Statistical models applied to manufacturing systems

Constantin Târcolea, Adrian Paris and Adina-Ioana Tarcolea

Abstract

Two aspects of manufacturing systems are explored by means of statistical models. Specifically, reliability growth models and Taguchi-type considerations about quality control are taken upon. The provided discussions help manufacturing engineers apply the presented models.

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Key words: Reliability growth models, maximum likelihood estimation, Taguchi methods, experimental design, control and noise factors.

1 Motivation

Statistical models are widely used in research, industrial manufacturing, reliability, security, safety, and this list is by no means exhaustive. A very important aspect of applied statistics concerns the statistical analysis and modelling of manufacturing systems. This contribution focusses on two aspects that are particularly well dealt with by means of statistical models: reliability and reliability growth, on one hand, as well as quality analysis, on the other.

A reliability growth model is an analytic tool that can be used in evaluating the time required to develop an acceptable level of reliability for a new product and the associated costs. During a few successive development stages, a learning curve is used to monitor the continuous reliability improvements.

The quality quantification paradigm proposed by Taguchi is based on two fundamental concepts: the quality loss function and the signal/noise ratio. According to the Taguchi approach, a manufacturing process has two complementary goals: zero bias and the smallest possible variance. Many forms of quality loss functions have been proposed in the literature. Notably, Feng and Kusiak extended the notion of quality loss function to multidimensional chains in the independence case. However, in many practical applications, one is also interested in the analysis of association between two or more factors, since a fraction of quality loss can be caused by some potential interactions. By examining the interactions between factors, statistical inferences become more conclusive. We therefore extended the Taguchi quality loss function to the multidimensional case with interactions.

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This contribution is structured as follows. In section 2, a few reliability growth models are described, together with an example illustrating their practical application. In the following section, the main ideas of Taguchi's quality viewpoint, followed by our proposals, are presented.

2 Reliability Growth Models

The reasoning behind reliability growth models is, that one wants to account for the *changes* in the reliability of a product, due to design modifications and corrective actions. This lies in contrast to the usual, static, reliability models. Reliability growth models can be used for achieving robust reliability. There are two broad types of reliability growth models, discrete models and continuous models.

The discrete models, which we discuss at first, are based on Bernoulli type data, using proportions of successes.

An example of discrete models is the Lloyd-Lipow model, which is expressed as

$$R_k = R_\infty - \frac{\alpha}{k},$$

where R_∞ is the final result of reliability, and α is an unknown constant. R_k denotes the reliability at the k-th (out of N) test, involving n_k tested items of which s_k are successfully functioning, and can be estimated by $\hat{R}_k = \dfrac{s_k}{n_k}$.

Denoting

$$P_k = \text{Prob}\ \{s_k \text{ successes of } n_k \text{ trials at the } k\text{-th test}\},$$

it follows that

$$P_k = \binom{n_k}{s_k} R_k^{s_k}(1-R_k)^{n_k-s_k}.$$

The likelihood function is

$$l = \ln \prod_{k=1}^{N} \binom{n_k}{s_k} + \sum_{k=1}^{N} s_k \ln\left(R_\infty - \frac{\alpha}{k}\right) + \sum_{k=1}^{N}(n_k - s_k)\ln\left(1 - R_\infty + \frac{\alpha}{k}\right).$$

The likelihood equations for R_∞ and α can be solved iteratively. Initial values for the iterative solution may be obtained from the following

$$\hat{\alpha}_0 = \frac{\dfrac{1}{\bar{n}}\left(\displaystyle\sum_{k=1}^{N} k s_k - \frac{N+1}{2}\sum_{k=1}^{N} s_k\right)}{\dfrac{N+1}{2}c_1 - N}$$

$$\hat{R}_\infty^0 = \frac{\dfrac{1}{\bar{n}}\left(\dfrac{c_1}{N}\displaystyle\sum_{k=1}^{N} k s_k - \sum_{k=1}^{N} s_k\right)}{\dfrac{N+1}{2}c_1 - N},$$

where

$$\bar{n} = \frac{1}{N}\sum_{k=1}^{N} n_k, \quad c_1 = \sum_{k=1}^{N}\frac{1}{k} \simeq \ln\left(N + \frac{1}{2}\right) + 0,5772.$$

Example. *The following table summarizes the results of 15 tests:*

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n_k	8	6	8	7	8	10	8	8	9	7	8	8	9	8	8
s_k	5	4	5	4	6	7	6	7	6	6	6	8	7	6	7

Based upon this data, the unknown parameters of the model are estimated. Following results are obtained: $c_1 = 3,32$; $\hat{\alpha}_0 = 0,541$; $\hat{R}^0_\infty = 0,869$, $\hat{\alpha}_1 = 0,594$; $\hat{R}^1_\infty = 0,873$ *and so on.*

Further, another discrete growth model is discussed. The model proposed by Chernoff and Woods is also based on Bernoulli type data. In this model, the dichotomy is: success, if the piece of equipment functions properly when operated, and failure, if the piece of equipment is not up to standard when operated. If P_k denotes the probability of success after k failures have been observed, then

$$P_k = 1 - \exp(-\alpha - \beta(k-1))$$

where $\alpha > 0$ and $\beta > 0$ are unknown parameters.

Another model is the Bonis model, given by

$$R_k = R_\infty - Qa^{k-1},$$

where a represents the rate of growth.

Further, two continuous growth models are succinctly described. The Duane model describes the instantaneous failure rate for a piece of equipment and is of the form

$$\lambda(t) = \frac{\alpha}{t^\beta},$$

where t is the time and α, β are unknown constants.

This model may be expressed in terms of the cumulative number of failures:

$$F = F_0 t^\gamma,$$

where F_0 and γ are unknown parameters. A modified reliability growth model based on the number of failures, but after (some) initial time t_0, is

$$F = F_0(t^\gamma - t_0^\gamma).$$

Weiss' proposal also models the failure rate for a piece of equipment, rate presumed constant between failures. If $m-1$ corrective actions have been performed, then the following form of the model emerges:

$$\lambda_m = \frac{\alpha + m}{\beta m},$$

where α and β are unknown parameters.

3 Taguchi-Methods on Manufacturing Systems

From the practical point of view, the goal of Taguchi methods is to find a trade-off between quality loss and product price. The equilibrium between levels of different factors, robust tolerance design, and costs is based on two main concepts proposed by Taguchi: quality loss function and signal/noise ratio.

According to Taguchi's quality engineering philosophy and methodology, there are three important steps in designing a product or process: system design, parameter design and tolerance design.

The aim of system design is to create a product that indeed possesses the properties intended for it at the planing stage. This involves the development of a prototype, choice of materials, parts, components, assembly system and manufacturing processes, so that the product fulfills the specified conditions and tolerances at the lowest costs.

Parameter design tries to determine the connections between controllable and noise factors, in order to ascertain the best combination of factor levels in the manufacturing process, having the purpose of achieving robustness, and improving quality, without increasing costs.

In the last stage, tolerance design tries to narrow the ranges of the operating conditions, so that the most economical tolerances are obtained.

According to Taguchi's viewpoint, the quality loss function is a measure for the evaluation of deviations from the target values of the product, even when these lie within specifications. The literature indicates three type of tolerances: "the nominal - the best", "the smaller - the better", "the larger - the better", and, therefore, there are three resulting classes of loss functions.

"The nominal - the best" type is required in many cases when a nominal characteristic can vary in two directions. Various studies proposed different loss functions for evaluating the quality level in one-dimensional case. For the multidimensional case with independent components, the following loss function is used:

$$L(x_1, x_2, \ldots, x_l) = \sum_{i=1}^{l} a_i (x_i - m_i)^2.$$

An extension is given by

$$L(x_1, \ldots, x_l) = b_1 (x_1 - m_1)^2 \sum_{i=2}^{l} a_i (x_i - m_i)^4 + b_l (x_l - m_l)^2 \sum_{i=1}^{l-1} (x_i - m_i)^4.$$

For products with several measurable functional quality characteristics, potential interactions may appear, interactions which can be used for improvements.

In case of a system with dependent components, the quality can be measured by

$$L(x_1, \ldots, x_p) = \sum_{i=1}^{p} c_i (x_i - m_i)^2 + 2 \sum_{1 \leq i < j \leq p} k_{ij} |(x_i - m_i)(x_j - m_j)|.$$

For equipment batches, or manufacturing systems and lines, the following formula of the average loss function results:

$$I = \sum_{1 \le i \le p} d_i [s_i^2 + (\bar{x}_i - m_i)^2] + 2 \sum_{1 \le i < j \le p} h_{ij} \left[|s_{ij}| + |(\bar{x}_i - m_i)(\bar{x}_j - m_j)| \right].$$

The signal/noise ratio for the dependent case has the expression

$$\frac{S}{N} = 10 \log_{10} \left(\prod_{1 \le i \le p} \frac{x_i^2}{s_i^2} \cdot 2 \prod_{1 \le i < j \le p} \frac{|\bar{x}_i \bar{x}_j|}{|s_{ij}|} \right).$$

The quality of products must be continuously improved, and these two concepts, loss function and signal/noise ratio, are of utmost importance in process-oriented manufacturing.

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Corresponding author: Constantin Târcolea
University "Politehnica" Bucharest, Department Mathematics I
Splaiul Independentei 313, Bucharest, Romania
e-mail: constantin.tarcolea@rdslink.ro