Geometry of Lagrangians and semisprays on Lie algebroids

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Abstract

One considers a regular Lagrangian L on the total space of a Lie algebroid and one associates to it a semispray suggested by the form of the Euler -Lagrange equations established by A. Weinstein, [5]. Some properties of this semispray are pointed out.

Mathematics Subject Classification: 53C60, 53C07.
Key words: regular Lagrangian, Euler-Lagrange equations, semisprays, Lie algebroids.

1 Introduction

In a paper appeared in 1996,[5], Alan Weinstein proposed a Lagrangian formalism for Lie algebroids. This is general enough to include several Lagrangian formalisms as those on tangent bundles, on tangent subbundles and on Lie algebras. He obtains the Euler - Lagrange equations using the Poisson structure on the dual of the given Lie algebroid and the Legendre transformation defined by a regular Lagrangian on it. He also defines a notion of semispray. Later on, E. Martinez,[3], develops a Lagrangian formalism for Lie algebroids that is similar to Klein's formalism,[2]. He mainly uses a vector bundle which replaces the double tangent bundle from the usual case. A notion of semispray appears in this setting,too.

In this paper we are mainly dealing with the notion of semipray in A. Weinstein' sense. In Section 2 we recall necessary facts from the theory of vector bundles and establish the notations following the monograph [4].

Section 3 is devoted to semisprays on Lie algebroids. We give a definition that is a direct generalization of the one used in tangent bundle case and we prove that this is equivalent with the definition given by A. Weinstein,[5]. A local characterization is also provided. Three invariants are associated to any semispray.

In Section 4 we show that any regular Lagrangian on a Lie algebroid induces a semispray. This is done on a direct way: the Euler - Lagrange equations obtained by A. Weinstein suggest the form of the local coefficients of a semispray and by a

The Fifth Conference of Balkan Society of Geometers, Aug. 29 - Sept. 2, 2005, Mangalia, Romania;
BSG Proceedings 13, Geometry Balkan Press pp. 10-17.

direct calculation we checked that those coefficients are the appropriate ones. Some examples are pointed out.

2 Vector bundles

Let $\xi = (E, \pi, M)$ be a vector bundle of rank m. Here E and M are smooth i.e. C^∞ manifolds with $\dim M = n$, $\dim E = n + m$, and $\pi : E \to M$ is a smooth submersion. The fibres $E_x = \pi^{-1}(x)$, $x \in M$ are linear spaces of dimension m which are isomorphic with the type fibre $\mathbb{R}^m$.

Let $\{(U_\alpha, \psi_\alpha)\}_{\alpha \in A}$ be an atlas on M. A vector bundle atlas is $\{(U_\alpha, \varphi_\alpha, \mathbb{R}^m)\}_{\alpha \in A}$ with the bijections $\varphi_\alpha : \pi^{-1}(U_\alpha) \to U_\alpha \times \mathbb{R}^m$ in the form $\varphi_\alpha(u) = (\pi(u), \varphi_{\alpha,\pi(u)})$, where $\varphi_{\alpha,\pi(u)} : E_{\pi(u)} \to \mathbb{R}^m$ is a bijection. The given atlas on M and a vector bundle atlas provide an atlas $\{(\pi^{-1}(U_\alpha), \Phi_\alpha)\}_{\alpha \in A}$ on E.
Here $\Phi_\alpha : \pi^{-1}(U_\alpha) \to \phi_\alpha(U_\alpha) \times \mathbb{R}^m$ is the bijection given by $\phi_\alpha(u) = (\psi_\alpha(\pi(u)), \varphi_{\alpha,\pi(u)}(u))$. For $x \in M$, we put $\psi_\alpha(x) = (x^i) \in \mathbb{R}^n$ and if (U_β, ψ_β) is another local chart such that $x \in U_\alpha \cap U_\beta \neq \phi$, we set $\psi_\beta(x) = \widetilde{x}^i$ and then $\psi_\beta \circ \psi_\alpha^{-1}$ has the form

$$\widetilde{x}^i = \widetilde{x}^i(x^1, \cdots, x^n), \ \text{rank}\left(\frac{\partial \widetilde{x}^i}{\partial x^j}\right) = n. \tag{1.1}$$

Let (e_a) be the canonical basis of $\mathbb{R}^m$. Then $\varphi_{\alpha,x}^{-1}(e_a) := \varepsilon_a(x)$ is a basis of E_x and $u \in E_x$ has the form $u = y^a \varepsilon_a(x)$.

We take (x^i, y^a) as coordinates on E. For the bundle chart $(U_\beta, \varphi_\beta, \mathbb{R}^m)$ we put $\varphi_{\beta,x}^{-1}(e_a) = \widetilde{\varepsilon}_a(x)$ and then $u = \widetilde{y}^a \widetilde{\varepsilon}_a(x)$. If we set $\varepsilon_a(x) = M_a^b(x)\widetilde{\varepsilon}_b$ with $\text{rank}(M_a^b(x)) = m$ it follows that $\widetilde{y}^a = M_b^a(x) y^b$. Thus the mapping $\Phi_\beta \circ \Phi_\alpha^{-1}$ has the form

$$\begin{aligned} &\widetilde{x}^i = \widetilde{x}^i(x^1, \cdots, x^n), \text{rank}\left(\frac{\partial \widetilde{x}^i}{\partial x^j}\right) = n \\ &\widetilde{y}^a = M_b^a(x) y^b, \text{rank}(M_b^a(x)) = m. \end{aligned} \tag{1.2}$$

The indices $i, j, k, ...$ and $a, b, c...$ will take the values $1, 2, ...n$ and $1, 2, ...m$, respectively. The Einstein convention on summation will be used.

We denote by $\mathcal{F}(M), \mathcal{F}(E)$ the ring of real functions on M and E respectively, and by $\chi(M)$, respectively $\Gamma(E), \chi(E)$ the module of sections of the tangent bundle of M, respectively of the bundle ξ and of the tangent bundle of E. On U_α, the vector fields $\left(\partial_k := \dfrac{\partial}{\partial x^k}\right)$ provide a local basis for $\chi(U_\alpha)$. The sections $\varepsilon_a : U_a \to \pi^{-1}(U_\alpha)$, $\varepsilon_a(x) = \varphi_{\alpha,x}^{-1}(e_a)$ provide a basis for $\Gamma(\pi^{-1}(U_\alpha))$ and a section $A : U_\alpha \to \pi^{-1}(U_\alpha)$ will take the form $A(x) = A^a(x)\varepsilon_a(x)$, $x \in U_\alpha$.

Let $\xi^* = (E^*, p^*, M)$ be the dual of the vector bundle ξ. We may also consider the tensor bundle $T_s^r(E)$ over E. The set of sections $\Gamma(T_s^r(E))$ are $\mathcal{F}(M)-$modules for any natural numbers r, s. On the sum $\oplus_{r,s}\Gamma(T_s^r(E))$ a tensor product can be defined and one gets a tensor algebra $T(E)$. For the tangent bundle (TM, τ, M) this reduces to the tensor algebra of the manifold M. The tensor algebra of the manifold E could be also involved. Its elements are sections in $\mathcal{T}_s^r(TE)$. The tensorial algebra of E contains the subset of $d-$tensor fields on E. For a general definition of these tensor fields we refer to [4], Ch. III. Shortly, these tensor fields are defined by components depending on (x^i, y^a) and transforming by a change of coordinates as tensors but with the matrices $\left(\dfrac{\partial \widetilde{x}^i}{\partial x^j}\right)$ and $(M^a_b(x))$ and their inverses, only. Notice that in the law of transformation of a tensor field on E could appear also the matrix $\left(\dfrac{\partial M^a_b(x)}{\partial x^i} y^b\right)$.

A large class of examples is provided by the sections in the vertical bundle over E. We recall that the vertical bundle $VE \to E$ is the union of the fibres $V_u E = \ker \pi_{*,u}$ over $u \in E$, where $\pi_{*,u}$ is the differential of π. A basis of local section of $VE \to E$ is given by $\left(\dfrac{\partial}{\partial y^a}\Big|_u \right)$ and its dual is $dy^a|_u$. The local components of any element in $\Gamma(T^r_s(VE))$, transform under a change of coordinates on E with the matrix $(M^a_b(x))$ and its inverse (W^a_b). We call such an element a vertical tensor field.

Now if $L : E \to M$ is a smooth function on E (called usually a Lagrangian) then it is easy to check that functions $\dfrac{\partial L}{\partial y^a}$, $g_{ab} = \dfrac{1}{2}\dfrac{\partial^2 L}{\partial y^a \partial y^b}$, $C_{abc} = \dfrac{1}{2}\dfrac{\partial g_{ab}}{\partial y^c}$ define vertical tensor fields of covariance indicated by the position and number of indices.

3 Semisprays for Lie algebroids

A vector bundle $\xi = (E, \pi, M)$ is called a Lie algebroid if it has the following properties:

1. The space of sections $\Gamma(\xi)$ is endowed with a Lie algebra structure $[,]$;
2. There exists a bundle map $\rho : E \to TM$ (called the *anchor map*) which induces a Lie algebra homomorphism (also denoted by ρ) from $\Gamma(\xi)$ to $\chi(M)$.
3. For any smooth functions f on M and any sections $s_1, s_2 \in \Gamma(\xi)$ the following identity is satisfied
$$[s_1, f s_2] = f[s_1, s_2] + (\rho(s_1)f)s_2.$$

Locally, we set

$$\rho(s_a) = \rho^i_a \frac{\partial}{\partial x^i},\ [\varepsilon_a, \varepsilon_b] = L^c_{ab} s_c, \tag{3.1}$$

A change of local charts implies

$$\widetilde{\rho}^i_a = W^b_a \rho^j_b \frac{\partial \widetilde{x}^i}{\partial x^j}, \tag{3.2}$$

where W^b_a is the inverse of the matrix (M^a_b).

Examples of Lie algebroids: the tangent bundle $\tau : TM \to M$ with $\rho =$identity, any integrable subbundle of TM with the inclusion as anchor map, TP/G for $P(M, G)$ a $G-$principal bundle, see [5].

For a function f on M one defines its vertical lift f^v on E by $f^v(u) = f(\pi(u))$ and its complete lift f^c on E by $f^c(u) = \rho^i_a y^a \frac{\partial f}{\partial x^i}$ for $u = (x, y)$ in E. If $A = A^a(x)\varepsilon_a$ is a section in ξ, the vertical lift A^v is a vector field on E defined by $A^v(x,y) = A^a(x)\frac{\partial}{\partial y^a}$ and the complete lift A^c is a vector field on E defined by

$$A^c(x,y) = A^a \rho_a^i \frac{\partial}{\partial x^i} + (\rho_b^i \frac{\partial A^a}{\partial x^i} - A^d L_{db}^a) y^b \frac{\partial}{\partial y^a}.$$

In particular, $\varepsilon_a^v = \frac{\partial}{\partial y^a}, \varepsilon_a^c = \rho_a^i \frac{\partial}{\partial x^i} - L_{ab}^d y^b \frac{\partial}{\partial y^d}$.

A semispray S for the tangent bundle $\tau : TM \to M$ is a vector field on TM which at the same time is a section in the vector bundle $\tau_* : TTM \to TM$, that is we have $\tau_{TM}(S(u)) = u$ and $\tau_{*,u}(S(u)) = u$, $\forall u \in TM$, where τ_{TM} is the vector bundle projection $TTM \to TM$. It follows that $\tau_{*,u}(S(u)) = \tau_{TM}(S(u))$, $\forall u \in TM$.

This equation suggests the following

Definition 3.1. *Let $\xi = (E, \rho, M)$ be a Lie algebroid with the anchor ρ. A vector field S on E will be called a semispray if*

$$\pi_{*,u}(S(u)) = (\rho \circ \tau_E)(S(u)), \ \forall u \in E \tag{3.3}$$

where $\tau_E : TE \to E$ is the natural projection.

Let $c : I \to M$, $I \subseteq \mathbb{R}$ be a curve on M and let $\widetilde{c} : I \to E$ be any curve on E such that $\pi \circ \widetilde{c} = c$. Denote by $\dot{\widetilde{c}}$ the vector field that is tangent to $\widetilde{c}$.

Definition 3.2. *We say that $\widetilde{c}$ is* **admissible** *if*

$$\pi_*(\dot{\widetilde{c}}) = \rho(\widetilde{c}).$$

In local charts on M and E, we have $c(t) = (x^i(t))$, $\widetilde{c}(t) = (x^i(t), y^a(t))$ and $\dot{\widetilde{c}}(t) = \frac{dx^i}{dt}\frac{\partial}{\partial x^i} + \frac{dy^a}{dt}\frac{\partial}{\partial y^a}$, $t \in I$.

It results

Lemma 3.1. *The curve $\widetilde{c}$ is admissible if and only if*

$$\frac{dx^i}{dt}(t) = \rho_a^i(x(t))y^a(t), \ \forall t \in I. \tag{3.4}$$

Again in local charts, let be $S = X^i \frac{\partial}{\partial x^i} + Y^a \frac{\partial}{\partial y^a}$ a vector field on E.

This is a semispray if and only if

$$X^i(x,y) = \rho_a^i(x) y^a. \tag{3.5}$$

Thus the coordinates $(Y^a(x,y))$ are not determined. We set for convenience $Y^a = -2G^a$. Furthermore, under a change of coordinates $(x^i, y^u) \to (\widetilde{x}^i, \widetilde{y}^a)$, the coordinates $(X^i), (G^a)$ have to change as follows:

$$\widetilde{X}^i = \frac{\partial \widetilde{x}^i}{\partial x^j}(x) X^j, \tag{3.6}$$

$$\widetilde{G}^a = M_b^a G^b - \frac{1}{2}\frac{\partial M_b^a}{\partial x^i} y^b \rho_c^i y^c. \tag{3.7}$$

Using (3.2) one easily sees that the coordinates $(X^i(x,y))$ given by (3.5) verify (3.6).

Concluding, we have

Theorem 3.1. *A vector field* $S = (\rho^i_a y^a)\dfrac{\partial}{\partial x^i} - 2G^a\dfrac{\partial}{\partial y^a}$ *on* E *is a semispray if and only if the coordinates* (G^a) *transform by (3.7).*

The integral curves of S are given by the system of differential equations

$$\frac{dx^i}{dt} = \rho^i_a(x)y^a,\ \frac{dy^a}{dt} + 2G^a(x,y) = 0. \tag{3.8}$$

It comes out these curves are all admissible. The converse is also true, that is we have

Theorem 3.2. *A vector field on* E *is a semispray if and only if all its integral curves are admissible.*

Remark 3.1. The characterization of a semispray provided by the Theorem 3.2 was taken by A. Weinstein ,[5], as definition for a semispray on E.

Remark 3.2. (i) Let us assume that $\rho = 0$. Then the admissible curves are all curves from the fibre E_{x_0}, $x_0(x^i_0) \in M$. The integral curves of a semispray S are given by the equations $\dfrac{dy^a}{dt} + 2G^a(x_0, y) = 0$.

(ii)The system of equations (3.8) is no longer equivalent with a second order differential equations as it happens for TM. Thus the term of "second order differential equations" used sometimes for a semispray is no longer appropriate.

(iii) Let D a distribution on M. We regard it as a subbundle of TM and so we may view it as a Lie algebroid with the natural inclusion as anchor map. Using a local basis on D one can see that the admissible curves are those that are tangent to the distribution D. For details we refer to [1].

Let $\widehat{S}$ be another semispray on E. Then $\widehat{S} = (\rho^i_a y^a)\dfrac{\partial}{\partial x^i} - 2\widehat{G}^a\dfrac{\partial}{\partial y^a}$, where the functions $(\widehat{G}^a(x,y))$ have to satisfy (3.7) under a change of coordinates on E. It follows that $\widehat{S} - S = 2(G^a - \widehat{G}^a)\dfrac{\partial}{\partial y^a}$ and the functions $D^a = G^a - \widehat{G}^a$ transform by the rule

$$\widehat{D}^a = M^a_b D^b. \tag{3.9}$$

So we have proved

Theorem 3.3. *Any two semisprays on* E *differ by a vertical vector field on* E.

A different point of view on semisprays for algebroids was proposed by E.Martinez,[3]. It can be shortly described as follows.

Let $\mathcal{L}^\pi E$ be the subset of $E \times TE$ defined by $\mathcal{L}^\pi E = \{(u,z)|\rho(u) = \pi_*(z)\}$ and denote by $\pi_L : \mathcal{L}^\pi E \longrightarrow E$ the mapping given by $\pi_L(u,z) = \tau_E(z)$. Then $(\mathcal{L}^\pi E, \pi_L, E)$ is a vector bundle over E of rank $2m$.One proves that this vector bundle is also a Lie algebroid.

One associates to a section A of ξ the vertical lift A^V and the complete lift A^C as sections of $\pi_L : \mathcal{L}^\pi E \longrightarrow E$ given by

$$A^V(u) = (0, A^v(u)), A^C(u) = (A(\pi(u)), A^c(u)), u \in E.$$

If $\{s_a\}$ is a local basis of $\Gamma(E))$, then $\{s^V_a, s^C_s\}$ is a local basis for $\Gamma(\mathcal{L}^\pi E)$.

The vector bundle $(\mathcal{L}^\pi E, \pi_L, E)$ admits a canonical section C called the *Liouville or Euler section* defined by $C(u) = (o, y^a \frac{\partial}{\partial y^a})$ for $u = y^a \varepsilon_a \in E$. A section J of the vector bundle $\mathcal{L}^\pi E \bigotimes (\mathcal{L}^\pi E)^* \longrightarrow E$ characterized by the conditions $J(A^V) = 0, J(A^C) = A^V, A \in \Gamma E$ is called the *vertical endomorphism.* We have that $J^2 = 0$. A section S of the vector bundle $(\mathcal{L}^\pi E, \pi_L, E)$ is said to be a semispray if it satisfies the condition $JS = C$. This definition is equivalent with the preceding one. Indeed, in local coordinates if we set $S = A^a \varepsilon^C_a + S^a \varepsilon^V_a$, the condition $JS = C$ gives $A^a = y^a$ and so $S = y^a(\rho^i_a \frac{\partial}{\partial x^i} - L^c_{ab} y^b \frac{\partial}{\partial y^c}) + S^a \frac{\partial}{\partial y^a} = y^a \rho^i_a \frac{\partial}{\partial x^i} + S^a \frac{\partial}{\partial y^a}$ since $L^c_{ab} y^a y^b = 0$.

For a semispray on TM, a case when this is equivalent with a system of second order differential equations (SODE), there exists a way to find geometric invariants that to determine, up to a change of coordinates, the solutions of the system.

This way led to a KCC-theory named so as after Kosambi, Cartan and Chern.

The KCC-theory apparently does not work for semisprays on Lie algebroids. However, at least formally we can associate to a semispray $S = (\rho^i_a y^a)\dfrac{\partial}{\partial x^i} - 2G^a(x,y)\dfrac{\partial}{\partial y^a}$, the following invariants:

$$\zeta^a = 2G^a - \frac{\partial G^a}{\partial y^b} y^b, \tag{3.10}$$

$$\Xi^a = \frac{\partial G^a}{\partial y^b} - \frac{\partial G^a}{\partial y^b \partial y^c} y^c, \tag{3.11}$$

$$\Gamma^a = 2G^a - 2\frac{\partial G^a}{\partial y^b} y^b + \frac{\partial G^a}{\partial y^b \partial y^c} y^b y^c. \tag{3.12}$$

Indeed, it is not difficult to check that all these sets of functions define vertical vector fields on E.

To find a complete list of such invariants could be a future task.

4 A semispray derived from a regular Lagrangian

Let $L : E \to R$ be a regular Lagrangian on the Lie algebroid $(E, [,], \rho)$, that is L is a smooth functions such that the matrix with the entries

$$g_{ab}(x,y) = \frac{1}{2} \frac{\partial^2 L}{\partial y^a \partial y^b}, \tag{4.1}$$

is of rank m.

In [5], one associates to L the Euler - Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial y^a}\right) = \rho^i_a \frac{\partial L}{\partial x^i} + L^c_{ba} y^b \frac{\partial L}{\partial y^c}, \tag{4.2}$$

for $c(t) = (x^i(t), y^a(t))$ an admissible curve.

Expanding the derivative in (4.2), using (4.1) and (3.4), we may put (4.2) in the form

$$\frac{dy^a}{dt} + 2G^a_L(x,y) = 0, \tag{4.3}$$

with the notation

$$G^a_L = \frac{1}{4} g^{ab} \left(\frac{\partial^2 L}{\partial y^b \partial x^i} \rho^i_c y^c - \rho^i_b \frac{\partial L}{\partial x^j} - L^c_{bd} y^d \frac{\partial L}{\partial y^c} \right). \tag{4.4}$$

We show that the function (G^a_L) verifies (3.7) under a change of coordinates on E.
We set

$$E_a = 4 g_{ab} G^b, \tag{4.5}$$

where

$$E_a = \frac{\partial^2 L}{\partial y^a \partial x^i} \rho^i_b y^b - \rho^i_a \frac{\partial L}{\partial x^i} - L^c_{ba} y^b \frac{\partial L}{\partial y^c}. \tag{4.6}$$

Then we use (3.2) as well as the following equations:

$$\frac{\partial L}{\partial x^i} = \frac{\partial L}{\partial \widetilde{x}^j} \frac{\partial \widetilde{x}^j}{\partial x^i} + \frac{\partial L}{\partial \widetilde{y}^a} \frac{\partial M^a_c}{\partial x^i} y^c$$

$$\frac{\partial^2 L}{\partial y^a \partial x^i} = M^b_a \left(\frac{\partial^2 L}{\partial y^b \partial \widetilde{x}^j} \frac{\partial \widetilde{x}^j}{\partial x^i} + 2\widetilde{g}_{db} \frac{\partial M^d_c}{\partial x^i} y^c \right) + \frac{\partial L}{\partial \widetilde{y}^d} \frac{\partial M^d_a}{\partial x^i}$$

$$L^c_{ab} M^e_c = M^c_a M^d_b \widetilde{L}^e_{cd} + \rho^k_a \frac{\partial M^e_b}{\partial x^k} - \rho^k_b \frac{\partial M^e_a}{\partial x^k}$$

in order to derive

$$E_a = M^b_a \widetilde{E}_b + 2 M^b_a \widetilde{g}_{bd} \frac{\partial M^d_c}{\partial x^i} y^c \rho^i_d y^d. \tag{4.7}$$

Using this in (4.5) one shows that $\widetilde{G}^a_L$ is related to G^a_L as in (3.7).
Thus we have proved

Theorem 4.1. *Let L be a regular Lagrangian on the Lie algebroid $(E, [,], \rho)$. Then L defines a semispray $S_L = (\rho^i_a y^a)\frac{\partial}{\partial x^i} - 2G^a_L(x,y)\frac{\partial}{\partial y^a}$, where the functions G^a_L are given by (4.4).*

Example 4.1. Let $g_{ab}(x)$ be the coefficients of a Riemannian metric in the Lie algebroid $(E, [,], \rho)$. Then

$$L(x,y) = g_{ab}(x) y^a y^b \tag{4.8}$$

is a regular Lagrangian on E. The semispray associated to it is determined by the functions

$$G^a = \frac{1}{2} g^{ab} \left(\frac{\partial g_{bc}}{\partial x^i} \rho^i_d - \frac{1}{2} \frac{\partial g_{cd}}{\partial x^i} \rho^i_b - L^e_{db} g_{ec} \right) y^c y^d. \tag{4.9}$$

Example 4.2. A more general example is provided by the regular Lagrangians which are homogeneous of degree 2 in (y^a). By the Euler theorem one obtains

$$L(x,y) = g_{ab}(x,y)y^a y^b, \tag{4.10}$$

where $(g_{ab}(x,y))$ are homogeneous functions of degree 0.

As $\frac{\partial}{\partial y^a}$ are homogeneous functions of degree 1 and the derivative with respect to (x^j) does not affect the degree of homogeneity, it results that the coefficients (G^a) from (4.4) are homogeneous of degree 2 in (y^a). This fact is equivalent with $\zeta^a = 0$ and so we have a meaning of the invariant ζ^a. The corresponding semispray is called a spray.

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