UNlFlCATION OF LIE ALGEBRAS $so(p, q)$, $p + q = n$, $p = \overline{1, n}$, $q = \overline{0, n}$

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Abstract

The aim of the present paper is to give a simple method in order to classify the Lie algebras $so(p, q)$, $p + q = n$, $p = \overline{1, n}$, $q = \overline{0, n}$

AMS Subject Classification: 17B30.
Key words: Lie algebra, isounit, isoset and special orthogonial group.

1 Introduction

It is known that the simple Lie algebras are A_n, B_n, C_n, D_n which are called *classical* and the *exceptional* G_2, F_4, E_6, E_7, E_8. There is an open problem which can be stated as follows: *Is there a vector space V such that if we apply different interior laws we can obtain those Lie algebras?*

From the Lie algebras B_n and D_n which are of the form $so(n)$, if n is odd we obtain B_n and if n is even we get D_n. However from these Lie algebras B_n and D_n we obtain some other of the form

$$so(p, q),\ p + q = n,\ p, q = .0, 1, \dots, n.$$

Therefore we obtain different Lie algebras and if $p = n$, $q = 0$ we have B_n and D_n. The purpose of this paper is to unify all the Lie algebras $so(p, q)$, $p + q = n$, $p, q = .0, 1, \dots, n$ with $p \geq q$, using isotopic theory.

The whole paper contains three sections. The first is the introduction. The general theory is included in the second section. The last section gives the method of the classification of the Lie algebras $so(p, q)$, $p + q = 3$, $p, q = 0, 1, 2, 3$.

Editor Gr.Tsagas *Proceedings of The Conference of Applied Differential Geometry - General Relativity and The Workshop on Global Analysis, Differential Geometry and Lie Algebras, 2000,* 184-186

2 Isostructures

Let $\{\Gamma,\circ,e\}$ be a *genetic structure* that means a set.Γ, an interior law of composition $\circ$ in Γ with the identity element e. We consider a set $s\neq\emptyset$ and suppose that the following set

$$\hat{S}=\left\{\ \hat{s}\ /\ \hat{s}=s\hat{I},\ s\in S,\ \hat{I}\in\Gamma\right\}$$

can be defined. The set $\hat{S}$is called *isotopic set*, the element $\hat{I}$ is called an *isounit* and the inverse element $\hat{I}^{-1}$of $\hat{I}$ is called an *isotopic element*: Therefore we can consider a map

$$I:S\rightarrow\hat{S},\ I:s\rightarrow\hat{s}=s\hat{I}$$

which is called an *isotopy*.

If S has some algebraic structures involving the isotropy, these structures can be transferred in $\hat{S}$. We are going to use this isostructure theory in order to unify some Lie algebras.

3 Unification of Lie algebras $so(3,0)$, $so(2,1)$

We consider the algebra $so(3,0)$ which consists of matrices of the following form:.

$$so(3,0)=\left\{A=\begin{bmatrix}0&\alpha&\beta\\-\alpha&0&\gamma\\-\beta&-\gamma&0\end{bmatrix}\ /\ \alpha,\beta,\gamma\in I\!R\right\}.$$

Now we consider an arbitrary matrix $\hat{I}\in GL(3,I\!R)$ (the group of invertible 3×3 real matrices), of the form:

$$\hat{I}=\begin{bmatrix}\alpha_1&\alpha_2&\alpha_3\\\beta_1&\beta_2&\beta_3\\\gamma_1&\gamma_2&\gamma_3\end{bmatrix}.$$

Now we create the isostructure set $\widehat{so}\,(3,0)$:

$$\widehat{so}\,(3,0)=\left\{\hat{A}=A\hat{I}\right\},$$

$$\widehat{so}\,(3,0)=\left\{\hat{A}=\begin{bmatrix}0&\alpha&\beta\\-\alpha&0&\gamma\\-\beta&-\gamma&0\end{bmatrix}\begin{bmatrix}\alpha_1&\alpha_2&\alpha_3\\\beta_1&\beta_2&\beta_3\\\gamma_1&\gamma_2&\gamma_3\end{bmatrix}\right\}.$$

Using the vector space $\widehat{so}\,(3,0)$ we can obtain the Lie algebras $so\,(3,0)$ and $so\,(2,1)$ as follows:

$$\widehat{so}\,(3,0)\,\hat{I}^{-1}=so\,(3,0)\,,$$

$$\widehat{so}\,(3,0)\,T_1=so\,(2,1)\,,$$

where $T_1 = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ -\gamma_1 & -\gamma_2 & -\gamma_3 \end{bmatrix}^{-1}$. Using the above constructions, the following result can be proved.

Proposition *The Lie algebras* $so(3,0)$ *and* $so(2,1)$ *can be obtained on the same vector space using different isounits and the isotopic theory.*

References

[1] Janussis A., Mignani R. and Santilli R.M., *Some problematic aspects of Weinberg's nonlinear theory*, Ann. Fond. De Broglie (to appear).

[2] Santilli R.M., *On the possible Lie -Admissible covering of the Galilei Relativity in Newtonian Mechanics for non conservative and Galilei non variant systems*, Hadronics (1978) 223 - 423.

[3] Sourlas D.S. and Tsagas G.T., *Mathematical foundations of the Lie Santilli theory*, Academy of sciences of Ukraine, 1993.

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