CLASSIFICATION OF ISOUNITS

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Abstract

The aim of the present paper is to gives some examples of isounits which are used in algebraic isostructures

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1 Introduction

The isostructure construction can be applied in algebraic, topological, differentiable and geometrical objects. The aim of the present paper is to study the classification of the isounits in some algebraic isostructures. The paper contains three sections. The first section is the introduction. The algebraic isostructures are studied in the second section. In the third section we study some special isostructures. From this study we give a classification of some isounits.

2 Algebraic structures

We consider a set A and an element $\hat{I}$ which does not need to belong in the set A. From A and $\hat{I}$ we can construct the set $\hat{A}$, which is defined as follows:

$$\hat{A} = \{\hat{\alpha} = \hat{\alpha}\hat{I} \parallel \alpha \in A\},$$

under the assumption that it's possible the $\hat{\alpha} = \alpha\hat{I}$ can be defined. The element $\hat{I}$ is called *isounit* and the set $\hat{A}$ is called *isoset associated* to A with isounit $\hat{I}$.

From the above we have a mapping:

$$l : A \to \hat{A}\ ,\ l : \alpha \longmapsto l(\alpha) = \hat{\alpha} = \alpha\hat{I},$$

which is called an *isotopic lifting*.

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If A has an algebraic structure, then $\hat{A}$ has another algebraic structure which is called *algebraic isostructure associated* to the first algebraic structure.

In order to be clear we give an example:

Let A be a group with operation $\square$, that means:

$$\square : A \times A \longrightarrow A,\ \square : (\alpha_1,\alpha_2) \longmapsto \alpha_1 \square \alpha_2 .$$

From A with isounit $\widehat{I}$ we obtain the isoset $\hat{A}$, as follows:

$$\hat{A} = \left\{ \hat{\alpha} = \alpha \hat{I} \parallel \alpha \in A \right\} .$$

Now, on A we can define the operation $*$ as follows:

$$* \ : \hat{A} \times \hat{A} \longrightarrow \hat{A},$$

$$* : (\hat{\alpha}_1,\hat{\alpha}_2) \longmapsto \hat{\alpha}_1 * \hat{\alpha}_2 \overset{def}{=} \alpha_1 \, \hat{I} \, \hat{I}^{-1} \, \alpha_2 \, \hat{I} \ = \alpha_1 \square \alpha_2 \, \hat{I} \ = \alpha_1 \, \widehat{\square} \, \alpha_2 \, .$$

Remark 2.1 We assume that the isounit $\hat{I}$ comes from an algebraic set Γ which has a unit e and there is another element $\hat{I}^{-1}$ such that:

$$\hat{I} \, \hat{I}^{-1} = e.$$

Remark 2.2 On the set A we can consider other algebraic structure which makes it a field, vector space, algebra, Lie algebra etc.

Therefore, we obtain the notions *isofield*, *isovector space*, *isoalgebra*, *Lie isoalgebra* etc.

3 Some special units

We consider the general I.ie group

$$GL(n, I\!R) = \{ A \in gl\,(n, I\!R) \parallel \ D\,(A) \ \neq 0 \} .$$

This set is a group with operation the multiplication of matrices, that is:

$$\begin{aligned} * \quad &: \quad GL\,(n, I\!R) \times GL\,(n, I\!R) \longrightarrow GL\,(n, I\!R)\,, \\ * \quad &: \quad (A, B) \longmapsto A \cdot B \ \text{(product of matrices)}. \end{aligned}$$

The unit of this group is:

$$I_n = \begin{bmatrix} 1\,0 \ldots \ 0 \\ 0\,1 \ldots \ 0 \\ \ldots\ldots\ldots \\ 0\,0 \ldots \ 1 \end{bmatrix},$$

which is the identity matrix.

Now, we obtain a symmetric matrix $B \in GL(n, I\!R)$, where $B \neq I_n$ *and* $D(B) \neq 0$, and we construct the isoset $\widehat{GL}\ (n, I\!R)$, with isounit B, that is:

$$\widehat{GL}_B\ (n, I\!R) = \left\{ \widehat{A} = AB \diagup A \in GL(n, I\!R) \right\}.$$

Proposition 1 *The set* $\widehat{GL}_B\ (n, I\!R)$ *can become a group with the following operation* $*$ *defined as follows*

$$\widehat{A}_1 * \widehat{A}_2 = A_1 B B^{-1} A_2 B = A_1 A_2 B = \widehat{A_1 A_2}\ .$$

Proof. *For the operation* $*$ *we have:*

$$* \ : \widehat{GL}_B\,(n, I\!R) \times \widehat{GL}_B\,(n, I\!R) \longrightarrow \widehat{GL}_B\,(n, I\!R)\,,$$

$$* \ : \left(\widehat{A}_1, \widehat{A}_2\right) \longmapsto \widehat{A}_1 \times \widehat{A}_2 \stackrel{def}{=} A_1 B B^{-1} A_2 B = A_1 A_2 B = \widehat{A_1 A_2}.$$

It is can easily be proved that this operation $*$ satisfies the properties for a group. Therefore, $\widehat{GL}_B\,(n, I\!R)$ is a group which is the isogroup associated to $GL(n, I\!R)$ with isounit B. $\square$

It is important to classify these isounits B. The classification is given by the following result.

Theorem 2 *The classification of isounits* B *is given by the following matrices:*

$$\widehat{I}_n\,(\varepsilon_1, \varepsilon_2\,, \ldots, \varepsilon_n) = \begin{bmatrix} \varepsilon_1\ 0\ \ 0 \ldots 0 \\ 0\ \varepsilon_2\ 0 \ldots 0 \\ 0\ \ 0\ \varepsilon_3 \ldots 0 \\ \ldots\ldots\ldots\ldots \\ 0\ \ 0\ \ 0 \ldots\ \varepsilon_n \end{bmatrix},$$

where $\varepsilon_i = 1$ *or* -1 *and* $i = 1, 2, ..., n$.

Proof. Since the matrix B is symmetric,there is a base in $I\!R^n$, such that B takes the form :

$$B = \begin{bmatrix} \varepsilon_1\ 0\ \ 0 \ldots\ 0 \\ 0\ \varepsilon_2\ 0 \ldots\ 0 \\ 0\ \ 0\ \varepsilon_3 \ldots\ 0 \\ \ldots\ldots\ldots\ldots \\ 0\ \ 0\ \ 0 \ldots\ \varepsilon_n \end{bmatrix},$$

where $\varepsilon_i = 1$ or -1 and $i = 1, 2, ...n$. Therefore,we have only the isounits $\dot{I}_n = (\varepsilon_1, \varepsilon_2\,, \ldots, \varepsilon_n)$. $\square$

The number of isounits of this case is finite, which is 2^n.

This theorem has some applications of some technical problems which are used matrices for models.

Problem 3.1 Classify the isounits when $D(B) \neq 0$, and B is not symmetric.

Problem 3.2 Classify the isounits when $D(B) = 0$.

Problem 3.3 Classify the íisounits for the Lie algebra $gl\,(n, I\!R)$.

We have some results on these problems, but they will be matter of an other paper..

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