi')$, tries to induce a mass quadrupole moment on Earth. However, Earth's crust is rigid by means of the chemical bond force, and resists such possible deformation. If a motionless pendulum (verticity meter) is placed on the surface of Earth, its pendulum-bob is free to move horizontally, and the gravito-radiative force at its location can be detected by measuring the resulting displacements, unless the gravito-radiative force at the location Φ happend to be equal to the average of the gravito-radiative force on the surface of Earth. Mizushima and Zimmerer constructed vertisity meters at Boulder, Colorado and recorded the displacements of the bobs [6,7]. Many of the observed signals were due to earthquakes, but some of them were not earthquakes [7].

The displacements of the bobs were measured to 1 μm, digitized, and averaged over

2 min. Both averages and mean-square deviations (msd) were recorded. Earthquakes were identified by comparing the published earthquake table of the time of occurrence and the location of the origin. It was observed that each earthquake was seen as an oscillation in the msd, but not in the average of the 2-min displacements. Because the diameter of the string of each pendulum is about 1 mm, Hooke's potential is modified at the origin of the displacement, so that the potential energy is nearly constant for about a 10 μm region near the origin (imperfect pendulum). Each gravitational pulse thus found comes as a pulse of gravito-radiative force, F_{rad2}, of duration time, Δt, much less than 2 min, giving an impulse of 10^{-8} m/s each to the bob relative to the surface of Earth [8,9].

If we take eq. (2.7) assuming that $\epsilon = Mc^2$ and $dt = \Delta t$, we obtain $F_{rad2}\Delta t/m = (V/r)\sqrt{\Delta t 45GM/c}$. Taking the observed value, $F_{rad2}\Delta t = 10^{-8}$ s, we obtain $M\Delta t = 10^{37}$ s-kg. If we take 10 s for Δt we obtain $M = 10^{36}$ kg, a fraction of a possible black-hole mass, but we do not know the value of Δt.

4 Rotation of Earth and gravitational wave

When the average of $F(E)_{rad2}$ over Φ is calculated using eq. (3.2), the first term, which depends on 2Φ, becomes zero, but the last term is zero only when $v \cdot \dot{v} = 0$. Because Earth's crust is rigid, we can still detect a gravito-radiative force by means of the verticity meter, but we expect that the rotational speed of Earth changes when a gravitational wave comes if $v \cdot \dot{v}$ is not zero. If the duration time of the galactic nuclear collision is Δt, then the rotational speed ω of Earth's surface changes by

$$\Delta\omega_z = \Delta t F(E)_{rad2}/(mR) = \frac{2GV\Delta t}{c^4 rR}\Sigma M v\dot{v}\sin\theta\sin\theta'\cos(\varphi-\varphi'), \tag{4.1}$$

where R is the radius of Earth. Because Earth is rigid, the ratio V/R stays the same when the rotational speed and the distance from the center of that part of Earth's crust are used for V and R, respectively, in this formula. That is,

$$\frac{\Delta\omega_z}{\omega} = \frac{2G\Delta t}{c^4 r}\Sigma M v\dot{v}\sin\theta\sin\theta'\cos(\varphi-\varphi') \tag{4.2}$$

gives the fractional change in ω_z due to the gravitational pulse with duration time Δt. The length of day (LOD) is defined as $2\pi/\omega$, and is nearly a constant, which used to be defined as 24 hours. Now LOD is measured continuously, in terms of the atomic clock, to an accuracy of one part in 10^{11} and is found to fluctuate [10]. The fluctuation is interpreted as due to the motion of the soft parts of Earth which changes the moment of inertia [11]. But a gravitational wave may contribute to the fluctuation [9].

If one of the signals we observed at Boulder ($\Theta = 40°$), with impulse 1×10^{-8} m/s, is due to a gravitational pulse with $v \cdot \dot{v} = 0$, then the associated gravito-radiative force must have produced the fractional change in the rotational speed of Earth as

$$\frac{\Delta\omega}{\omega} = 3\times10^{-11}. \tag{4.3}$$

The corresponding change in LOD is 3×10^{-6} s on that day. This is within the accuracy, but smaller than the fluctuation of the LOD measurement. The fractional change in ω means a fractional change in the rotational angular momentum, and the changed angular momentum is conserved until the next gravitational pulse comes in. Because of the moving mantle, ω itself would fluctuate, but the average ω would keep the new value. In 1 year, for example, the shift of $3 \times 10^7 \times 3 \times 10^{-11} = 1$ ms would result. It has been reported [10] that the adjustment time, called a leap second, appeared almost 2 s each year until January 1, 1999, but then disappeared until January 1, 2006. The adjustment has been necessary because the definition of LOD in terms of the atomic clock, which has an accuracy of one part in 10^{13} to 10^{15}, was not good enough. We see that the change in ω as expected by eq. (4.3) would be measurable as a leap second in a few decades, when the adjustment is done with enough accuracy already, assuming that the gravitational pulses are coming with a polarization $v \cdot \dot{v} \simeq 0$. This polarization corresponds to the head-on collision of massive objects at the galactic center.

References

[1] A. Einstein, *Die Grundlage der allgemeinen Rlarivitaetstheorie*, Ann. Phys. 49 (1916), 769-822.

[2] L. Landau and E. Lifshitz, *The Classical Theory of Fields*, Addison-Wesley, Cambridge, 1951.

[3] J. Weber, *General Relativity and Gravitational Waves*, Interscience Publishing Inc, New York 1962.

[4] M. Mizushima, *Gravito-magnetic forces and gravito-radiaive forces due to the galactic center*, Hadronic J. 18 (1995), 577-584.

[5] J. H. Taylor and J. M. Weisberg, *A new test of General Relativity -gravitational radiation and the binary pulsar PSR-1913+16*, Astrophys. J. 253 (1982), 908-920.

[6] M. Mizushima and R. W. Zimmerer, *Gravito-radiative forces due to galactic nucleus (veritcity experiment)*, Hadronic J. 20 (1997), 163-172.

[7] M. Mizushima and R. W. Zimmerer, *Detection of a gravitational wave through the length of day*, Hadronic J. 28 (2005), 187-196.

[8] M. Mizushima, *Detection of gravitational waves*, Nonlinear Analysis 47 (2001), 5513-5524.

[9] M. Mizushima, *Gravitational wave and spiral galaxy (gravito-raditive force)*, In: Froniters of Fundamental Physics, 4-th ed., Siderarth and Alitaisky, Kluwer Acad./Plenum Pub. NewYork 2001, 179-188; *Creation of spiral galaxies*, Hadronic J. 28 (2005), 474-478.

[10] www.iers.org

[11] F. D. Stacey, *Physics of the Earth*, Brookfield Press, Brisbane, Australia, 1992.

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