“CASIMIR-LIKE“ CONTRIBUTION TO THE SPECTRAL ENERGY DENSITY OF PHOTONS DUE TO TSALLIS STATISTICS

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Abstract

By calculating the average thermal energy of photons at high frequencies we demonstrate that a Tsallis like multi-fractal distribution of energy levels of the oscillators making up the photon gas generates a Casimir-Like diminution of the vacuum energy at high frequencies. Such a result points to the importance of including the vacuum energy in all cosmological considerations.

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Key words: Tsalis statistics, energy.

1 Introduction

Classical statistics is based on the assumption of equi-probablity in phase space [1, 2] and quantum statistics is based on the results of the spin statistics connection whose origin is rooted in relativistic quantum field theory [3, 4, 5]. It has been pointed out that of all of the tenets of quantum theory the spin statistics connection is probably the most mysterious [6, 7]. In fact, Geroch, et. al. [8] have pointed out that the spin statistics connection is the result of topological properties in spin space [9] and is not a space time generated property of fermions. In recent years because of studies in non-linearity and multi-fractality [10, 11] there has been a new attitude promulgated by theorists suggesting a new expression for the entropy of a system admitting a multi-fractal structure [12]. Such systems have non-Markovian memory associated with collisions or can admit to long range interactions [13]. The above statistics has been applied to the solar plasma [14, 15, 16], a generalized H theorem [17, 18, 19], the fluctuation dissipation theorem [20], the Langevin and Fokker-Planck equation [21], the equipartition theorem [22], the Ising chain [23, 24], paramagnetic systems

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[25], and the Planck radiation law [26, 27]. Limits on the anomalous statistics parameter can be set by calculating how the new Tsallis statistics affects the primordial helium abundance [28]. In a recent study we have calculated the average thermal energy of each mode (given frequency) of isotropic radiation and demonstrated that because of Tsallis statistics the spectrum cuts off at a given frequency [29]. In the present study we show that if one goes beyond this frequency we may interpret the diminuation of the thermal energy as a Casimir-like diminution of the vacuum state $\left(\frac{\hbar\omega}{2}\right)$.. Traditionally, the Casimir effect is a diminuation of the vacuum energy either due to confining boundary conditions or to a non-trivial topology of the space-time containing the quantum field [30]. The Casimir effect has widespread applications to the bag-model of quarks [31], the Kaluza Klein theory [32], black hole emissions [33] and to cosmologies admitting a non-trivial topology [34]. In what follows we show that even without boundaries or non-trivial topology the radiation field acquires a Casimir-like contribution to the vacuum energy at any temperature. Such an effect could lead to both cosmological consequences and the possible self-binding of radiation which is ordinarily forbidden by conventional physics [35]. It also suggests that multi-fractality generates a coupling of the energy levels of a harmonic oscillator that simulates the presence of boundaries to a system that does not have any geometric boundaries.

2 Casimir-Like Contribution to the Thermal Energy of Photons

We begin by writing the expression for the entropy of N particles within the framework of Tsallis statistics

$$S = \frac{kN}{q-1}\left(\sum \frac{N_i}{N} - \sum \left(\frac{N_i}{N}\right)^q\right) \tag{2.1}$$

$$q = \text{non-extensive parameter.}$$

Varying Eq. (2.1) with respect to N_i and using the constraints

$$N_i = const. \tag{2.2}$$

$$N_i \varepsilon_I = N \tag{2.3}$$

with Lagrange multipliers $\left(\frac{\mu}{\tau}, \frac{-1}{\tau}\right)$ we find

$$N_i = Ne^{-1} e^{\frac{\mu_0 - \varepsilon_i}{\tau}} + \alpha N e^{-1} \left[\begin{array}{c} \frac{1}{2\tau^2} \frac{e^{\frac{\mu_0-\varepsilon_i}{\tau}} \sum (\mu_0 - \varepsilon_i)^2 e^{\frac{\mu_0-\varepsilon_i}{\tau}}}{\sum e^{\frac{\mu_0-\varepsilon_i}{\tau}}} \\ -\frac{1}{2\pi} e^{\frac{\mu_0-\varepsilon_i}{\tau}} \frac{(\mu_0-\varepsilon_i)^2}{2\tau^2} \end{array} \right], \tag{2.4}$$

$$e^{\frac{\mu_0}{\tau}} = \frac{e}{\sum e^{-\frac{\varepsilon_i}{\tau}}}, \text{ (here } \tau = kT).$$

Eq. (2.4) follows from writing $\alpha = q - 1$, $N_i = N_{i0} + \alpha N_{i1} + \alpha^2 N_{i2} \cdots$, $\mu = \mu_0 + \mu_1 + \alpha^2 \mu_2$ and using perturbation theory to evaluate N_{i1} and μ_i [36]. From Eq. (2.4) we find for the average energy of an oscillator [29]

$$\langle \varepsilon \rangle = \frac{\sum \varepsilon_i N_i}{N} = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\frac{\hbar\omega}{\tau}} - 1} - \frac{\alpha\,(\hbar\omega)^3}{8\tau^2} + \alpha\frac{(\hbar\omega)^2}{\tau}. \tag{2.5}$$

In [29] we showed that apart from the vacuum energy we have a cut-off at

$$\frac{\hbar\omega_c}{\tau} = X_c,$$

where $e^{-X_c} = \frac{\alpha X_c^2}{8} - \frac{\alpha X_c}{8}$, such that

$$\langle \varepsilon \rangle - \frac{\hbar\omega}{2} = 0. \tag{2.6}$$

$$\left(\text{Note } \frac{1}{e^{\frac{\hbar\omega}{\tau}} - 1} \cong e^{\frac{\hbar\omega}{\tau}} \text{ for } \frac{\hbar\omega}{\tau} > 1.\right)$$

If instead of just cutting off the spectrum at ω_c we consider the total energy for $\omega > \omega_c$ we have

$$\langle \varepsilon \rangle = \frac{\hbar\omega}{2} + \hbar\omega e^{-\frac{\hbar\omega_T}{\tau}} - \alpha\frac{(\hbar\omega)^3}{8\tau^2} + \alpha\frac{(\hbar\omega)^2}{8\tau} < \frac{\hbar\omega}{2} \tag{2.7}$$

for $\omega_c < \omega < \omega_T$, where ω_T obeys

$$\frac{\hbar\omega_T}{2} + \hbar\omega_T e^{-\frac{\hbar\omega_T}{\tau}} - \alpha\frac{(\hbar\omega_T)^3}{8\tau^2} + \alpha\frac{(\hbar\omega_T)^2}{8\tau} = 0. \tag{2.8}$$

Thus for $\omega_c < \omega < \omega_T$ the thermal energy is less than the vacuum energy and can be interpreted as a Casimir energy. Eq. (2.8) can be written as $\left(X_T = \frac{\hbar\omega_T}{\tau}\right)$:

$$\frac{1}{2} + e^{-X_T} - \frac{\alpha}{8}(X_T)^2 + \frac{\alpha}{8}(X_T) = 0. \tag{2.9}$$

The Casimir contribution to the total energy per unit volume is found from the second, third and fourth terms in Eq. (2.7) for $\omega_c < \omega < \omega_T$:

$$U_C = \int_{v_C}^{v_T} \left(\hbar\omega e^{-\frac{\hbar\omega_T}{\tau}} - \alpha\frac{(\hbar\omega)^3}{8\tau^2} + \alpha\frac{(\hbar\omega)^2}{8\tau}\right) \frac{8\pi v^2 dv}{C^3}.$$

Eq. (2.10) must have a negative sign and can be viewed as a binding energy per unit volume brought about by the use of the non-extensive statistics of Tsallis.

3 Conclusion

The fascinating result that a multi-fractal distribution of states applied to the harmonic oscillator gives rise to a thermal energy having a Casimir contribution at high suggests that radiation can condense without the presence of matter or catalysts such as strings, domain walls or monopoles. It also suggests that fluctuations in the CBR might be induced by a Tsallis parameter varying with position. For decades it was believed that embryonic structure of matter emerged from quantum field theory whose primitive components were Fermi and Bose fields. The present study suggests that in fact many properties of matter and radiation in the large may derive their properties from an anomalous statistical behavior whose primitive origin lies in the domain of scale invariance and the multi-fractral distribution of energy levels.

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