Gauge theory of Gravitation

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Abstract

A gauge theory of gravitation having the de-Sitter group $SO\,(1,4)$ as local symmetry is presented. The strength tensor field of the gravitational gauge potentials is obtained and then the field equations are written. In order to obtain an unified model of the gravitation with other interactions we will consider the group $SU\,(2) \times SO\,(1,4)$ as gauge symmetry. Some analytical solutions of the field equations are obtained and a comparison with the General Relativity is made. An analytical computing program based on MAPLE Platform, with emphasis on the application to the gauge theories, is also presented.

1 Introduction

The gauge theory of gravitation has been considered by many authors in order to describe the gravity in a similar way with other interactions (electromagnetic, weak or strong)[1]. Some authors consider the Poincairé group (PG) or de-Sitter (DS) group as "active" symmetry groups, i.e. acting on the space time coordinates [2]. Others adopt the "passive" point of view when the space-time coordinates are not affected by group transformations [3,4]. Only the fields change under the action of the symmetry group.

Although the Poincaré gauge theory leads to a satisfactory classical theory of gravity, the analogy with gauge theories of internal symmetries is not a satisfactory one because of the specific treatment of translations [5]. It is possible, however, to formulate the gauge theory of gravity in a way that treats the whole PG in a more unified framework. The approach is based on the DS group and the Lorentz and translation parts are distinguished through a mechanism of spontaneous symmetry breaking [6]. An immediate consequence of replacing PG by the DS group as the symmetry underlying the Universe is the appearance of a non-vanishing cosmological constant Λ, which is determined by a real parameter λ of deformation. When we consider the limit $\lambda \longrightarrow 0$, i.e. the group contraction process, the DS group reduces

to the PG, and the corresponding gravitation theory can not describe the cosmological constant [7]. The matter fields are described by an action that is invariant under the global DS symmetry and the gravity is introduced as a gauge field in the process of localization of this symmetry.

In this work, we adopt the "passive" point of view for the symmetry group in order to develop a DS gauge theory of gravitation over a spherical symmetric Minkowski space-time. Therefore, we restrict ourselves to recast DS symmetry and its consequences used to specify the space-time events is no longer affected by DS transformations. In order to obtain an unified model of the gravitation with other interactions we will consider the group $SU(2) \times SO(1,4)$ as gauge symmetry, where $SO(1,4)$ denotes the DS group.

2   The gauge theory

The DS group has the dimension equal to ten and the $SU(2)$ group is non-abelian, three-dimensional. The infinitesimal generators of the DS group are denoted by M_{ab}, $a,b = 0,1,2,3,5$ and those of $SU(2)$ group by $T_\alpha, \alpha = 1,2,3$. The equations of structure have the form [4,6]:

2.1a
$$[M_{ab}, M_{cd}] = \eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac} + \eta_{ad}M_{bc},$$

2.1b
$$[T_\alpha, T_\beta] = \varepsilon_{\alpha\beta\gamma}T_\gamma,$$

2.1c
$$[M_{ab}, T_\alpha] = 0,$$

where $\eta_{ab} = (1,-1,-1,-1,-1)$ is the five-dimensional Lorentz metric. A matter field $\phi(x)$ is always referred to a local frame L of the Minkowski space-time. In general, it is a multicomponent object which can be represented as a vector-column. The action of the global de-Sitter group, in the tangent space, transforms an L frame into another L frame and determine an appropriate transformation of the field $\phi(x)$ [4]:

2.2
$$\phi'(x') = \left(1 + \frac{1}{2}\lambda^{ab}\Sigma_{ab}\right)\phi(x').$$

Here Σ_{ab} is the spin matrix related to the multicomponent structure of $\phi(x)$.

We define now the gauge covariant derivative, associated to the local group of symmetry $SU(2) \times SO(1,4)$:

2.3
$$\nabla_\mu \phi(x) = \left(\partial_\mu + \frac{g'}{2}A_\mu^{ab}\Sigma_{ab} + g''A_\mu^\alpha T_\alpha\right)\phi(x),$$

where $A_\mu^{ab}(x) = -A_\mu^{ba}(x)$ are the gauge potentials describing the gravitational field and $A_\mu^\alpha(x)$ are the internal gauge potentials associated to the group $SU(2)$. The quantities g' and g'' denote the coupling constants of the gravitational and respectively internal $SU(2)$ gauge fields. Now, we calculate the commutator $[\nabla_\mu, \nabla_\nu]$ in order to obtain the expressions of the strength tensors. We have:

2.4 $[\nabla_\mu, \nabla_\nu]\,\phi\,(x) \;\;=\;\; \{\frac{g'}{2}\left[\partial_\mu A_\nu^{ab} - \partial_\nu A_\mu^{ab} + g'(A_{c\mu}^{a}A_\nu^{cb} - A_{c\nu}^{a}A_\mu^{cb})\right]\Sigma_{ab}$
$+\left(\partial_\mu A_\nu^{\alpha} - \partial_\nu A_\mu^{\alpha} + g''\varepsilon^{\alpha\beta\gamma}A_\mu^{\beta}A_\nu^{\gamma}\right)T_\alpha\}\phi\,(x)\,.$

If we use the general definition

2.5 $$[\nabla_\mu, \nabla_\nu]\,\phi\,(x) = \left(\frac{g'}{2}F_{\mu\nu}^{ab}\Sigma_{ab} + g''G_{\mu\nu}^{\alpha}T_\alpha\right)\phi\,(x)$$

and identify the Eqs. (2.4) and (2.5), we obtain:

2.6 $$F_{\mu\nu}^{ab} = \partial_\mu A_\nu^{ab} - \partial_\nu A_\mu^{ab} + g'\left(A_{c\mu}^{a}A_\nu^{cb} - A_{c\nu}^{a}A_\mu^{cb}\right),$$

2.7 $$G_{\mu\nu}^{\alpha} = \partial_\mu A_\nu^{\alpha} - \partial_\nu A_\mu^{\alpha} + g''\varepsilon^{\alpha\beta\gamma}A_\mu^{\beta}A_\nu^{\gamma}.$$

If we chose $a = i, 5$, $b = j, 5$, $c = m, 5$ with $i, j, m = 0, 1, 2, 3$, and denotes $A_\mu^{i5} = 2\lambda e_\mu^{i}$, then the Eq. (2.6) becomes:

2.8 $$F_{\mu\nu}^{ij} = \partial_\mu A_\nu^{ij} - \partial_\nu A_\mu^{ij} + g'\left(A_{s\mu}^{i}A_\nu^{sj} - A_{s\nu}^{i}A_\mu^{sj}\right) - 4\lambda^2 g'\left(e_\mu^{i}e_\nu^{j} - e_\nu^{i}e_\mu^{j}\right),$$

2.9 $$F_{\mu\nu}^{i} = \partial_\mu e_\nu^{i} - \partial_\nu e_\mu^{i} + g'\left(A_{s\mu}^{i}e_\nu^{s} - A_{s\nu}^{i}e_\mu^{s}\right),$$

In a Riemann-Cartan model the quantities $F_{\mu\nu}^{i}$ are interpreted as the components of the torsion tensor, and $F_{\mu\nu}^{ij}$ as the components of the curvature tensor associated to the gravitational field whose gauge potentials are $e_\mu^{i}\,(x)$ and $A_\mu^{ij}\,(x)$.

3   Model with spherical symmetry

We consider now a particular form of spherically gauge fields of the $SU(2) \times SO(1,4)$ group given by the following ansatz:

3.1 $e_\mu^{0} = (A, 0, 0, 0)\,, e_\mu^{1} = (0, B, 0, 0)\,, e_\mu^{2} = (0, 0, rC, 0)\,, e_\mu^{3} = (0, 0, 0, rC\sin\theta)\,,$

and

3.2a $A_\mu^{01} = (U, 0, 0, 0)\,, A_\mu^{12} = (0, 0, W, 0)\,, A_\mu^{13} = (0, 0, 0, Z\sin\theta)\,,$

3.2b $A_\mu^{23} = (0, 0, 0, V\cos\theta)\,, A_\mu^{02} = \omega_\mu^{03} = 0,$

where A, B, C, U, V, Z and W are functions only of the three-dimensional radius r. In addition, the spherically symmetric $SU(2)$ gauge fields will be parametrized as (Witten ansatz):

3.3 $A = uT_3 dt + w\,(T_2 d\theta - T_1\sin\theta d\varphi) + T_3\cos\theta d\varphi,$

where u and w are functions also depending only on r.

We use the above expressions to compute the components of the tensors $F_{\mu\nu}^{i}$ and $F_{\mu\nu}^{ij}$. Their's non-null components are:

3.3a
$$F_{10}^0 = A' + g'UB, \quad F_{12}^2 = C + rC' - g'WB,$$

3.3b
$$F_{13}^3 = (C + rC' - g'ZB)\sin\theta, \quad F_{23}^3 = rC\cos\theta\,(1 - g'V),$$

and respectively:

3.4a
$$F_{10}^{01} = U' + 4g'\lambda^2 AB, \; F_{20}^{02} = g'(UW + 4\lambda^2 rAC),$$

3.4b
$$F_{30}^{03} = g'\sin\theta\,(UZ + 4\lambda^2 rAC), \quad F_{21}^{21} = W' - 4g'\lambda^2 rBC,$$

3.4c
$$F_{31}^{31} = (Z' - 4g'\lambda^2 rBC)\sin\theta, \quad F_{31}^{32} = V'\cos\theta,$$

3.4d
$$F_{32}^{31} = (Z - g'VW)\cos\theta, \; F_{32}^{23} = (V - g'ZW + 4g'\lambda^2 r^2 C^2)\sin\theta,$$

where A', C', U', V', W', and Z' denotes the derivatives with respect to the variable r. Analogously, we obtain the following non-nul components of the $SU(2)$ stress tensor $G_{\mu\nu}^\alpha$:

3.5a
$$G_{02}^1 = -uw, \; G_{13}^1 = -w'\sin\theta, \; G_{03}^2 = -uw\sin\theta,$$

3.5b
$$G_{12}^2 = -w', \; G_{01}^3 = -u', \; G_{23}^3 = (w^2 - 1)\sin\theta,$$

with $u' = \frac{du}{dr}$ and $w' = \frac{dw}{dr}$.

The integral action of our model is:

3.6
$$S_{EYM} = \int d^4x e \left\{ -\frac{1}{16\pi G}F - \frac{1}{4Kg''^2}Tr\,(T_\alpha T_\beta)\,G_{\mu\nu}^\alpha G^{\beta\mu\nu} \right\},$$

where $F = F_{\mu\nu}^{ij}\bar{e}_i^\mu \bar{e}_j^\nu$, $e = \det\left(e_\mu^i\right)$. We choose $Tr\,(T_\alpha T_\beta) = K\delta_{\alpha\beta}$; for $SU(2)$ group we have $T_a = \frac{1}{2}\tau_a$ (τ_a being the Pauli matrices) and then $K = \frac{1}{2}$. The gravitational constant G is the only dimensional quantity in action (the units $\hbar = c = 1$) are understood) and is connected with the coupling constant g'. Taking $\delta S_{EYM} = 0$ with respect to A_μ^α, e_μ^i and A_μ^{ij}, we obtain respectively the following field equations [9]:

3.7
$$\frac{1}{e}\partial_\mu\left(eG^{\alpha\mu\nu}\right) + \varepsilon^{\alpha\beta\gamma}A_\mu^\beta G^{\gamma\mu\nu} = 0,$$

3.8
$$F_\mu^i - \frac{1}{2}Fe_\mu^i = 8\pi G T_\mu^i,$$

where T_μ^i is the energy-momentum tensor of the $SU(2)$ gauge fields

3.9
$$T_\mu^i = \frac{1}{Kg''^2}\left(-G_{\mu\rho}^\alpha G_\alpha^{i\rho} + \frac{1}{4}e_\mu^i G_{\rho\lambda}^\alpha G_\alpha^{\rho\lambda}\right),$$

and

3.10 $$F^i_{\mu\nu} = 0.$$

Then, introducing (3.4) and (3.5) into these field equations and imposing the constraints $C = 1$, $A = \frac{1}{B} = \sqrt{N}$ with $N(r)$ a new unknown positive defined function, we obtain:

3.11a $$(Nw')' = \frac{w(w^2 - 1)}{r^2} - \frac{u^2 w}{N},$$

3.11b $$(r^2 u')' = \frac{2uw^2}{N},$$

3.11c $$\frac{w'^2}{r} + \frac{u^2 w^2}{rN^2} = 0,$$

3.11d $$\frac{1}{2}(rN' + N - 1) + \frac{r^2 u'^2}{2} + \frac{u^2 w^2}{N} + Nw'^2 + \frac{(w^2 - 1)^2}{2r^2} + \frac{\Lambda r^2}{2} = 0,$$

where we used $K = \frac{1}{2}$ and $\frac{4\pi G}{g''} = 1$ units. These equations admit the following solution (Schwarzschild-Reissner-Nordstrom-de-Sitter) with a nontrivial gauge field describing colored black holes [ ]:

3.12a $$u(r) = u_0 + \frac{Q}{r}, w(r) = 0, N(r) = 1 - \frac{2m}{r} + \frac{Q^2 + 1}{r^2} - \frac{\Lambda}{3}r^2,$$

where $\Lambda = -12\lambda^2$ is the cosmological constant of the model. They admit also the self-dual solution (Schwarzschild):

3.12b $$u(r) = 0, w(r) = \pm 1, N(r) = 1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2.$$

But, the solution (3.12a) is not a self-dual one.

4 Renormalizability and the minimal gravitational action

In this section we study the scaling behavior of the one-loop partition function for a spinor field $\psi(x)$ in the presence of the gravitational field $e^i_\mu(x)$, $A^{ij}_\mu(x)$ in terms of the $\zeta-$function belonging to the appropriate matter fluctuation operators. The contribution of the spinor field $\psi(x)$ to the partition is given by the Grassmannn functional integral[13]:

4.1 $$Z_\psi[e, A] = \int D\overline{\psi}D\psi \exp\left(iS\left(\overline{\psi}, \psi; e, A\right)\right).$$

The integral of action S is already of the usual quadratic form we may perform the Grassmann integral and formally obtain:

4.2
$$Z_\psi [e, A] = \exp \left[\frac{1}{2} \log \det M_\psi (e, A) \right].$$

The hyperbolic fluctuation operator in the spinor case is obtained as usual by squaring the Dirac operator:

4.3
$$M_\psi (e, A) = -\nabla_i \nabla^i + \frac{i}{2} \Sigma^{ij} F_{ij} - m^2,$$

where $\nabla_i = \bar{e}_i^\mu \nabla_\mu$ and $F_{ij} = \frac{i}{4} F_{ij}^{mn} \Sigma_{mn}$, with

4.4
$$F_{ij}^{mn} = F_{\mu\nu}^{mn} \bar{e}_i^\mu \bar{e}_j^\nu.$$

The tensor $F_{\mu\nu}^{mn}$ is obtained from Eq. (2.8) considering $\lambda = 0$; the cosmological constant $\Lambda = -12\lambda^2$ will be introduced finally as a term into the integral of the action [see Eq. (4.11)]. The spinor contribution to the partition function normalized at scale μ becomes then:

4.5
$$Z_\psi [\mu; e, A] = \exp \left[-\frac{1}{2} \zeta' (0; \mu; M_\psi (e, A)) \right].$$

If we change the scale $\widetilde{\mu} = \lambda\mu$, then the partition function transforms as:

4.6
$$Z_\psi [\widetilde{\mu}; e, A] = Z_\psi [\mu; e, A] \exp \left[- \log \lambda \zeta (0; \mu; M_\psi (e, A)) \right].$$

Renormalizability of any theory including dynamical gauge fields requires now at least that these anomalous contributions, which are local polynomials in e_μ^i and A_μ^{ij} and their derivatives, may be absorbed in the classical action for the gauge fields e_μ^i and A_μ^{ij}. Hence, to determine explicitly a minimal gauge field dynamics consistent with renormalizability we finally have to evaluate the corresponding ζ-function. To evaluate $\zeta (0; \mu; M)$ we can use the following representation:

4.7
$$\zeta (u; \mu; M) = \frac{i\mu^{2u}}{\Gamma (u)} \int_0^\infty ds \, (is)^{u-1} Tr e^{-isM}.$$

where $\Gamma (u)$ is the gamma function, and expand the "heat kernel"

4.8
$$Tr e^{-isM} |_{s \longrightarrow 0} = \frac{i}{(4\pi is)^{d/2}} \sum_{k=0}^\infty (is)^k \int d^D x Tr \, c_k (x).$$

Performing the s-integration in (4.7) we obtain the contribution for $k = \frac{D}{2}$:

4.9
$$\zeta (0; \mu; M) = \frac{i}{(4\pi)^{d/2}} \int d^D x \, e Tr \, [c_{D/2} (x)].$$

Therefore, we have to calculate the coefficient $Tr[c_{D/2} (x)]$ in order to obtain the value of $\zeta (0; \mu; M_\psi (e, A))$ which finally yields the anomalous term in (4.6). We obtain:

4.10
$$Tr \, [c_2] = \frac{1}{30} \nabla_i \nabla^i F^{ij}{}_{ij} + \frac{1}{72} F_{ij}{}^{ij} F_{rs}{}^{rs}$$
$$- \frac{1}{360} F_{ijrs} F^{ijrs} - \frac{1}{45} F_{ij}{}^i{}_s F_r{}^{jrs} + \frac{1}{3} m^2 F^{ij}{}_{ij} + 2m^4.$$

We explicitly obtained the anomalous contribution to the rescaled partition function as local DS gauge invariant polynomials in the fields $e^i_\mu(x)$ and $A^{ij}_\mu(x) = -A^{ji}_\mu(x)$. The anomalous contributions are present in any classical gauge field dynamics consistent with renormalizability of the matter sector. Hence, we are finally led to construct a minimal action for the gauge fields just in terms of these DS gauge invariant polynomials. In our case, if we restrict ourselves to the contributions of second order in the derivatives $O(\partial^2)$ we obtain as minimal classical action to this order:

4.11

$$S_G(e) = \int d^4x \; e \left[\Lambda - \frac{1}{k^2} F^{ij}_{\;\;ij} + \alpha_1 F_{ij}^{\;\;ij} F_{\;\;rs}^{rs} + \alpha_2 F_{ijrs} F^{ijrs} + \alpha_3 F_{ij}^{\;\;i}_{\;\;s} F_r^{\;\;jrs} \right].$$

The couplings k, α_1, α_2, α_3 and the constant Λ obtain again contributions from the one-loop scale anomalies which have been determined above. We emphasize that S_G is an action for the gauge fields defined on the Minkowski space-time (M_4, η) and is invariant on one hand under local DS gauge transformations, and on the other hand under the global Poincare transformations, reflecting the symmetries of the underlying space-time.

4.1   The analytical program

All the calculations in Section 3 have been performed using an analytical program working on the MAPLE platform. The computer language of MAPLE includes facilities for interactive algebra, calculus, discrete mathematics, graphics, numerical computation, and many other areas of mathematics. It provides also a unique environment for rapid development of mathematical programs using its vast library of built-in functions and operations.

We used the GRTensorII [16] which is a package for the calculation and manipulation of tensor components and related objects. In GRTensorII, when the goal is the calculation of components and indexed objects (in particular tensors) or the defining new tensors, first of all, we must specify the space geometry. The simplest way to specify this geometry is to use the **makeg( )** facility. This function can be used to enter all the information needed to specify a coordinate metric (a $n \times n$ - dimensional 2-tensor) or a basis (a set of n linearly independent vectors related by a user-defined inner product). The metrics created can be saved to ASCII files. These files can be then loaded into GRTensorII using either the **qload( )** or **grload( )** commands. For example, in our model, we used the spherical symmetric Minkowski metric.

Our analytical program allows to calculate: the components $G^\alpha_{\mu\nu}$ $F^{ij}_{\mu\nu}$ and $F^i_{\mu\nu}$ of the strength tensor fields corresponding, respectively, to the $SU(2)$ and $SO(1,4)$ groups, the components T^i_μ of the energy-momentum tensor, and the field equations for the gauge potentials.

Below we list the part of program which allows to define and calculate the previously specified quantities.

Program "GAUGE THEORY.mws"

>restart: grtw( ):
>grload(minkowski,'C:/maple/sferice.mpl');
>grdef('ev{^i miu}'); grcalc(ev(up,dn));
>grdef('evinv(i ^miu}'); grcalc(evinv(dn,up));

```
>grdef{'A{^i ^j miu niu}'); grcalc(A(up,up,dn,dn));
>grdef('A1{miu}:= [0, 0, 0, -w(r)*sin(theta)]'):
 grdef('A2{miu}:= [0, 0, w(r), 0]'):
>grdef('A3{miu}:= [u(r), 0, 0, cos(theta)]'):
>grdef('G1{[i j]}:=2*(A1{[j,i]}+A2{[i}*A3{j]})');
>grdef('G2{[i j]}:=2*(A2{[j,i]}+A3{[i}*A1{j]})');
>grdef('G3{[i j]}:= 2*(A3{[j,i]}+A1{[i}*A2{j]})');
>grcalc(G1(dn,dn),G2(dn,dn),G3(dn,dn));
>gralter(G1(dn,dn),G2(dn,dn),G3(dn,dn),1,3,5,6,10):
>grdef('F{^i ^j miu niu}:=A{^i ^j niu, miu}- A{^i ^j miu, niu} +
 (A{^i ^s miu}*A{^m ^j niu} - A{^i ^s niu}*A{^m ^j miu})*
 eta1{s m}-4*lambda^2*(ev{^i miu}*ev{^j niu}-
ev{^i niu}*ev{^j miu})'); grcalc(F(up,up,dn,dn));
>grdef('gb{miu niu}:=eta1{a b}*ev{^a miu}*ev{^b niu}');
>grcalc(gb(dn,dn,));
>grdef('gbinv{^miu ^niu}:=eta1inv{^a ^b}*evinv{a ^miu}*
evinv{b ^niu}'); grcalc(gbinv(up,up));
> grdef('Gb1{^miu ^niu}:=gbinv{^miu ^rho}*gbinv{^niu ^sigma}*
 G1{rho sigma}'); grcalc(Gb1(up,up));
>grdef('Gb2{^miu ^niu}:=gbinv{^miu ^rho}*gbinv{^niu ^sigma}*
 G2{rho sigma}'); grcalc(Gb2(up,up));
>grdef('Gb3{^miu ^niu}:=gbinv{^miu ^rho}*gbinv{^niu ^sigma}*
 G3{rho sigma}'); grcalc(Gb3(up,up));
>grdef('detev:=r^2*sin(theta)'); grcalc(detev);
>grdef('Gbb1{^miu ^niu}:=detev*Gb1{^miu ^niu}'); grcalc(Gbb1(up,up));
>grdef('Gbb2{^miu ^niu}:=detev*Gb2{^miu ^niu}'); grcalc(Gbb2(up,up));
>grdef('Gbb3{^miu ^niu}:=detev*Gb3{^miu ^niu}'); grcalc(Gbb3(up,up));
>grdef('EQ1{^niu}:=(1/detev)*Gbb1{^miu ^niu,miu}+
 A2{miu}*Gb3{^miu ^niu}-A3{miu}*Gb2{^miu ^niu}');
>grcalc(EQ1(up)); grdisplay(_);
>grdef('EQ2{^niu}:=(1/detev)*Gbb2{^miu ^niu,miu}+
 A3{miu}*Gb1{^miu ^niu}-A1{miu}*Gb3{^miu ^niu}');
>grcalc(EQ2(up)); grdisplay(_);
>grdef('EQ3{^niu}:=(1/detev)*Gbb3{^miu ^niu,miu}+
 A1{miu}*Gb2{^miu ^niu}-A2{miu}*Gb1{^miu ^niu}');
>grcalc(EQ3(up)); grdisplay(_);
>grdef('T{^i miu}:=(1/(4*pi*G))*(-G1{miu rho}*ev{^i niu}*
 Gb1{^niu ^rho}-G2{miu rho}*ev{^i niu}*Gb2{^niu ^rho}-
 G3{miu rho}*ev{^i niu}*Gb3{ ^niu ^rho}+1/4*ev{^i miu}*
 (G1{rho lambda}*Gb1{^rho ^lambda}+G2{rho lambda}*
 Gb2{^rho ^lambda}+G3{rho lambda}*Gb3{^rho ^lambda}))');
>grcalc(T(up,dn)); grdisplay(_);
>grdef('F{^i miu}:=F{^i ^j miu niu}*evinv{j ^niu}');
>grcalc(F(up,dn)); grdisplay(_);
>grdef('F:=F{^i ^j miu niu}*evinv{i ^miu}*evinv{j ^niu}');
>grcalc(F); grdisplay(_);
>grdef('EQ{^i miu}:=F{^i miu}-1/2*F*ev{^i miu}-8*pi*G*T{^i miu}');
>grcalc(EQ(up,dn)); grdisplay(_);
```

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