A computer aided study of field interactions

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Abstract

Gauge theory describe the interactions of fields and gives unified model both at classical and quantum levels. The basic physical quantity of such a theory is the tensor of gauge potentials describing the interacting fields. The tensorial calculus involve a great number of calculations, that allows computer implementation. Symbolic and numerical programs, like Maple, are appropriate from this point of view. In this paper we present some computer algebra procedures applied to the two models of gauge theory for gravitation. Firstly, we develop a model of Poincaré gauge self-dual theory and secondly, a deSitter gauge theory. Both models use the Minkowski space-time, endowed with spherical symmetry. In the first case the self-duality conditions are imposed and the equations for the gauge fields are obtained while, in the second case we compute the strength tensor. For these models we obtain analytical solutions. Also, we develop a method for obtaining solutions without singularities of the gauge field equations.

All the calculations, including the integration of the field equations, are performed using analytical procedures conceived in GRTensorII for MapleV. The program allows to compute (without using a metric) the strength tensor, the Riemann tensor, the Ricci tensor, the curvature scalar, the self-duality equations, the field equations and the integration of these equations.

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1 Introduction

The study of field interactions using gauge theories requires a great volume of symbolic calculations. For such a purpose, there are many versatile algebraic computing systems. The most important of these are MAPLE, MATHEMATICA, REDUCE, etc. In this paper we use the computer algebra language MAPLE, which is a comprehensive computer system for advanced mathematics. It includes facilities for interactive algebra, calculus, discrete mathematics, graphics, numerical computation, and many other areas of mathematics. It also provides a unique environment for rapid development of mathematical programs using its vast library of built-in functions and operations.

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On the MAPLE platform we use the package GRTensor [7]. GRTensor is a package for the calculation and manipulation of components of tensors and related objects. The program is designed to operate efficiently for a wide range of applications and allows the use of a number of different mathematical formalisms. The algorithms for this package are optimized for the individual formalisms and transformations between formalisms are simple and intuitive. Additionally, the package allows for customization and expansion with the ability to define new objects, user define algorithms, and add-on libraries. Especially, we use this property to define new objects. In designing the package, emphasis has also been placed on the interface allowing simple user input, as well as presenting readable output. The results of calculations are printed on the screen in the usual form.

Recently, many works have been given with intention to develop a gauge theory of gravitation [5]. Some authors consider the Poincaré group or deSitter group as "active" symmetry groups, i.e. acting on the space-time coordinates [1]. Other authors adopt the "passive" point of view when the space-time coordinates are not affected by group transformations [8, 11]. Only the fields change under the action of the symmetry group.

In this paper we adopt the second point of view to develop a deSitter (DS) gauge theory of gravitation and a Poincaré (P) self dual theory of gravitation over a spherical symmetric Minkowski space-time. Therefore we restrict ourselves to recast DS symmetry or P symmetry, and its consequences in the form of an inner symmetry. The coordinate system used to specify the space-time events is not affected anymore by DS or Poincaré transformations. In the particular case, when the constants of structure vanish, we can obtain the theory of electromagnetic field.

The Section 2 presents the facilities of GRTensorII for tensorial calculus. The Section 3 is devoted to the formulation of the Poincaré and deSitter gauge models on a spherical symmetric Minkowski space-time. The general expressions for the components $F^A_{\mu\nu}$ of the strength tensor of the gauge fields are obtained. In Section 4 we choose a model of theory for the Poincaré group, with spherical symmetry for potentials, which has four independent functions, each depending only the 3D radius r. We give also a part of the analytical program conceived by us in GRTensor and used to obtain the self-duality equations and field equations. Finally, an analitical solution of these equations are obtained. The Section 5 is devoted to the formulation a deSitter gauge model. The case of null torsion is considered and an analytical solution of Scwarzschild-deSitter type is given. The conclusion is that the deSitter group can be considered as a "passive" gauge symmetry group for gravitation. Therefore, the gravitation can be described by gauge potentials defined on a Minkowski space-time and we have not to use Riemann or Riemann-Cartan theories. The main part of the analytical program used for these calculations is given in this section. A method for obtaining solutions without singularities for gauge field equations is presented in Section 6.

The tensorial calculus involves a great number of calculations and sometime it is impossible to effectuate, them by hand. But, we can use the computer implementation for such a purpose. From this point of view, the symbolic programs, as Maple, are appropriate. In this paper all the calculations are performed using the GRTensorII computer algebra package, which runs within the MapleV environment.

2 GRTensorII, an ideal tool for work with tensors

GRTensorII is a computer algebra package for performing calculations in the general area of differential geometry. Its purpose is the calculation of tensor components on curved spacetimes specified in terms of a metric or a set of basis vectors. The package contains a library of standard definitions of a large number of commonly used curvature tensors. The standard object libraries are easily expandable by a facility for defining new tensors. Calculations can be carried out in spaces of arbitrary dimension, and in multiple spacetimes simultaneously. Though originally designed for use in the field of general relativity, GRTensorII is useful in many other fields. GRTensorII is not a stand alone package, but requires an algebraic engine. The program was originally developed for MapleV. GRTensorII runs with all versions of Maple, MapleV Release 3 to Maple 9. A limited version (GRTensorM) has been ported to Mathematica.

GRTensorII and related software and documentation are distributed free of charge as an aide for both research and teaching.

In GRTensor, when the goal is the calculation of components of indexed objects (in particular tensors) or the defining new tensors, first of all, we must to specify the space geometry. The simplest way to specify a space geometry is to use the `makeg()` facility. This function can be used to enter all information needed to specify a coordinate metric (a $n \times n$ dimensional 2-tensor) or basis (a set of n linearly independent vectors related by a user-defined inner product). The metrics created can be saved to ASCII files. These files can be loaded into GRTensor using either the `qload()` or `grload()` commands. For example, in our models we use the metric from relation (1) in the Section 3. After the first three commands from analytical program, we start the GRTensor package in a new MapleV session and we load the metric from file "spheric.mpl". On screen the metric is printed in usual form (like writing by hand).

```
restart:
grtw():
grload(minkowski, ‘c:/maple/spheric.mpl‘);
```

GRTensorII Version 1.77 (R5)
3 May 2000
Developed by Peter Musgrave, Denis Pollney and Kayll Lake
Copyright 1994-2000 by the authors.
Latest version available from: http://grtensor.phy.queensu.ca/
Default spacetime = minkowski
For the minkowski spacetime:
Coordinates
x(up)

$$x^a = [t, r, \theta, \phi]$$

Line element

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin(\theta)^2 d\phi^2$$

coordonate sferice, 4 dim.

The main command provided by GRTensor to calculate and operate on tensors with any given index configuration is `grcalc()`. Tensors are specified by their name and index configuration, using abbreviations dn and up to indicate covariant (down indices) and contravariant (up indices) indices. For instance, in program we use the command: `grcalc(Famn(up,dn,dn));` which requests the calculation of the tensor $F^a_{\mu\nu}$ from Eq. (6). Of course, before this command we must define this tensor.

In general, the commands which act on tensors take the form:

$$commandName(tensorSeq, [other\_Arguments])$$

Here *tensorSeq* is a sequence of GRTensor objects each of which has the following form:

$$tensorName(indexSeq).$$

For above example, the name tensor is Famn. The argument *indexSeq* is a sequence of names giving the configuration of the tensor. Available index type are: up, dn - covariant/contaravariant indices; pup, pdn - partial derivatives; cup, cdn - covariant derivative, etc. Thus Famn(up,dn,dn,pdn) refers to the objects:

$$F^a_{\mu\nu,\gamma} = \frac{\partial F^a_{\mu\nu}}{\partial x^\gamma}$$

where x^γ are the coordinates of metric.

The `grdef()` command is included to facilitate the specification of new tensor in a simple and natural manner. It allows tensors to be defined either as an equation in terms of previously defined tensors, or by manual entry of the components values. Inner and outer products of tensors, symmetrizations, and derivatives can all be specified as part of the tensor definitions. Furthermore, index symmetries of the newly defined tensor can be included.

There are two ways to define a new tensor using the command `grdef()`. The first method is to simply state the name of tensor, including the index structure and, eventual, the symmetrizations. For instance, the command:

```
grdef(‘eta1{(a b)}‘);
```

used in the analytical program define a tensor with the name eta1 whose component values are arbitrary and can be manually specified. Specifically, it creates a definition of a covariant two-index object, eta1. The indices are listed in curly braces, {}, and assigned the labels a and b. The tensor would be accessed as `eta1(dn,dn)` in commands such as `grcalc()`. The round braces ( ) indicate that this tensor is symmetric in enclosed indices. Symmetries among tensor indices can be used in calculation programs to significantly reduce the time it takes for a calculation by recognizing redundant components. If we define an antisymmetric object, we use the square braces [ ].

The second method for using the command `grdef()` provides a complete definition in term of previously defined tensors. For example, the tensor $F^a_{\mu\nu}$ from Eq. (6) is defined in the form:

```
grdef(‘Famn{^a miu niu}:=ev{^a niu, miu}-omega{^a ^b niu}*
          ev{^c niu}*eta1{b c}-ev{^a miu,niu}+
             omega{^a ^b miu}*eta1{b c}‘);
```

Here we have two classes of indices: covariant indices, specified by miu, niu, etc. and contravariant indices specified by ^a, ^b, etc.

Note that the summation over the range of an index is specified by repeating the index name, once in the covariant and once in the contravariant positions. In the above expression we have summation over index b and c.

Certainly commonly used tensors require extra information (in addition to the background geometry) in order to be calculated. For instance, consider the definitions of the 'electric' and 'magnetic' parts of the Weyl tensor [1]:

$$E_{ab} = C_{abcd} v^c v^d, \qquad H_{ab} = C^*_{abcd} v^c v^d.$$

In addition to the Weyl tensor C_{abcd} (which can be calculated directly from the metric), it is required the specification of a vector field v^a. In this case, assuming a single-index tensor (a vector field) v(up) had been defined, the electric and magnetic Weyl tensors would be referenced using:

`E[v](dn,dn),` and `H[v](dn,dn).`

A more general example is the d'Alambertian derivative operator, $\Box := \nabla^a \nabla_a$. This definition, in GRTensor, applies to a tensor with an arbitrary number of indices as argument. We could reference $\Box R_{abcd}$ using

```
Box[R(dn, dn, dn, dn)].
```

In GRTensor the operators can be used in grdef() just as they are used in calculations using grcalc(). The argument of operator is placed in square braces. Thus, the command:

```
grdef('X:=R{^a ^b}*Box[R{a b}]');
```

serve to define the scalar:

$$X = R^{ab} \Box R_{ab}.$$

A limitation on the use of operators, however, is that they can only be used on individual objects, not for functions of objects. To define, for instance, the object:

$$T_{ab} = \Box(R_{acdb} R^{cd}),$$

a two stage definition is needed. First we define an intermediate tensor:

```
grdef('Tint{a b}:=R{a c d b}*R{^c ^d}');
```

and afterthat:

```
grdef('T{a b}:=Box[Tint{a b}]');
```

GRTensor allows the definition of objects which depend on multiple background geometries. Such objects arise, for instance, when one considers the junction between two spacetimes described by different metrics. The objects which are to use alternate background geometries are indexed using angle braces, $\langle\ \rangle$. For example, in the following definition of tensor Dg, we use two different metrics:

```
grdef('Dg{a b} := g⟨1⟩{a b}-g⟨2⟩{a b}');
```

Here, the indices $\langle 1\rangle$ and $\langle 2\rangle$ in the objects g{a b} indicate that the components of g(dn, dn) should be taken from metrics specified by the user at calculation time. To calculate the object Dg we use the command `grcalc()` in the form:

```
grcalc(1=s1,2=s2,Dg(dn,dn));
```

In this example s1 and s2 are the name of spacetimes that have been previously loaded in the session.

3 Models of gauge theory

We will present two models of gauge theory: a self-dual theory of a Poincaré group and a theory of the deSitter group. For both models we use a 4-dimensional Minkowski space-time, endowed with spherical symmetry:

$$ds^2 = dt^2 - dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{3.1}$$

The groups P and DS are both 10-dimensional and theirs infinitesimal generators are denoted by P_a and $M_{ab} = -M_{ba}$, $a, b = 0, 1, 2, 3$ [4, 6]. In order to give a general formulation of the gauge theory for the Poincaré and deSitter group, we will denote these generators by X_A, $A = 1, 2, ..., 10$. Then, the equations of structure can be written under the general form:

$$[X_A, X_B] = i f^C_{AB} X_C, \tag{3.2}$$

where $f^C_{AB} = -\ f^C_{BA}$ are the constants of structure whose concrete expressions will be given below (see Eq.(3.5) and Eq.(3.6)).

Let us suppose now that the deSitter group DS and Poincaré groups are gauge groups for gravitation; corresponding, we introduce 10 gauge fields $h^A_\mu(x)$, $A = 1, 2, ..., 10$, $\mu = 0, 1, 2, 3$. Then, we construct the tensor of the gauge fields (strength tensor) $F_{\mu\nu} = F^A_{\mu\nu} X_A$ which takes its values in the Lie algebra of the deSitter group DS (Lie algebra-valued tensor). The components of this tensor are given by:

$$F^A_{\mu\nu} = \partial_\mu h^A_\nu - \partial_\nu h^A_\mu + f^A_{BC} h^B_\mu h^C_\nu. \tag{3.3}$$

We notice that if the constants of structure f^A_{BC} vanish, the tensor $F^A_{\mu\nu}$ become analogous to the tensor of the electromagnetic field and we will obtain a theory of electromagnetic field in the Minkowski space.

In order to write the constants of structure f^C_{AB}, we use the following notation for the index A:

$$A = \begin{cases} a = 0, 1, 2, 3, \\ [ab] = [01], [02], [03], [12], [13], [23]. \end{cases} \tag{3.4}$$

This means that we have $X_a = P_a$, $X_{[bc]} = M_{ab}$. We find the following expressions for the constants of structure :

$$\begin{aligned} f^a_{bc} &= f^{[ab]}_{c[de]} = f^a_{[bc][de]} = f^{[ab]}_{cd} = 0, \\ f^a_{b[cd]} &= -f^a_{[cd]b} = \frac{1}{2}\left(\eta_{bc}\delta^a_d - \eta_{bd}\delta^a_c\right), \\ f^{[ef]}_{[ab][cd]} &= \frac{1}{4}\left(\eta_{bc}\delta^e_a\delta^f_d - \eta_{ac}\delta^e_b\delta^f_d + \eta_{ad}\delta^e_b\delta^f_c - \eta_{bd}\delta^e_a\delta^f_c\right) - e \longleftrightarrow f, \end{aligned} \tag{3.5}$$

for the case of P group. In the case of deSitter group we have the same thing, with an exception:

$$f^{[ab]}_{cd} = 4\lambda^2\left(\delta^b_c\delta^a_d - \delta^a_c\delta^b_d\right) = -f^{[ab]}_{dc}, \tag{3.6}$$

where λ is a deformation parameter. When $\lambda \to 0$, we obtain the Poincaré-Lie algebra. Here η_{ab} =diag$(1, -1, -1, -1)$ is the Minkowski metric used on the Poincaré or deSitter group manifold and δ^a_b is the usual Kronecker symbol.

We will denote the gauge fields $h_\mu^A(x)$ by $e_\mu^a(x)$ (tetrad fields) if $A = a$ and by $\omega_\mu^{ab}(x) = -\omega_\mu^{ba}(x)$ (spin connection) if $A = [ab]$. Then, introducing the relations (3.5), respectively (3.5) and (3.6), into the definition (3.3), we find the following expressions of the strength tensor components:

$$F_{\mu\nu}^a = \partial_\mu e_\nu^a - \partial_\nu e_\mu^a + \left(\omega_\mu^{ab} e_\nu^c - \omega_\nu^{ab} e_\mu^c\right)\eta_{bc} = T_{\mu\nu}^a, \tag{3.7}$$

$$F_{\mu\nu}^{ab} = \partial_\mu \omega_\nu^{ab} - \partial_\nu \omega_\mu^{ab} + \left(\omega_\mu^{ac}\omega_\nu^{db} - \omega_\nu^{ac}\omega_\mu^{db}\right)\eta_{cd} = R_{\mu\nu}^{ab} \tag{3.8}$$

for P group and, in the case of DS group, the same relation for $F_{\mu\nu}^a$ but for $F_{\mu\nu}^{ab}$ we obtain.

$$F_{\mu\nu}^{ab} = \partial_\mu \omega_\nu^{ab} - \partial_\nu \omega_\mu^{ab} + \left(\omega_\mu^{ac}\omega_\nu^{db} - \omega_\nu^{ac}\omega_\mu^{db}\right)\eta_{cd} - 4\lambda^2\left(e_\mu^a e_\nu^b - e_\nu^a e_\mu^b\right). \tag{3.9}$$

These relation are calculated using the analytical program which is presented bellow. We remark the complexity of calculations, for example, here, the indices μ and ν are indices for space and a, b are indices for group. From the point of geometrical significance, the quantity $F_{\mu\nu}^a = T_{\mu\nu}^a$ is interpreted as the torsion tensor and $F_{\mu\nu}^{ab} = R_{\mu\nu}^{ab}$ as the curvature tensor of a Riemann-Cartan space-time defined by the gravitational gauge fields e_μ^a and ω_μ^{ab}.

In the following section, we present the two models and also the main parts of analytical programs used to perform all calculations.

4 Model I. Results and analytical program

We consider a particular form of spherically gauge fields of the Poincaré group given by the following ansatz:

$$\begin{aligned} e_\mu^0 &= (A, 0, 0, 0)\,, \quad e_\mu^1 = \left(0, \frac{1}{r^2 A}, 0, 0\right), \\ e_\mu^2 &= (0, 0, rC, 0)\,, \quad e_\mu^3 = (0, 0, 0, rC\sin\theta) \end{aligned} \tag{4.1}$$

and

$$\begin{aligned} \omega_\mu^{01} &= (U, 0, 0, 0)\,, \quad \omega_\mu^{02} = \omega_\mu^{03} = \omega_\mu^{12} = \omega_\mu^{13} = (0, 0, 0, 0)\,, \\ \omega_\mu^{23} &= (iV, 0, 0, \cos\theta)\,, \end{aligned} \tag{4.2}$$

where A, C, U and V are functions only of the $3D$ radius r. We use the above expressions to compute the components of the tensors $F_{\mu\nu}^a$ and $F_{\mu\nu}^{ab}$. We remark that we performed all the calculations using an analytical program conceived by us and given in this section.

In order to obtain a self-dual model, first of all, we consider the dual tensor ${}^*F_{\mu\nu}$ [5, 6, 9, 10]. In our case, the components of the dual tensor ${}^*F_{\mu\nu}$ are:

$${}^*F_{\mu\nu}^a = \frac{1}{2}\sqrt{-g}\varepsilon_{\mu\nu\rho\sigma}F^{a\rho\sigma} \qquad \text{and} \qquad {}^*\mathrm{F}_{\mu\nu}^{\mathrm{ab}} = \frac{1}{2}\sqrt{-\mathrm{g}}\varepsilon_{\mu\nu\rho\sigma}\mathrm{F}^{\mathrm{ab}\rho\sigma}, \tag{4.3}$$

where "$*$" is the Hodge dual map and $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita symbol of rank four, with $\varepsilon_{0123} = 1$.

The field equations are solved for arbitrary gauge fields, which satisfy the self-duality condition [5, 6, 9, 10]:

$$ {}^{*}F^{a}_{\mu\nu} = iF^{a}_{\mu\nu}, \qquad {}^{*}F^{ab}_{\mu\nu} = iF^{ab}_{\mu\nu}. \tag{4.4} $$

Now, we write the self-duality equations. The calculations are performed using the same analytical program and we obtain equations (4.5)-(4.6). For the first set of equations (4.4), we have obtained only two independent equations:

$$ A' + \frac{U}{r^2 A} = 0, \qquad rC' + (1 - rV)\, C = 0. \tag{4.5} $$

The second set of equations (4.4) reduces too only to the following two independent equations:

$$ U' = 0, \qquad V' = -\frac{1}{r^2}. \tag{4.6} $$

The equations (4.5) and (4.6) are the self-duality equations on the Minkowski space-time endowed with spherical symmetry and with the Poincaré group as gauge group. We remark that these equations are of the first order unlike the Y-M equations which are of the second order. From this reason, the search of solutions is easier. We remember that, for the Minkowski space-time, the solutions of self-duality equations are automatically solutions for the Y-M equations [1].

For example, the field equation in this case are [9]:

$$ E^{a\nu} \equiv \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} F^{a\mu\nu} \right) + f^{a}_{b[cd]}\, e^{b}_{\mu}\, F^{cd\mu\nu} + f^{a}_{[bc]d}\, \omega^{bc}_{\mu} F^{d\mu\nu} = 0, \tag{4.7} $$

$$ E^{ab\nu} \equiv \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} F^{ab\mu\nu} \right) + f^{[ab]}_{[cd][ef]}\, \omega^{cd}_{\mu} F^{ef\mu\nu} = 0. \tag{4.8} $$

¿From here, with the analytical program, we find only four independent equations, which are:

$$ \begin{aligned} & A'' + \frac{2A'}{r} + \frac{2U'}{r^2 A} - \frac{UA'}{r^2 A^2} = 0, \\ & r^2 C'' + 2rC' + \left(1 - r^2 V^2\right) C = 0, \\ & AU' - UA' - \frac{U^2}{r^2 A} = 0, \\ & rV'' + 2V' = 0. \end{aligned} \tag{4.9} $$

The equations (4.9) are the Y-M equations for the our ansatz. These equations can be obtained from the self-duality equations if we derive the first equation from (4.5) with respect to r, respectively the second equation, and using the equations (4.6). As a consequence, we proved that the solutions of the self-duality equations are also solutions for the Y-M equations.

If we define, as usually, a new metric $\overline{g}$ by the formula:

$$ \overline{g}_{\mu\nu} = e^{a}_{\mu} e^{b}_{v} \eta_{ab}, \tag{4.10} $$

then we obtain the following expression for the square of the line element:

$$d\sigma^2 = \left(a + \frac{2\alpha}{r}\right) dt^2 - \frac{1}{r^4 \left(a + \frac{2\alpha}{r}\right)} dr^2 - b^2 r^2 e^{2\beta r} \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right). \tag{4.11}$$

In particular, if we choose the constants of integration α, β, a and b equal to: $\alpha = -m$, $\beta = 0$, $a = 1$, $b = 1$, then the expression of the square of the line element can be considered as solution for the gravitational field in vacuum, created by a mass distribution m with spherical symmetry, which is valid only in the region $0 < r < 2m$.

The calculations of this paper, starting with the relations (3.7), were made using an analytical program written by us in the package GRTensor. We used GRTensorII version 1.77 which run on the MapleV platform. Because the group index a takes the values $0, 1, 2, 3$ and the spatial index μ takes the same values, $0, 1, 2, 3$, there have not appeared problems with the indices and we have not need to work with each component for the group index [6]. For the raising and lowering the group indices a we use the Minkowski (flat) metric $\eta_{ab} = (1, -1, -1, -1)$, whereas for the spatial indices μ we used the metric $g_{\mu\nu}$ given by the relation (1). The analytical program allows to calculate: the components of the strength tensor field $F^a_{\mu\nu}$, respectively $F^{ab}_{\mu\nu}$, the components ${}^*F^a_{\mu\nu}$, respectively ${}^*F^{ab}_{\mu\nu}$, the self-duality equations and the Y-M equations. In program we denoted $F^a_{\mu\nu}$ by Famn, ${}^*F^a_{\mu\nu}$ by Famndual, $F^{ab}_{\mu\nu}$ by Fabmn, ${}^*F^{ab}_{\mu\nu}$ by Fabmndual, the self-duality equations by SDamn and respectively by SDabmn, the Y-M equations by Ean respectively Eabmn. The metric $g_{\mu\nu}$ is loaded from the file "spheric.mpl" and the potentials e^a_μ, $\omega^{ab}_{\mu\nu}$ are introduced during of the running of program (by the command "grcalc"). Below, we list the part of program, which allows to define and to calculate the quantities previously specified.

Program "Poincare.mws"

```
restart: grtw():
    grload (minkowski, `c:/maple/spheric.mpl`);
    grdef(`ev{^a miu}`); grcalc(ev(up,dn));
    grdef(`omega{[^a ^b] miu}`); grcalc(omega(up,up,dn));
    grdef(`eta1{(a b)}`); grcalc(eta1(dn,dn));
    grdef(`Famn{^a miu niu}:= ev{^a niu, miu} - omega{^a ^b niu}*
            ev{^c niu}*eta1{b c}-ev{^a miu,niu} +
            omega{^a ^b miu}* eta1{b c}`);
    grcalc(Famn(up,dn,dn));  grdisplay(_);
    grdef(`Famndual{^a miu niu}:=1/2*r^2*sin(theta)*
             LevCS{miu niu rho sigma}*g{^rho ^c}*g{^sigma ^d}
             *Famn{^a c d}`);
    grdef(`SDamn{^a miu niu}:=Famndual{^a miu niu}-
             I*Famn{^a miu niu}`);
    grcalc(Famndual(up,dn,dn), SDamn(up,dn,dn)); grdisplay(_);
    grdef{`Fabmn{^a ^b miu niu}:=omega{^a ^b niu, miu}-
          omega{^a ^b miu, niu}+ (omega{^a ^c miu}*omega{^d ^b niu}-
          omega{^a ^c niu}*omega{^d ^b miu})*eta1{c d}`);
    grcalc{Fabmn(up,up,dn,dn)); grdisplay(_);
    grdef(`Fabmndual{^a ^b miu niu}:=1/2*r^2 sin(theta)*
             LevCS{miu niu rho sigma}*g{^rho ^gamma}*g{^sigma ^tau}*
```

```
        Fabmn{^a ^b gamma tau}`);
grdef(`SDabmn{^a ^b miu niu}:=Fabmndual{^a ^b miu niu}-
        I*Fabmn{^a miu niu}`);
grcalc(SDabmn(up,up,dn,dn)); grdisplay(_);
grdef(`GFamn{^a ^miu ^niu}:=r^2*sin(theta)*Famn{^a ^miu ^niu}`);
grcalc(GFamn(up,up,up));
grdef(`Ean{^a ^niu}:=1/(r^2*sin(theta))*GFamn{^a ^miu ^niu, miu}+
        1/2*(eta1{b c}*kdelta{^a d} - eta1{b d}*kdelta{^a c})*
        ev{^b miu}*Fabmn{^c ^d ^miu ^niu}-
        1/2*(eta1{d b}*kdelta{^a c} - eta1{d c}*kdelta{^a b})*
        omega{^b ^c miu}*Famn{^d ^miu ^niu}`);
grcalc(Ean(up,up);  grdisplay(_);
grdef(`GFabmn{^a ^b ^miu ^niu}:=
        r^2*sin(theta)*Fabmn{^a ^b ^miu ^niu}`);
grcalc(GFabmn(up,up,up,up));
grdef(`Eabn{^a ^b ^niu}:=
        1/(r^2*sin(theta))*GFabmn{^a ^b ^miu ^niu, miu} +
        ( 1/4*(eta1{d e}*kdelta{^a c}*kdelta{^b f} - eta1{c e}*
        kdelta{^a d}* kdelta{^b f} + eta1{c f}*kdelta{^a d}*
        kdelta{^b e} - eta1{d f}*kdelta{^a c}*kdelta{^b e})
        -1/4*(eta1{d e}*kdelta{^b c}*kdelta{^a f} -
        eta1{c e}*kdelta{^b d}* kdelta{^a f} +
        eta1{c f}*kdelta{^b d}*kdelta{^a e} -
        eta1{d f}*kdelta{^a c}*kdelta{^b e} ) )*
        omega{^c ^d miu}*Fabmn{^e ^f ^miu ^niu} `);
grcalc(Eabn(up,up,up)); grdisplay(_);
```

5 Model II. Results and analytical program

We consider a particular form of spherically gauge fields of the deSitter group DS given by the following ansatz:

$$\begin{aligned} e^0_\mu &= (A,0,0,0)\,, \quad e^1_\mu = \left(0,\frac{1}{A},0,0\right), \\ e^2_\mu &= (0,0,rC,0)\,, \quad e^3_\mu = (0,0,0,rC\sin\theta) \end{aligned} \tag{5.1}$$

$$\begin{aligned} \omega^{01}_\mu &= (U,0,0,0)\,,\ \omega^{12}_\mu = (0,0,W,0)\,,\ \omega^{13}_\mu = (0,0,0,Z\sin\theta)\,, \\ \omega^{23}_\mu &= (V,0,0,\cos\theta)\,,\ \omega^{02}_\mu = \omega^{03}_\mu = (0,0,0,0)\,, \end{aligned} \tag{5.2}$$

where A, C, U, V, W, and Z are functions only of the $3D$ radius r. We use the above expressions to compute the components of the tensors $F^a_{\mu\nu}$ and $F^{ab}_{\mu\nu}$. From this point at the end we performed all the calculations using an analytical program conceived by us and presented bellow.

In the following, we develop an Einstein model for gravitation, i.e. we will suppose that the torsion vanishes. First of all, we calculate the Riemann tensor of the model, defined by the formula [3]:

$$\widetilde{R}^{\rho\sigma}_{\mu\nu} = F^{ab}_{\mu\nu} e^{\rho}_{a} e^{\sigma}_{b}. \tag{5.3}$$

Then, we calculate the components of the Ricci tensor, defined as:

$$\widetilde{R}^{\nu}_{\mu} = R^{ab}_{\mu\rho} e^{\nu}_{a} e^{\rho}_{b} = \widetilde{R}^{\nu\rho}_{\mu\rho}, \tag{5.4}$$

where the sum over the index $\rho = 0, 1, 2, 3$ is understand in the last equality.

In order to write the equations of Einstein, we calculate the curvature scalar $\widetilde{R} = \widetilde{R}^{\mu}_{\mu}$ (sum over $\mu = 0, 1, 2, 3$). The equations of Einstein for the vacuum can be written in the form:

$$\widetilde{R}^{\nu}_{\mu} - \frac{1}{2}\delta^{\nu}_{\mu}\widetilde{R} = 0. \tag{5.5}$$

For the above model, if we consider the case of null torsion ($F^{a}_{\mu\nu} = 0$), that is:

$$U = -AA', V = 0, W = Z = A\left(C + rC'\right) \tag{5.6}$$

and the supplementary condition $C = 1$, we obtained only two independent equations:

$$\begin{aligned} -\frac{2AA'}{r} + \frac{1 - A^2}{r^2} + 12\lambda^2 &= 0, \\ -\frac{2AA'}{r} + U' + 12\lambda^2 = 0. \end{aligned} \tag{5.7}$$

The equations (5.7) are compatible if we chose the function $A(r)$ so that:

$$U' = \frac{1 - A^2}{r^2}. \tag{5.8}$$

Tacking into account first condition (5.6), the condition (5.8) becomes:

$$r^2\left(A^2\right)'' - 2A^2 + 2 = 0. \tag{5.9}$$

The solution of the equation (5.9) is the following:

$$A^2 = 1 + \frac{\alpha}{r} + \beta r^2, \tag{5.10}$$

where α and β are two arbitrary constants. This solution verifies also the equations of Einstein (5.7) if and only if $\beta = 4\lambda^2$. But, according to the result of MacDowell-Mansouri [6], the cosmological constant of the model is identified as $\Lambda = -12\lambda^2$. Then, the solution (5.10) have the form:

$$A^2 = 1 + \frac{\alpha}{r} - \frac{\Lambda}{3} r^2. \tag{5.11}$$

In particular, if we chose $\alpha = -2M$, then we obtain the Scwarzschild-deSitter solution:

$$A^2 = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2. \tag{5.12}$$

In the limit $\lambda \to 0$, we obtain the Schwarzschild solution:

$$A^2 = 1 - \frac{2M}{r}, \tag{5.13}$$

and for $\alpha = 0$ the solution (5.11) is that of deSitter.

The spin connection components are determined by tetrads e^a_μ (i.e. they are not independent fields):

$$U = -\frac{M}{r^2} + \frac{\Lambda}{3}r, \quad W = Z = \left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)^{1/2}. \tag{5.14}$$

The previous results show that the model presented in this paper can be considered as a gauge theory for the "active" deSitter symmetry group.

All the calculations of this section, inclusive the integration of the field equation (5.9), were made using an analytical program written in the package GRTensorII, running on the MapleV platform. The analytical program allows to calculate: the components of the strength tensor field $F^a_{\mu\nu}$, respectively $F^{ab}_{\mu\nu}$, the components of the Riemann tensor $\widetilde{R}^{\rho\sigma}_{\mu\nu}$, the components of the Ricci tensor $\widetilde{R}^\nu_\mu$, the curvature scalar $\widetilde{R}$, the field equations (the Einstein tensor). We remark that the Riemann tensor, the Ricci tensor, the curvature scalar and the field equations are computed without using a metric. For this purpose we use only the tetrad fields e^a_μ and their inverses e^μ_a. In program we denoted $F^a_{\mu\nu}$ by Famn, $F^{ab}_{\mu\nu}$ by Fabmn, $\widetilde{R}^{\rho\sigma}_{\mu\nu}$ by Rtetrad, $\widetilde{R}^\nu_\mu$ by Ricci, $\widetilde{R}$ by Rtilda, the field equation (the equation of Einstein) by Eq. The initial metric is loaded from the file "spheric.mpl" and the potentials e^a_μ (denoted by ev) and ω^{ab}_μ(denoted by omega) are introduced during of the running program (by the command "grcalc"). The integration of equation (5.9) has been done also in the analytical program, where we denoted A^2 with y. Below we list the part of program which allows to define and to calculate the quantities previously specified.

Program "DeSitter.mws"

```
restart: grtw():
grload(minkowski, `c:/maple/spheric.mpl`);
grdef(`ev{^a miu}`);  grcalc(ev(up,dn));
grdef(`omega{[^a ^b] miu}`);  grcalc(omega(up,up,dn));
grdef(`eta1{(a b)}`);  grcalc(eta1(dn,dn));
grdef(`Famn{^a miu niu} := ev{^a niu,miu} - ev{^a miu,niu}
        + omega{^a ^b miu}*ev{^c niu}*eta1{b c}
        - omega{^a ^b niu}*ev{^c miu}*eta1{b c}`);
grcalc(Famn(up,dn,dn)); grdisplay(_);
grdef(`Fabmn{^a ^b miu niu} := omega{^a ^b niu, miu}
        - omega{^a ^b miu, niu} + (omega{^a ^c miu}
        *omega{^d ^b niu} - omega{^a ^c niu}
        *omega{^d ^b miu})*eta1{c d} +4*lambda^2*(kdelta{^b c}
        *kdelta{^a d} - kdelta{^a c}*kdelta{^b d})*ev{^c miu}
        *ev{^d niu} `);
grcalc(Fabmn(up,up,dn,dn)); grdisplay(_);
grdef(`evi{^miu a}`);  grcalc(evi(up,dn));
grdef(`Rtetrad{^rho ^sigma miu niu }:=Fabmn{^a ^b miu niu}*
        evi{^rho a}*evi{^sigma b}`);
grcalc(Rtetrad(up,up,dn,dn));  grdisplay(_);
grdef(`Ricci{^niu miu}:=Rtetrad{^niu ^rho miu rho}`);
```

```
grcalc(Ricci(up,dn)); grdisplay(_);
grdef(`Rtilda:=Ricci{^miu miu}`); grcalc(Rtilda); grdisplay(_);
grdef(`Eq{^niu miu}:= Ricci{^niu miu} - 1/2*kdelta{^niu miu}*Rtilda`);
grcalc(Eq(up,dn)); grdisplay(_);
with(DEtools, odeadvisor); ode1 := diff(y(r),r,r) = 2*y(r)/r^2-2/r^2;
odeadvisor(ode1); dsolve(ode1);
```

6 Solutions without singularities

We use the method of Lagrange-multipliers in order to obtain solutions without singularities of DS-gauge theory of gravitation. Namely, we impose some restrictions [2] on two invariants I_1 and I_2 of the theory. Introducing the Lagrange-multiplier $\varphi_1(t)$ and $\varphi_2(t)$, the integral of action can be rewritten as:

$$S_g = -\frac{1}{16\pi G}\int d^4xe\left[F + \varphi_1(t)\, f_1(I_1) + \varphi_2(t)\, f_2(I_2) + V(\varphi_1,\varphi_2)\right], \tag{6.1}$$

where

$$F = F^{ab}_{\mu\nu}\bar{e}^{\mu}_{a}\bar{e}^{\nu}_{b}, \quad e = \det\left(e^{a}_{\mu}\right). \tag{6.2}$$

and $\bar{e}^{\nu}_{b}$ is the inverse of e^{a}_{μ}. The quantities $f_i(I_i)$, $i = 1,2$ are functions which must be chosen in an appropriate form in order to obtain solutions without singularities of the corresponding field equations. Thus, the potential $V(\varphi_1,\varphi_2)$ have to satisfy the constraint equations [2]:

$$f_1(I_1) = -\frac{\partial V}{\partial \varphi_1}, \quad f_2(I_2) = -\frac{\partial V}{\partial \varphi_2}, \tag{6.3}$$

The model can be simplified further if we assume:

$$V(\varphi_1,\varphi_2) = V_1(\varphi_1) + V_2(\varphi_2), \tag{6.4}$$

and chose the invariants I_1, I_2 in the form

$$I_1 = F - \sqrt{3}\left(4F^{a}_{\mu}F^{\mu}_{a} - F^2\right)^{1/2}, \tag{6.5}$$

respectively

$$I_2 = 4F^{a}_{\mu}F^{\mu}_{a} - F^2. \tag{6.6}$$

In these expressions, the quantities F^{a}_{μ} are defined by

$$F^{a}_{\mu} = F^{ab}_{\mu\nu}\bar{e}^{\nu}_{b}. \tag{6.7}$$

As an example, we chose the functions f_1 and f_2 in the simple form [2]:

$$f_1(I_1) = I_1, \qquad f_2(I_2) = -\sqrt{I_2}. \tag{6.8}$$

Then, the action S_g in Eq. (6.1) becomes:

$$S_g = -\frac{1}{16\pi G}\int d^4xe\left[F + \varphi_1 I_1 - \varphi_2\sqrt{I_2} + V_1(\varphi_1) + V_2(\varphi_2)\right]. \tag{6.9}$$

Now, all we have to do is to write the variational field equations which follow from (6.9) and search their solutions without singularities.

We choose a particular form of spherically gauge fields $e_\mu^a(x)$ and $\omega_\mu^{ab}(x)$ given by the following ansatz:

$$\begin{aligned} e_\mu^0 &= (N(t),0,0,0), \quad e_\mu^1 = \left(0, \frac{a(t)}{\sqrt{1-kr^2}}, 0, 0\right), \\ e_\mu^2 &= (0,0,ra(t),0), \quad e_\mu^3 = (0,0,0,ra(t)\sin\theta), \end{aligned} \tag{6.10}$$

respectively

$$\begin{aligned} \omega_\mu^{01} &= \left(0, -\frac{a'(t)}{N(t)\sqrt{1-kr^2}}, 0, 0\right), \quad \omega_\mu^{02} = \left(0,0,-\frac{ra'(t)}{N(t)},0\right), \\ \omega_\mu^{03} &= \left(0,0,0,-\frac{ra'(t)\sin\theta}{N(t)}\right), \qquad \omega_\mu^{12} = \left(0,0,\sqrt{1-kr^2},0\right), \\ \omega_\mu^{13} &= \left(0,0,0,\sqrt{1-kr^2}\sin\theta\right), \quad \omega_\mu^{23} = (0,0,0,\cos\theta), \end{aligned} \tag{6.11}$$

where $N(t)$ and $a(t)$ are functions only of the time variable, k is a constant, and a' is the derivative of $a(t)$ with respect to the variable t. The choice (6.11) of gauge fields $\omega_\mu^{ab}(x)$ assures that all components of the strength tensor $F_{\mu\nu}^a$ vanish. If we remember the Riemann-Cartan theory of gravitation, then this result implies the vanishing of the torsion tensor $T_{\mu\nu}^\rho = \bar{e}_a^\rho F_{\mu\nu}^a$, in accord with GR theory. Here, $\bar{e}_a^\rho$ denotes the inverse of e_μ^a with the properties:

$$e_\mu^a \bar{e}_b^\mu = \delta_b^a, \quad e_\mu^a \bar{e}_a^\nu = \delta_\mu^\nu. \tag{6.12}$$

¿From this point to the end we performed all the calculations using an analytical program conceived by us which is presented in the final part of this Section.

Using the Eqs. (6.10) and (6.11), we obtain the following expressions of the invariants F, I_1 and I_2 above defined:

$$F = -6\frac{aa''N - aa'N' + kN^3 + a'^2N + 8\lambda^2a^2N^3}{a^2N^3}, \tag{6.13}$$

$$I_1 = -12\frac{kN^2 + a'^2 + 4\lambda^2a^2N^2}{a^2N^2}, \tag{6.14}$$

and respectively

$$I_2 = 12\frac{\left(kN^3 + a'^2N - aa''N + aa'N'\right)^2}{a^4N^6}. \tag{6.15}$$

where a'' is the second derivative of $a(t)$ with respect to the variable t. Introducing these expressions into Eq. (6.9) and imposing the variational principle $\delta S_g = 0$ with respect to $N(t)$, $\varphi_1(t)$ and $\varphi_2(t)$, we obtain the corresponding field equations. We write now these equations for the particular case $N(t) = 1$ which is of interest in our model:

$$\begin{aligned} &-\frac{1}{2}(V_1+V_2) + 3H^2(1-2\varphi_1) + 3\frac{k}{a^2}(1+2\varphi_1) - 2\Lambda \\ &= \sqrt{3}\left(\varphi_2' + 3H\varphi_2 - \frac{k}{Ha^2}\varphi_2\right), \end{aligned} \tag{6.16}$$

$$\frac{k}{a^2} + H^2 - \frac{\Lambda}{3} = \frac{1}{12}\frac{dV_1}{d\varphi_1}, \quad H = \frac{a'}{a}, \tag{6.17}$$

$$H' - \frac{k}{a^2} = -\frac{1}{2\sqrt{3}}\frac{dV_2}{d\varphi_2}, \quad H' = \frac{dH}{dt} = \frac{a''a - a'^2}{a^2}, \tag{6.18}$$

where φ_2' is the derivative of $\varphi_2(t)$ with respect to t, and $\Lambda = -12\lambda^2$ is interpreted as cosmological constant [4, 9].

If we consider the limit $\lambda \longrightarrow 0$, or equivalently $\Lambda = 0$, we obtain the results in Ref. [2], but, for $\Lambda \neq 0$ we can study in addition the dependence on the cosmological constant of the solutions (without singularities) obtained by solving the Eqs.(6.16)-(6.18). We make also the mention that the Eqs. (6.17) and (6.18) are identically with the constraints (6.3) introduced into the integral of action S_g by means of the Lagrange-multiplier fields $\varphi_1(t)$ and $\varphi_2(t)$.

Below, we list down the part of program which allows to define and to calculate the quantities needed in obtaining of Eqs. (6.16)-(6.18).

Program "DS_gauge_theory.mws"

```
restart: grtw( ):
grload(minkowski, `c:/maple/spheric.mpl`);
grdef(`ev{^a mu}`); grcalc(ev(up,dn));
grdef(`eta1{(a b)}`); grcalc(eta1(dn,dn))
grdef(`omega{[^a ^b] mu}`); grcalc(omega(up,up,dn));
grdef(`Famn{^a mu nu} := ev{^a nu,mu} - ev{^a mu,nu}
      + omega{^a ^b mu}*ev{^c nu}*eta1{b c}
      - omega{^a ^b nu}*ev{^c mu}*eta1{b c}`);
grcalc(Famn(up,dn,dn));
grdef(`Fabmn{^a ^b mu nu} := omega{^a ^b nu, mu}-
      omega{^a ^b mu, nu}+ (omega{^a ^c mu}*omega{^d ^b nu} -
      omega{^a ^c nu}*omega{^d ^b mu})*eta1{c d}-
      4*lambda^2*( ev{^a mu}*ev{^b nu} - ev{^b mu}*ev{^a nu})`);
grcalc(Rabmn(up,up,dn,dn));
grdef(`R:=Rabmn{^a ^b mu nu}*einv{a ^mu}*einv{b ^nu}`);
grcalc(R);  grdef(`F{^a mu}:=Rabmn{^a ^b mu nu}*einv{b ^nu}`);
grcalc(F(up,dn));
grdef(`I2:=4*F{^a mu}*Finv{a ^mu}-(R)^2`); grcalc(I2);
grdef(`I1:=R-sqrt(3)*sqrt(I2)`);  grcalc(I1);
grdef(`de`); grcalc(de);
grdef(`Lg:=(R+phi1(t)*I1-phi2(t)*sqrt(I2)+V1(phi1)+
      V2(phi2))*de`); grcalc(Lg); grdisplay(_);
```

The solution of Eqs. (6.16)-(6.18) includes a dependence on the cosmological constant Λ. We suppose that the Lagrange-multiplier function $\varphi_1(t)$ is absent, and consider the case when $k = 0$. Then, denoting $\varphi_2(t) = \varphi(t)$ and $V_2(\varphi_2) = V(\varphi)$, the Eqs. (6.16)-(6.18) become:

$$H' = -\frac{1}{2\sqrt{3}}\frac{dV}{d\varphi}, \tag{6.19}$$
$$\varphi' = -3H\varphi + \sqrt{3}H - \frac{1}{2\sqrt{3}H}V - \frac{2\Lambda}{\sqrt{3}H}.$$

We consider the potential $V(\varphi)$ of the simple form:

$$V(\varphi) = 2\sqrt{3}\lambda^2\left(\frac{\varphi^2}{1+\varphi^2} + \frac{24}{\sqrt{3}}\right), \tag{6.20}$$

where λ is the real parameter that determines the cosmological constant Λ. This parameter coincides with the constant H_0 in Ref. [2] that has been interpreted as a Planck scale of the model. Therefore, in our example the Planck scale is related to the cosmological constant Λ. For small values of H and φ, the Eqs. (6.19) can be written as:

$$H' \simeq -2\lambda^2\varphi, \qquad \varphi'(t) \simeq \frac{\sqrt{3}H^2 - \lambda^2\varphi^2}{H}. \tag{6.21}$$

These equations have the periodic solution

$$\varphi(t) = \varphi_0 \sin(\omega t), \quad H(t) = \frac{\omega\varphi_0}{2\sqrt{3}}\left[\cos(\omega t) - 1\right], \tag{6.22}$$

where φ_0 is an integration constant and $\omega = 2 \times 3^{1/4}\lambda$ is the frequency of oscillation of the corresponding gravitational field described by the gauge potentials $e^a_\mu(x)$ and $\omega^{ab}_\mu(x)$. This solution has no singularities and it is valid if the cosmological constant is negative ($\Lambda < 0$). The case with positive cosmological constant ($\Lambda > 0$) can be studied choosing the anti-de-Sitter group as gauge group. But, the deformation parameter λ will be then pure imaginary. We emphasize that there are possible also periodic solutions if we suppose a time-dependent cosmological "constant". In particular, we can consider a cosmological constant which is itself a periodic function on time. It will be also of interest to apply the previous method in obtaining non-singular solutions of the gauge theories with internal groups of symmetry.

7 Concluding Remarks

In this paper, we developed two model of gauge theory of gravitation, one for Poincaré group and another for deSitter group. The space-time Minkowski with spherical symmetry is considered. In these cases, if all the constants of structure vanish, then we obtain a analogous theory for the electromagnetic field. For both models we have obtained analytical solutions.

Also, we developed a de-Sitter gauge theory of gravitation on a spherical symmetric Minkowski space-time as base manifold. This theory allows a complementary description of the gravitational effects in which the mathematical structure of the underlying space-time is not affected by physical events [11]. Only the gauge potentials $e^a_\mu(x)$ and $\omega^{ab}_\mu(x)$ of the gravitational field change as functions of coordinates. This is important when we consider a quantum gauge theory of gravitation.

In order to obtain solutions without singularities, we imposed constraints on some invariants of the gauge model we considered. The solutions in this paper are time-periodic and correspond to a fixed (negative) cosmological constant Λ whose value is related to the Planck scale and that determines the frequency of the corresponding gravitational field.

All the calculations from this paper were performed using analytical procedures, which are written in GRTensorII computer algebra system. The procedures are conceived for a general form of the potentials. The main part of these procedures are given in the Section 3, Section 4 and Section 6. Because the tensorial operations from this paper are very difficult and sometime impossible to perform, the utilization of a computer algebra system is not only useful but also necessary.

References

[1] N. Batakis, *General relativity reformed to a genuine Yang-Mills gauge theory for gravity*, arXiv: gr-qc/9711054 (1997).

[2] R. Brandenberger, M. Mukhanov, A. Sornborger, *Phys. Rev.* D48 (1993), 1629.

[3] K. Dietz, R. Flume, G. V. Gehlen, V. Rittenberg, (editors) *Supersymmetry*, Plenum Press, N.Y., 1985.

[4] P. Freund, *Introduction to supersymmetry*, Cambridge University Press, 1986.

[5] F. Gronwald, Fr. Hehl, *On the gauge aspects of gravity,* arXiv: gr-qc/9602013 (1996).

[6] S. W. MacDowell, F. Mansouri, *Phys. Rev. Lett.* 38 (1977), 739.

[7] P. Musgrave, D. Pollney, K. Lake, *GRTensorII*, Queens's University at Kingston, Ontario, 2000.

[8] G. Zet, V. Manta, *Self-dual Poincaré gauge theory of graviation*, IJMPC (computational physics) Vol.13, No. 4 (2002), 509-516.

[9] G. Zet, V. Manta, S. Babeti, *DeSitter gauge theory of gravitation*, IJMPC (computational physics) Vol.14, No. 1 (2003), 41-48.

[10] G. Zet, V. Manta, *Self-dual model of SU(2) gauge theory in a space-time with axial simmetry*, Mathematical and Computer Modelling, 34 (2001), 37-43.

[11] C. Wiesendanger, *Classical and Quantum Gravity* 13, 681, 1996.

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