Atlas of Magnetic Geometric Dynamics (Book Presentation)

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1 Preface

Since 1925 Magnetic Fields around picewise rectilinear electric circuits and their applications to Geodynamics, Thermonuclear Fusion, and Confinement of Plasma have been intensively studied by a leading scientist in the field, the former Acad. Sabba Ştefănescu (1902-1994). Just him suggested some open problems and conjectures to be studied because, in his opinion, the behaviour of elementary magnetic fields is not yet well understood and fructified in practical problems being able to generate important mathematical surprises. After ten years of research work, we can offer the first version of the MAGNETIC DYNAMICS ATLAS which includes our original ideas.

Chapter 1 concerns with history and background regarding the morphology of Biot-Savart fields produced by suitable spatial configurations. The topics include: Sabba Ştefănescu conjectures, numerical and graphical simulation, morphology of magnetic lines and surfaces, energy density of magnetic field, equilibrium sets, phase portraits, geometric magnetic dynamics, heteroclinic connections, homoclinic connections, Riemann-Jacobi or Riemann-Jacobi-Lagrange structure, Finsler-Jacobi structure, magnetic traps, and much more. At the end of the chapter we introduce the reader into single-time and multi-time Lagrangian dynamics.

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Chapters 2 and 3 summarize my recent discovery of CONTINUOUS AND DISCRETE GEOMETRIC DYNAMICS, as a framework for the science and engineering of the future. This will allow to predict and simulate new processes of states in the biological systems we consist of, in the ecological and economical systems we live in, and in the technical systems we make use of, via generalized Lorentz laws.

As it is shown in our papers (see the references at the end of each chapter), the philosophy of geometric dynamics has two sources:

- a vector field and a Riemannian metric produce a dynamics (single-time geometric dynamics), described by the "rot" of the field and the "grad" of the energy density, whose trajectories are harmonic maps;

- a differentiable distribution and two Riemannian metrics produce a dynamics (multi-time geometric dynamics), described by the "rot" of the distribution and the "grad" of the energy density, whose parametric sheets are harmonic maps.

All this theory leads to the following conjecture: FOR EACH PAIR (VECTOR FIELD, METRIC) OR (DISTRIBUTION, PAIR OF METRICS) THERE EXISTS A "PARTICLE" WHICH IS SENSITIVE TO THE PAIR ("ROT", "GRAD").

Chapters 4 and 6 use our "mind's eye" to visualize the abstract objects and processes that are connected directly with the geometric dynamics. It benefits of the remarkable improvements in MAPLE 6 who permit now to externalize some vague and subjective pictures that we "see" in our heads, replacing them with concrete visualizations that can be shared with others.

One important lesson I have learned from my own experience is that a new theory must be checked using as starts classical models. That is why we comprise some classical flows into geometric dynamics, displaying the mathematical objects with MAPLE 6-software techniques.

We have been working on a mathematical visualization program regarding magnetic geometric dynamics for more than ten years now using different kinds of computer programs. The highly interactive nature of the images produced by us in MAPLE 6 will allow to the scientists to do mathematical and physical experiments with an accuracy never before possible. Also the animation produced in our laboratory of MATHEMATICAL VISUALIZATION AND COMPUTER GRAPHICS shows a related family of mathematical objects that arise by some procedures naturally associated to magnetic fields around piecewise rectilinear wires. Besides scientific products, we hope that our magnetic geometric visualization will be used in future by talented graphic artists and sculptors, to create new paintings and sculptures. In this sense, I agree with Prof. Dr. Richard S. Palais [see Notices of the AMS, 46, 6(1999), 647-658] who proposes to create an online, interactive gallery of mathematical visualization and art under the name MATHEMATHICAL EXPLORATORIUM.

Chapter 5 shows that now we can use mathematical visualization software to obtain fresh insights concerning hidden conservation laws in classical field theory. It was competently realized by Prof. Dr. D. Opriş & his PhD student P. Zinca in a stimulating and enthusiastic collaboration between our group of research in UNIVERSITY POLITEHNICA OF BUCHAREST and a group in WEST UNIVERSITY OF TIMIŞOARA.

2 Contents

1. History and Background (Sabba Ştefănescu conjectures on magnetic lines, Magnetic flow in MR and Zbl reviews on results of our research team, Single-time Lagrangian dynamics, Multi-time Lagrangian dynamics, References);

2. Single-time Geometric Dynamics (First-order jet bundle, Second-order jet bundle, Least squares problem for ODEs and geometric dynamics, Discrete single-time geometric dynamics, Single-time case studies, (Pendulum geometric dynamics, Lorenz geometric dynamics, ABC geometric dynamics, Henon geometric dynamics, Maxwell-Bloch geometric dynamics, Chemical reaction geometric dynamics, 2-Coral geometric dynamics, Social ecology geometric dynamics, Population competition geometric dynamics, Goodwin geometric dynamics, 3-μm erbium laser geometric dynamics, Tzitzeica geometric dynamics, Plasma geometric dynamics, Magnetic geometric dynamics), References);

3. Multi-time Geometric Dynamics (First-order jet bundle, Second-order jet bundle, Least squares problem for PDEs and geometric dynamics, Discrete multi-time geometric dynamics, Multi-time case studies, Pendulum-Pendulum geometric dynamics, Pendulum-Maxwell-Bloch geometric dynamics, Magnetic-Magnetic geometric dynamics, Saddle-Saddle geometric dynamics, References);

4. Atlas of Magnetic Geometric Dynamics (Several samples of geometric dynamics, Atlas of magnetic geometric dynamics (PWSS magnetic geometric dynamics, PWOS magnetic geometric dynamics, RASS magnetic geometric dynamics, RAOS magnetic geometric dynamics, SPCO magnetic geometric dynamics), Analysis of figures, References);

5. Variational Integrators and Discrete Hidden Conservation Laws in Classical Field Theory (Introduction, Hidden conservation laws for classical mechanical systems, Hidden conservation laws for classical field theory, Discrete variational principle for classical field theory, Discrete hidden conservation laws, References);

6. Graphic Illustrations (Pendulum geometric dynamics, Lorenz geometric dynamics, ABC geometric dynamics, Maxwell-Bloch geometric dynamics, Chemical reaction geometric dynamics, 2-Coral geometric dynamics, Goodwin geometric dynamics, Laser geometric dynamics, Tzitzeica geometric dynamics, Plasma geometric dynamics, PWSS magnetic geometric dynamics, PWOS magnetic geometric dynamics, RASS magnetic geometric dynamics, RAOS magnetic geometric dynamics, SPCO magnetic geometric dynamics, Linear wave equation, Nonlinear wave equation, Klein-Gordon nonlinear equation).

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