Extraction of B-H hysteresis model from measurements with non-uniform magnetic field[1]

Daniel Ioan, Member, IEEE and Mihai Rebican

Abstract. The paper describes a magnetic field inverse problem and an efficient numerical method to solve it. Starting from the experimental flux-current $\Phi - I$ data obtained using a non-uniform magnetic field device with known geometry, the $B - H$ nonlinear characteristic of the homogeneous magnetic core is extracted. Both functions $B - H$ and $\Phi - I$ are piecewise linear approximated with the same number of breakpoints. The method was successfully applied to characterize the ribbon core material of a FLUXSET magnetic field sensor. In this case, the hysteresis loop and the equivalent lumped magnetic nonlinear circuit were extracted. Comparison with experimental results validates the proposed method.

1. Introduction

The classical methods to measure the $B - H$ relations of magnetic materials preferably use devices with uniform magnetic field, such as: coils with toroidal core or Epstein frames. In this case the $B - H$ characteristic is obtained by appropriate scaling of the flux-current $\Phi - I$ relation. Due to technological restrictions (e.g. parasitic air gaps), not even these devices can ensure a perfectly uniform magnetic field in the ferromagnetic material sample.

If the sample has small dimensions or an irregular shape, it can be tested only in an open magnetic circuit. In these particular cases the magnetic field is strongly nonuniform and the measured data are influenced by on the shape-dependent demagnetization factor. To eliminate this drawback it is necessary to use a method able to extract the $B - H$ relation, even if the magnetic field is a non-uniform one, such as that described in the present paper.

2. Problem Formulation

Let us consider (Fig. 1) a homogeneous nonlinear ferromagnetic body $\mathcal{D}_m$ (having $\mathbf{B}\|\mathbf{H}$ and a monotonic, bijective $B - H$ relation) surrounded by air, and a non-magnetic coil $\mathcal{C}$ with the magnetic flux φ generated by the current i.

[1] Proceedings of The First French-Romanian Colloquium of Numerical Physics, October 30-31, 2000, Bucharest, Romania.
Manuscript received October 24, 1999. This work was supported by the INCO-COPERNICUS MANODET project (Contract no. ERBIC15CT969703) financed by the European Commission and by the Romanian ANSTI Agency, under the contract no. 836/A21/1999.

The magnetostatic field equations are:

$$\begin{array}{ll} \operatorname{div}\mathbf{B} = 0, & \text{in } \mathbb{R}^3; \\ \operatorname{curl}\mathbf{H} = \mathbf{J}, & \text{with } \mathbf{J} = 0 \text{ in } \mathbb{R}^3 \backslash \mathcal{C}; \\ \mathbf{B} = f(H) \cdot \mathbf{H}/H, & \text{in } \mathcal{D}_m; \\ \mathbf{B} = \mu_0 \mathbf{H}, & \text{in } \mathbb{R}^3 \backslash \mathcal{D}_m. \end{array} \tag{1}$$

Considering a given geometrical configuration and knowing function $g : \mathbb{R} \to \mathbb{R}$ with $\varphi = g(i)$, the problem is to find the function $f : \mathbb{R} \to \mathbb{R}$, which is the magnetic characteristic $B = f(H)$ of the ferromagnetic material in the domain $\mathcal{D}_m$, assuming that f is not dependent on the point $P \in \mathcal{D}_m$, and $f(0) = 0$.

The current density $\mathbf{J}$ depends linearly on the total current i, and for the simplicity we assume that the coil wire is relatively thin. Therefore the current is distributed uniformly across its section.

The **formulated problem is an inverse one**, because it is not the field but the material characteristic that must be found, starting from the "external" data. This problem has a unique solution [1].

3. B-H Characteristic Extraction Algorithm

The algorithm we propose to solve the problem (1) has the following steps:

- **Generate a piece-wise linear approximation** of the $\varphi - i$ characteristic with an imposed error, by the selection of appropriate m points (φ_k, i_k) from the experimental data (Fig. 2).
- Consider a linear magnetostatic field problem with $B = \mu H$ in D_m, and find $\mu = \mu_1$ such that the magnetic flux due to the current i_1 is φ_1. This is accomplished **by iteratively solving the nonlinear equation**: $\varphi(i_1) = \varphi_1$, where φ is the numerical solution of the forward magnetostatic field problem (1), linear at low field.
- Compute the maximum magnetic field in D_m (H_{max}, $B_{max} = \mu_1 H_{max}$) and set the first two points on the extracted $B - H$ characteristic: $H_0 = 0$, $B_0 = 0$ and **the first breakpoint** $H_1 = H_{max}$, $B_1 = B_{max}$.

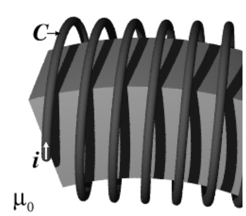

Fig. 1 A device with non-uniform magnetic field

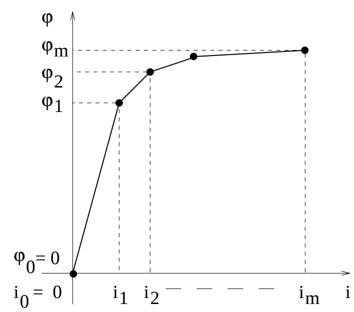

Fig. 2 Piece-wise linear $\varphi - i$ characteristic

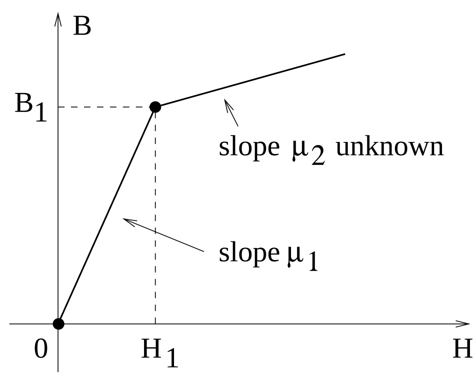

Fig. 3 The extracted $B - H$ characteristic

- **Iteratively solve the forward magnetostatic field** problem (1) with a non-linear $B - H$ characteristic with $k = 1$ breakpoints (as in Fig. 3) and **find** μ_2 such that the magnetic flux due to the current i_2 is: $\varphi(i_2) = \varphi_2$.

- Each point of $\mathcal{D}_m$ has different static permeability. The maximum magnetic field H_{max}, B_{max} due to i_2 represents **the second breakpoint** of the $B - H$ characteristic.

- **Repeat iteratively the last two steps** considering $k = 2, 3, \ldots$, until the breakpoint number m of the $\varphi - i$ characteristic is reached.

The computing effort to solve the inverse problem is directly linked to the number m of breakpoints on the $\varphi - i$ characteristic. This number is related to the approximation error generated by linearization on each segment. If the imposed error ε is known (usually of the order of measurement error), the number of breakpoints can be reduced by the selection step 1 of the algorithm. Starting from 418 experimental data points, we obtained $\varepsilon = 1\%$ with $m = 5$ breakpoints. Roughly speaking, the halving of the approximation error leads the doubling of the number of breakpoints.

4. Field computation and results

An important decision to be taken in order to apply the proposed algorithm concerns the numerical method used to compute the magnetic field.

In the case of the FLUXSET Sensor (Fig. 4), the nonlinear magnetostatic field problem was solved with different methods: FEM [2] [3], hybrid numerical–analytical method [4], and integral equations method [5]. Comparing the necessary computing resources (CPU time and memory), accuracy and reliability of the code, we can conclude that the integral equation method seems to be the most suitable for this problem.

Satisfactory numerical results were obtained by discretizing the ribbon core along the axis in a 1D uniform grid of N rectangles, each having as unknown the axial magnetization M_i. The nonlinear system of equations:

$$M_i + \chi_i(M_i) \sum_{j=1}^{N} a_{ij} M_j = b_i, \quad i = 1, N \tag{2}$$

was successfully solved with a low computing effort, by using an adapted version of the Katzenelson method [4].

The ribbon was discretized in 15 rectangular elements, the coil was discretized in the axial direction in 10 slides and each oval slide in 24 segments.

The nonlinear scalar equation $\varphi(i_k) = \varphi_k$ used to find μ_k for $k = 1, m$, was efficiently solved with the “secant” method. The algorithm was implemented in Scilab programming language [6] and was run on a Pentium PC/160MHz.

Table 1 presents the main numerical results (saturation flux density B_s, the relative initial permeability μ_{ri} and CPU time) for different numbers of breakpoints.

The B_s value is not sensitive to the number of breakpoints (the modification is less than 0.3%) but the initial permeability has values within ±8% range.

Less than 11 iterations (field computations) were enough for each k to obtain the solution with a numerical relative error less than the experimental one. Starting from

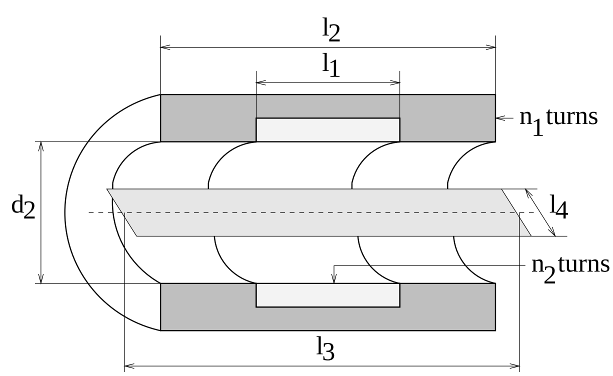

Fig. 4 The FLUXSET Sensor

Table 1: Numerical results

m	5	10	23	36
B_s [T]	0.7043	0.7042	0.7057	0.7068
μ_{ri}	40235	46485	48047	45703
CPU Time [s]	120	260	840	1520

the $\varphi - i$ dependence with $m = 10$ breakpoints represented in Fig. 5, the $B - H$ characteristic of the core's material presented in Fig. 6 was extracted.

5. Fundamental Hysteresis Loop Extraction

In the particular case of the 1D FLUXSET model, the vectors **B**, **H** and **M** in the magnetic ribbon-core are oriented along the same axial direction (Ox). Let us consider the dependence $\varphi = g_1(i)$, valid for a decreasing current i, after a strong positive saturation.

The algorithm presented in the previous section, applied in reversed order $k = m, m-1, \ldots, 1$, can be used to extract the descendent branch f_1 of the fundamental $B - H$ hysteresis loop. To extract the breakpoint $k = m$, we will consider a strong saturation, therefore the differential permeability is $\mu_d = \mu_0$ in the entire domain $\mathcal{D}_m$. The $B - H$ relation is $B_x = \mu_0 \, H_x + I_m$ and the first solved nonlinear equation provides the value of I_m. The first extracted breakpoint (B_m, H_m) corresponds to the minimum field values in $\mathcal{D}_m$.

This approach represents an improvement of the method presented in [1].

The hysteresis loop of the FLUXSET sensor's ribbon, extracted with this technique, is presented in Fig. 7.

At the driving frequency of 100kHz the remanent flux density is $B_r = 0.2245\,\mathrm{T}$

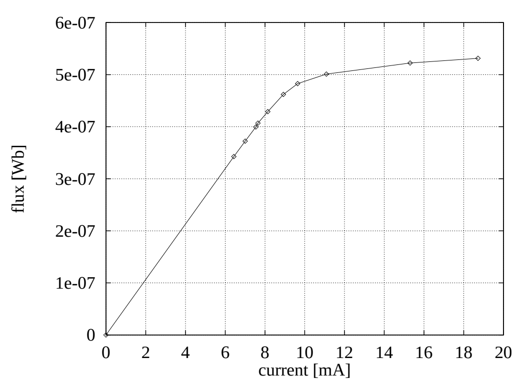

Fig. 5 Piece-wise linear $\varphi - i$ characteristic

and the coercive field is $H_c = 8.0966\,\mathrm{A/m}$. At the frequency 10kHz the obtained values are $B_r = 0.048\,\mathrm{T}$, $H_c = 0.425\,\mathrm{A/m}$.

6. Dynamic hysteresis model

One of the simplest hysteresis models of the dynamic $B - H$ relation is the Chua-Stromsmoe model [7]. It is completely specified by two monotonous increasing functions: f-the "*restoring function*" which describes the energy storage and g-the "*dissipation function*" which describes the energy dissipation. The increase of the loop area with frequency is provided by the model, but in order to predict dc behavior a more sophisticated "generalized hysteresis model" is needed.

The Chua-Stromsmoe model is based on the differential equation:

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{dt}} = \mathbf{g}[\mathbf{H}(t) - \mathbf{f}(\mathbf{B})] \tag{3}$$

where **B** and **H** have the same direction.

The dynamic system associated to an iron-core inductor has the state equation:

$$\frac{\mathrm{d}\varphi}{\mathrm{dt}} = g[i(t) - f(\varphi)], \tag{4}$$

where f and g are computed based on the material characteristic functions **f** and **g** and on the geometrical dimensions of the inductor. Both f and g are odd functions with $f(0) = g(0) = 0$ in the case of 180° rotational symmetric hysteresis loop. Considering $\varphi(t) = \varphi_{max} \sin \omega t$, the corresponding current can be decomposed in its odd and even components $i(t) = i_o(t) + i_e(t)$ with

$$i_o(t) = f(\varphi(t)),\ i_e(t) = g^{-1}(\varphi'(t)).$$

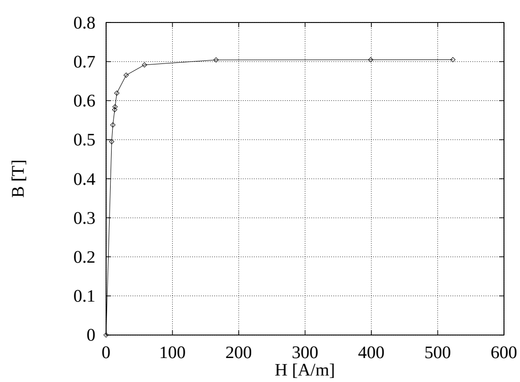

Fig. 6 The extracted $B - H$ characteristic

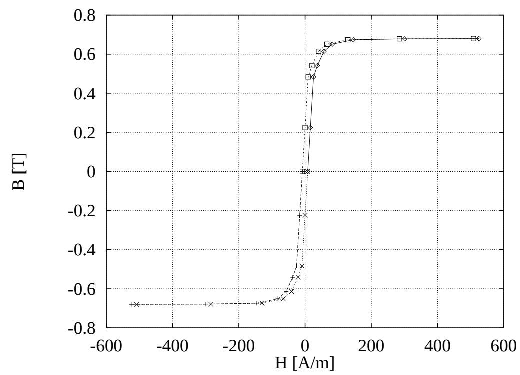

Fig. 7 The $B-H$ hysteresis loop of the FLUXSET ribbon

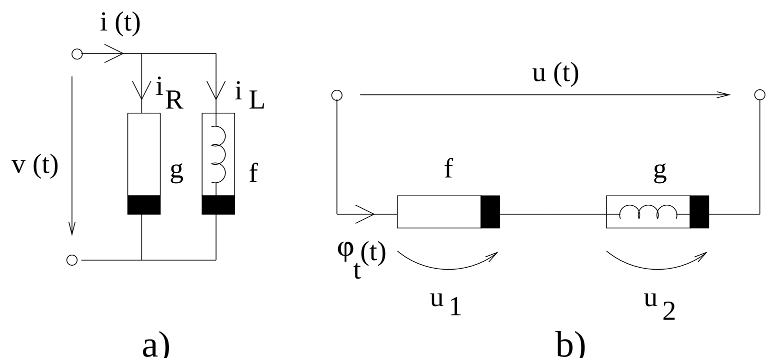

Fig. 8 Lumped-circuit models

The two characteristic functions can be constructed using the two $i-\varphi$ dependences of a fundamental loop namely $i_2(\varphi)$-the descendent branch and $i_1(\varphi)$-the ascendent branch:

$$\varphi = f^{-1}[i_1(\varphi) + i_2(\varphi))/2],$$
$$\varphi' = \varphi_{max}\,\omega\cos\omega t = g[(i_2(\varphi) - i_1(\varphi))/2].$$

Considering an iron-core inductor and using the consequence of (4):

$$i(t) = g^{-1}[v(t)] + f[\varphi(t)], \tag{5}$$

where $v(t) = \mathrm{d}\varphi/\mathrm{d}t$ is the induced voltage, the lumped-circuit model in Fig. 8a is obtained. The $v - i_R$ relationship of the resistor is given by $v = g(i_R)$ and the $i_L - \varphi$ relationship of the inductor is given by $i_L = f(\varphi)$.

The relation (5) could have another interpretation: from a magnetic circuit point of view, the lumped magnetic circuit model consists of two elements connected in series (Fig. 8b). One is a nonlinear magnetic reluctance with $u_1 - \varphi_t$ constitutive relation given by $u_1 = n\, f(n\varphi_t)$ and the other is a nonlinear "dumping inductor" with the $u_2 - \varphi_t$ constitutive relation given by:

$$\frac{\mathrm{d}\varphi_t}{\mathrm{d}\,\mathrm{t}} = \frac{1}{n} g\left(\frac{u_2}{n}\right), \tag{6}$$

where n is the number of turns, $\varphi_t = \varphi/n$ is the one-turn magnetic flux and $u = u_1 + u_2 = n\,i$ is the total current. The two representations in Fig. 8 satisfy the duality theorem between the electric and the magnetic circuits of an inductor [8].

7. Experimental verification

Because there are no available experimental data to check directly the local $B - H$ relation, the only possibility is to use an indirect verification based on measured global quantities (φ, i). This approach is correct due to the uniqueness theorem of inverse problem solution.

Figures 9 and 10 show the numerical $\varphi - i$ hysteresis loops obtained at two different frequencies (100kHz and 10kHz) together with the experimental ones.

Numerical data are obtained by SPICE simulation of the lumped circuit built using the $B - H$ loop of the core material, extracted from field solution.

Using this lumped-circuit model, the global behavior of the FLUXSET sensor can be predicted by SPICE simulation in a couple of seconds.

Fig. 11 presents the FLUXSET pick-up voltage, when the external field is increasing with a constant slope of $1.25\, T/s$ and the driving current is triangular-shaped with period $T = 10\, \mu s$ and $I_{max} = 20\, mA$.

Notice that the magnetic field spatial distribution, magnetic nonlinearity and especially the hysteresis play important roles in this simulation. To obtain a similar result, the "brute force" approach based on numerical solution of electromagnetic field equations such as in [2] would need at least a couple of hours.

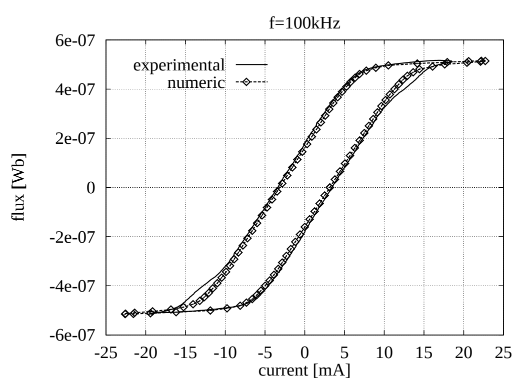

Fig. 9 The $\varphi - i$ hysteresis loop at 100 kHz

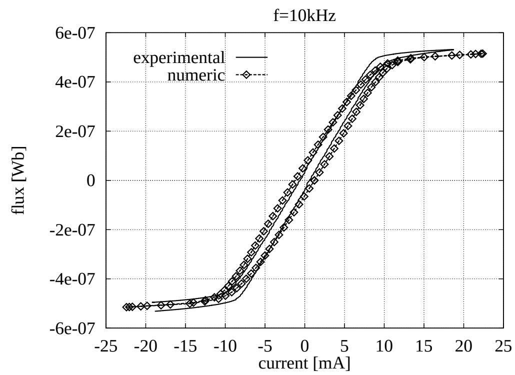

Fig. 10 The $\varphi - i$ hysteresis loop at 10 kHz

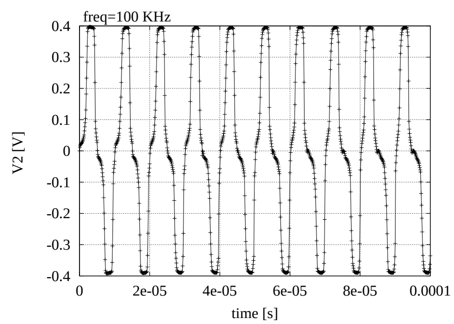

Fig. 11 The output pick-up voltage of FLUXSET

The pick-up voltage in Fig. 11 allows to compute the FLUXSET transducer characteristic: $\Delta T(H_{ext})$, where ΔT is the pick-up pulse time shift and H_{ext} is the external measured field (Fig. 12).

8. Conclusions

The paper presents an algorithm able to solve the inverse magnetostatic nonlinear problem and to extract the $B - H$ characteristic of the tested ferromagnetic samples from the $\varphi - i$ dependence, regardless the sample's shape and field uniformity. The non-uniform magnetic field in the nonlinear core was numerically computed using the integral equations method. An adapted version of Katzenelson method was successfully used to solve the nonlinear system of equations obtained by discretisation. This approach of forward problem solving proved to be more efficient in our case than other method such as: FEM or hybrid numerical-analytical techniques.

A modified version of the algorithm, dedicated to the fundamental $B - H$ loop extraction for wire-shaped samples, is presented in the paper. It was successfully applied to extract the $B - H$ fundamental hysteresis loop of the ribbon-shaped core

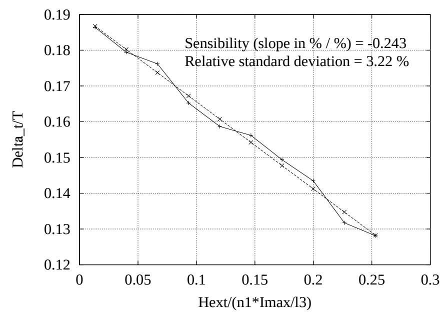

Fig. 12 The FLUXSET response

of a FLUXSET magnetic field sensor. Based on it, a hysteresis model for cores material and a lumped magnetic circuit for sensor are proposed.

Using SPICE simulation of the extracted lumped circuit, good agreement with the experimental data was obtained. The proposed method is able to determine in a very effective manner the sensibility and frequency bandwidth of a newly designed FLUXSET sensor.

Acknowledgments

The authors would like to express their thanks to Dr. Irina Munteanu for her valuable help and comments.

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Daniel Ioan and Mihai Rebican
University "POLITEHNICA" of Bucharest,
Electrical Engineering Department,
Numerical Methods Laboratory,
Splaiul Independentei 313,
RO-77206 Bucharest, Romania.
E-mail: [daniel, mihai_r]@lmn.pub.ro;