New results on Ioffe-Ştefănescu magnetic trap

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Abstract

We analyze a magnetic trap based on a magnetic field whose density of energy has an isolated minimum point where the field is non-vanishing. Such elementary magnetic fields appear around suitable configurations of electric circuits.

Section 1 presents some ideas of Ioffe and Ştefănescu for realizing a magnetic trap using a union of electric circuits (an anti-Helmholtz pair and six rectilinear bars), and informs the readers about the actual theories and experiments regarding such devices. Section 2 gives the components of the magnetic field. Section 3 reproduces some simulations with MATHEMATICA that confirm the trapping phenomena.

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Key words: magnetic flow, Ioffe - Ştefănescu magnetic trap, subharmonic functions.

1 Morphology of the magnetic trap

Any magnetic trap is intended to confine electrical charged particles in a small spatial region. This work follows the papers [1], [11], where the authors describe in detail the magnetic trap proposed in 1962 by Ioffe and co-workers [4]. We use a Biot-Savart-Laplace magnetic field $H$ with a local minimum in the magnetic density of energy $f = \frac{1}{2}||H||^2$ (sub-harmonic function) at a point where $H$ remains nonzero. The magnetic field is produced by a union of simple piecewise linear electric circuits and two circular coils, with suitable geometry. Such geometrical patterns were studied in Romania by Ştefănescu [6-8] and by Udriște et. al. [8]-[15].

The configuration of the trap is described in Fig. 1. There are two main magnetic fields which superpose:
- a vertical magnetic mirror field produced by two anti-Helmholtz coils where flows the electric current $I_1$ (the mirror axis is denoted by $Oz$);
- a magnetic field generated by a piecewise rectilinear circuit made from six straight lines parallel to the magnetic mirror axis; the electric current flowing through is $I_2$.

Let us denote by $R$ the radius of the coils. In a Helmholtz disposition the two coils are separated by a distance $d = 2R$. Such an arrangement insures the best homogeneity of the field in the central region. Being interested to build a magnetic trap,
we choose opposite electric currents in the two coils. In this mirror design particles moving in the vertical direction are trapped indeed. The remainder straight lines insure trapping in the horizontal plane. They are tangent to the two circular coils as in Fig. 1 and intersect the $xOy$ plane in six points, namely $(R, 0, 0)$, $(R/2, R\sqrt{3}/2, 0)$, $(-R/2, R\sqrt{3}/2, 0)$, $(-R, 0, 0)$, $(-R/2, -R\sqrt{3}/2, 0)$, $(R/2, -R\sqrt{3}/2, 0)$.

Fig. 1. Ioffe-Ștefănescu configuration of magnetic trap (after [4])

In an actual circuit the six vertical lines are linked by transverse electric lines. We assume that these lines as well as the supply battery are far enough and we approximate the magnetic field of the structure with the field given by six infinite vertical lines. The study of a real finite structure requires additional calculus. We did not take this approach because the horizontal segments connecting the vertical lines would produce an inhomogeneous field in the central indeed very far, or one uses additional compensating coils to annihilate this inhomogeneous field.

Some experiments in physics demand a homogeneous non-zero magnetic field in the central region. This is so, in particular, for Bose-Einstein condensation experiments (BEC), where spin-flip losses would appear in regions with zero magnetic field [3], [16]. The paper [2] presents three designs for a magnetic trap that will simultaneously confine neutral atoms and non-neutral plasma. Various configurations, containing static as well as rotating magnetic fields were proposed. They may easily be taken account for. For our study it is sufficient to add a constant magnetic field in the central region, and then we obtain a total magnetic field $H$ with the density of energy, $f = \frac{1}{2}||H||^2$, satisfying

$$H(0, 0, 0) \neq 0, \quad df(0, 0, 0) = 0, \quad d^2f(0, 0, 0) > 0.$$ 

In this way, the density of energy of the magnetic field would have an isolated non-zero minimum in this region, its trapping characteristic being realized.

Our main concern is to further investigate the Ioffe - Ștefănescu magnetic trap, using the tools in the papers [1], [9-15]. While the papers [1],[11] used an approximation for the intervening elliptic integrals, this paper presents a new approach, using elliptic integrals. We have also changed some geometric factors, trying to obtain the greatest field homogeneity in the central region that contains plasma.
2 Components of the total magnetic field

The computations are made in the particular case \( R = 1 \). To have the better field homogeneity, the distance between the two coils must be \( d = 2R \), but it may be changed in order to see the variations. The two electric currents may be equal, or they may be adjusted to achieve desired magnetic fields.

The 2D magnetic field:

\[
H_x = \frac{I_2}{2\pi} \left( \frac{y - \sqrt{3}/2}{(x - 1/2)^2 + (y - \sqrt{3}/2)^2} - \frac{y}{(x + 1/2)^2 + (y - \sqrt{3}/2)^2} \right) - \frac{y + \sqrt{3}/2}{(x + 1/2)^2 + (y + \sqrt{3}/2)^2} \right)
\]

\[
H_y = \frac{I_2}{2\pi} \left( \frac{x - 1/2}{(x - 1/2)^2 + (y - \sqrt{3}/2)^2} + \frac{x - 1/2}{(x - 1/2)^2 + (y + \sqrt{3}/2)^2} \right)
\]

where \( I_2 \) is the electric current.

For the magnetic field given by the two coils it is better first to compute its vectorial potential given by

\[
A = \frac{\mu_0 I_1}{4\pi} \int \frac{dl}{||r||},
\]

where \( \mu_0 \) is the vacuum magnetic permeability, \( I_1 \) the electric current, \( dl \) the element of arc, \( ||r|| \) the distance between a point on the coil and the observation point \( M(x, y, z) \) and the integral is taken around one or the other of the two coils. It is easier to work in cylindrical coordinates \((\rho, \varphi, z)\). Because \( dl \) rests in a plane parallel with the plane \( xOy \), we find \( A_z = 0 \). A little computation [5] shows that \( A_\rho \) is also zero and the only non-zero component of \( A \) is \( A_\varphi \). Its expression is

\[
A_\varphi(\rho, z) = \frac{\mu_0 I_1}{\pi \xi} \sqrt{\frac{R}{\rho}} \left[ \left( 1 - \frac{\xi^2}{2} \right) F(\xi^2) - E(\xi^2) \right],
\]

where \( R \) is the common radius of the coils and \( \xi \) is given by

\[
\xi^2 = \frac{4\rho R}{(R + \rho)^2 + (z \pm d)^2}.
\]

The sign \( \pm \) at the denominator is – for the upper coil and + for the other one. The functions \( F(\xi^2) \) and \( E(\xi^2) \) are the complete elliptic integrals of the first kind, given
by
\[
F(\xi^2) = \frac{\xi}{\sqrt{1 - \xi^2 \cos^2 \psi}} \quad \text{and} \quad E(\xi^2) = \frac{\xi}{\sqrt{1 - \xi^2 \cos^2 \psi}} d\psi
\]
(in the papers written in English, the elliptic function \( F(\xi^2) \) is denoted by \( K(\xi^2) \).

From the expression of \( A_\varphi(\rho, z) \) we obtain the cylindrical components of the magnetic field:
\[
H_\rho = -\frac{1}{\mu_0} \frac{\partial A_\varphi}{\partial z}, \quad H_\varphi = 0 \quad \text{and} \quad H_z = \frac{1}{\mu_0} \frac{\partial}{\partial \rho} (\rho A_\varphi).
\]
The Cartesian components of the mirror magnetic field are \( H_x = H_\rho \cos \varphi \) and \( H_y = H_\rho \sin \varphi \).

For all these functions, the origin belongs to the domain of definition. But the program, based on local formulas, may give errors for small values of the coordinate \( x \). To avoid this, for \( |x| < 10^{-4} \), we approximate the field by the leading terms in the Taylor developments.

3 Simulations using MATHEMATICA

The field of the infinite vertical lines

ClearAll \([x, y, z, h1x, h1y, iel2] \]
iel2 = 1;
h1x[x_, y_, z_] := iel2*
(y/((x - 1)^2 + y^2) - (y - 0.5 \sqrt{3})/
((x - 0.5)^2 + (y - 0.5 \sqrt{3}))^2) +
(y - 0.5 \sqrt{3})/((x + 0.5)^2 + (y - 0.5 \sqrt{3})^2) -
y/((x + 1)^2 + y^2) + (y + 0.5 \sqrt{3})/
((x + 0.5)^2 + (y + 0.5 \sqrt{3})^2) -
(y + 0.5 \sqrt{3})/((x - 0.5)^2 + (y + 0.5 \sqrt{3})^2));
h1y[x_, y_, z_] := iel2*
(-((x - 1)^2 + y^2) + (x - 0.5)/((x - 0.5)^2 + (y - 0.5 \sqrt{3})^2) -
(x + 0.5)/((x + 0.5)^2 + (y - 0.5 \sqrt{3})^2) + (x + 1)/((x + 1)^2 + y^2) -
(x + 0.5)/((x + 0.5)^2 + (y + 0.5 \sqrt{3})^2) +
(x - 0.5)/((x - 0.5)^2 + (y + 0.5 \sqrt{3})^2));

The mirror field given by the two coils

ClearAll[a1, a2, b1x, b1y, b1z, b1r, b1zpol, b1theta, b2x, b2y, b2z, b2zpol, b2r, b2z, b2theta, dist, x, y, z, r, z, R, theta, ksi1, ksi2, iel1, iel2]  
R = 1.0; iel1 = 1; iel2 = 1; dist = 1;
r[x_, y_, z_] = Sqrt[x^2 + y^2];
ksi1[r_, z_] = 4 * r * R/((r + R)^2 + (z - dist * R)^2);
a1[r_, z_] = 2 * iel1 * Sqrt[R/((r * ksi1[r, z])]*
((1 - 0.5 * ksi1[r, z]) * EllipticK[ksi1[r, z]] - EllipticE[ksi1[r, z]]);
b1r[r_, z_] = -D[a1[r, z], z];
b1zpol[r_, z_] = D[r * a1[r, z], r]/r;
New results on Ioffe–Ștefănescu magnetic trap

\[\begin{align*}
ksi2[r_-, z_-] &= 4 * r * R / ((r + R)^2 + (z + \text{dist} * R)^2);
\end{align*}\]
\[\begin{align*}
a2[r_-, z_-] &= -2 * \text{ie}1 * \text{Sqrt}[R / (r * ksi2[r, z])] * 
((1 - 0.5 * ksi2[r, z]) * \text{EllipticK}[ksi2[r, z]] - \text{EllipticE}[ksi2[r, z]])
\end{align*}\]
\[\begin{align*}
b2r[r_-, z_-] &= -D[a2[r, z], z];
b2zpol[r_-, z_-] &= D[(r * a2[r, z]), r]/r;
\end{align*}\]
\[\begin{align*}
b1x[x_, y_, z_] &= b1r[r[x, y], z] * \frac{1}{\sqrt{1 + \frac{y^2}{x^2}}}
\end{align*}\]
\[\begin{align*}
b1y[x_, y_, z_] &= b1r[r[x, y], z] * \frac{y}{x \sqrt{1 + \frac{y^2}{x^2}}}
\end{align*}\]
\[\begin{align*}
b1z[x_, y_, z_] &= b1zpol[r[x, y], z]
\end{align*}\]
\[\begin{align*}
b1[x_, y_, z_] &= \{\text{b1x}[x, y, z], \text{b1y}[x, y, z], \text{b1z}[x, y, z]\}
\end{align*}\]
\[\begin{align*}
b2x[x_, y_, z_] &= b2r[r[x, y], z] * \frac{1}{\sqrt{1 + \frac{y^2}{x^2}}}
\end{align*}\]
\[\begin{align*}
b2y[x_, y_, z_] &= b2r[r[x, y], z] * \frac{y}{x \sqrt{1 + \frac{y^2}{x^2}}}
\end{align*}\]
\[\begin{align*}
b2z[x_, y_, z_] &= b2zpol[r[x, y], z]
\end{align*}\]
\[\begin{align*}
b2[x_, y_, z_] &= \{\text{b2x}[x, y, z], \text{b2y}[x, y, z], \text{b2z}[x, y, z]\}
\end{align*}\]

The total field

\begin{verbatim}
ClearAll [htottruncx, htottruncy, htottruncz]
htottruncx[x_, y_, z_] =
Normal[Series[b1x[x, y, z]+b2x[x, y, z], {x,0,1}, {y,0,1}, {z,0,1}]] ;
htottruncy[x_, y_, z_] =
Normal[Series[b1y[x, y, z]+b2y[x, y, z], {x,0,1}, {y,0,1}, {z,0,1}]] ;
htottruncz[x_, y_, z_] =
Normal[Series[b1z[x, y, z]+b2z[x, y, z], {x,0,1}, {y,0,1}, {z,0,1}]] ;
htotx[x_, y_, z_] = Which[Abs[x] >= 0.00001, h1x[x, y, z]+b1x[x, y, z]+b2x[x, y, z],
Abs[x] < 0.00001, h1x[x, y, z]+htottruncx[x, y, z]] ;
htoty[x_, y_, z_] = Which[Abs[x] >= 0.00001, h1y[x, y, z]+b1y[x, y, z]+b2y[x, y, z],
Abs[x] < 0.00001, h1y[x, y, z]+htottruncy[x, y, z]] ;
htotz[x_, y_, z_] = Which[Abs[x] >= 0.00001, b1z[x, y, z]+b2z[x, y, z],
Abs[x] < 0.00001, htottruncz[x, y, z]] ;
htot[x_, y_, z_] := {htotx [ x, y, z ], htoty [ x, y, z ], htotz [ x, y, z ]};
htotmod[x_, y_, z_] := (htotx [ x, y, z ])^2 + (htoty [ x, y, z ])^2 +
(htotz [ x, y, z ])^2;
\end{verbatim}
The modulus of the total field

\[
\text{ClearAll} \{\text{datamod}, \text{g}\} \\
\text{datamod} = \text{Table}[\text{htotmod}[x, y, z], \{x, -0.015, 0.015, .0015\}, \{y, -0.015, 0.015, .0015\}, \{z, -0.015, 0.015, .0015\}] ; \\
g = \text{ListContourPlot3D}[\text{datamod}, \text{Contours} \to \{0.0005\}, \text{ViewPoint} \to \{-1.5, 1, 1\}] 
\]

The energetic function

\[
\text{ClearAll}[\text{energ1}, \text{energ2}, \text{energ3}, \text{energ}] \\
\text{energ1}[x_, y_, z_] = D[(\text{htotmod}[x, y, z])^2, x] ; \\
\text{energ2}[x_, y_, z_] = D[(\text{htotmod}[x, y, z])^2, y] ; \\
\text{energ3}[x_, y_, z_] = D[(\text{htotmod}[x, y, z])^2, z] ; \\
\text{energ}[x_, y_, z_] = \text{energ1}[x, y, z]^2 + \text{energ2}[x, y, z]^2 + \text{energ3}[x, y, z]^2 ; \\
\text{ClearAll}[\text{dataenerg}, \text{generg}] \\
\text{dataenerg} = \text{Table}[\text{energ}[x, y, z], \{x, -0.01, 0.01, .002\}, \{y, -0.01, 0.01, .002\}, \{z, -0.01, 0.01, .002\}] ; \\
\text{generg} = \text{ListContourPlot3D}[\text{dataenerg}, \text{Contours} \to \{0.00000001, 0.0000000005\}, \text{PlotRange} \to \{1, 7\}, \text{ViewPoint} \to \{0.2, 8, 4\}] 
\]

Fig. 2. Constant level set of magnetic energy, \(|H|^2 = 40\) (arbitrary units)

Field lines

\[
\text{ClearAll}[x, y, z, y1, y2, y3, t] \\
\text{NDSolve}\{\{x'[t] == \text{htotx}[x[t], y[t], z[t]], y'[t] == \text{htoty}[x[t], y[t], z[t]], z'[t] == \text{htotz}[x[t], y[t], z[t]], x[0.00] == .2, y[0.00] == 0.2, z[0.00] == 0.2\}, \{x, y, z\}, \{t, 0, 400\}, \text{MaxSteps} \to 100000\} ; \\
\text{ParametricPlot3D}[\text{Flatten}[\{x[t] / .\%, y[t] / .\%, z[t] / .\%\}], \{t, 0, 14, 0.1\}, \text{BoxRatios} \to \{1, 1, 2\}, \text{PlotRange} \to \{-2, 2\}, \{-2, 2\}, \{-10, 10\}] , \\
\text{ViewPoint} \to \{1.0, -0.6, 0.6\}] 
\]
New results on Ioffe-Ştefănescu magnetic trap

Fig. 3. The total field of the trap in the central region for three values of the energetic function $\|H\|^2$; from the inside outwards:

$\|H\|^2 = 2 \cdot 10^{-4}, 5 \cdot 10^{-4}, 10^{-3}$ (arbitrary units)

Fig. 4. The variation of the energetic function $\|\nabla \|H\|^2\|$ in the central region for two values: from the inside outwards, $\|\nabla \|H\|^2\| = 5 \cdot 10^{-10}$ and $\|\nabla \|H\|^2\| = 10^{-8}$ (arbitrary units)
Fig. 5. Field lines for three different initial conditions: curve 1 \((x = 0.331, y = z = 0)\); curve 2 (dashed, \(x = 0.332, y = z = 0\)); curve 3 (in the plane \(z = 0; x = 0.35, y = z = 0\)).

Fig. 6. Field lines for the initial conditions \(x = y = z = 0.1\).

Fig. 7. Field lines for the initial conditions \(x = y = z = 0.2\).

Fig. 8. Field lines for the initial conditions \(x = y = 0.2, z = 0.5\).
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References


New results on Ioffe-Ştefănescu magnetic trap

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