NEW GENERALIZED COHERENT STATES

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Abstract

Recently Penson and Solomon have constructed a new family of bosons coherent states by using a specially designed function which is a solution of a functional equation $dE(q,x)/dx = E(q,qx)$, with $0 \leq q \leq 1$ and $E(q,0) = 1$. The above authors by using this function in place of the usual exponential and generate now coherent states $|q,z>$ from the vacuum , which are normalized and continuous in their label z. In this paper we use the same procedure of the above authors, but for an other functional equation of the form $D_q E(q,x) = E(q,x)$,where D_q is the $q - differential\ operator$ and we obtain new coherent states $|q,z>$ which are more convenient for the q-bosons. Also we will find the solution of the general commutation relation (3.1) and the coherent states of the coherent states of the corresponding annihilation operator.Especially we investigated the coherent states in the isobosonic and genobosonic formulation of Hadronic Mechanics .

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1 Introduction

Resently Penson and Solomon [1] have constructed a new family of boson coherent states by using a specially designed function,which is a solution of a functional equation $dE(q,x)/dx = E(q,qx)$ with $0 \leq q \leq 1$and $E(q,0) = 1$.If we substituting the differential operator D with the q-differential operator D_q,we obtain a new functional equation of the form:

$$D_q E(q,x) = \frac{1}{x}\frac{E(q,qx) - E(q,x)}{q-1} = E(q,qx) \tag{1.1}$$

which is more convenient to establish the q-bosons.Before studying the above equation ,we will mention briefly the coherent states of the annihilation operator a_q, inasmuch

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they have been studied by many researches [?], and we will compare them with the corresponding ones which arise from the solution of equation (1.1).

According to ref 2, the operators a_q, a_q^+ satisfay the q-commutation relation:

$$a_q a_q^+ - q a_q^+ a_q = 1 \tag{1.2}$$

and take the following forms:

$$a_q = \sqrt{\frac{[\widehat{n}+1]}{\widehat{n}+1}} a, a_q^+ = a^+ \sqrt{\frac{[\widehat{n}+1]}{\widehat{n}+1}} \tag{1.3}$$

Also the following relations are valid:

$$a_q^+ a_q \left| n \right> = [n] \left| n \right> \tag{1.4}$$

$$a_q \Big| n > = \sqrt{[n]} \left| n-1 \right> \tag{1.5}$$

$$a_q^+ \Big| n > = \sqrt{[n+1]} \left| n+1 \right> \tag{1.6}$$

where $[n] = \dfrac{q^n - 1}{q-1}$ and $a, a^+$are the usual boson operators.

The coherent states:

$$a_q \left| z \right> = z \left| z \right> \tag{1.7}$$

are given by the expression:

$$||z\ > = \frac{1}{\sqrt{e\left|z\right|^2\Big|}} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{[n]!}} \Big| n > = (E(|z|^2))^{-\frac{1}{2}} E(\lambda a^+) \left| 0 \right> \tag{1.8}$$

or:

$$|z\ > = (e(|z|^2))^{-\frac{1}{2}} e(z a_q^+) e(-z^* a_q) \left| 0\ \right> \tag{1.9}$$

where $e(x)$ are the q-exponential function.

The operator $\Big|(e(|z|^2))^{-\frac{1}{2}}\ e(z a_q^+) e(-z^* a_q) \left| 0 \right>$ is denoted by $D(q, z)$ and defined as q-Weyl displacement operator, because for $q = 1$ it coincides with the well-known Weyl operator[6].

In addition the calculation of the probability amplitute supplies the following result:

$$|< z \left| n > \right|^2 = (e(|z|^2))^{-1} \frac{|z|^n}{[n]!}, \tag{1.10}$$

that is the q-deformed Poisson distribution which for $q = 1$ coincides with the classical one.

From the calculation of the q-coherent states (1.8),the resolution of non orthogonality of these states is obtained, i.e.

$$< z^{'} \left| z \right> = \left\{ e(q, |z|^2\, e(q, \left| z^{'} \right|^2) \right\}^{-\frac{1}{2}} e(q, z^* z^{'}) \tag{1.11}$$

The relation (1.8), (1.9) and (1.10) for the case $q = 1$ coincide with the know relation of the coherent states Glauber[7]. In section 3 we investigated the cohernt states of the correspondings annihilations operators in the isobosonic and genobosonic formulation of hadronic mechanics of Santilli[8]. Section 4 is devote to concluding remarks.

2 Study of Equation (1.1)

According to ref. 1 we consider the following functional equation for a function of the complex variable z:

$$\frac{1}{z}\frac{E(q,qz)-E(q,z)}{q-1} = E(q,qz) \tag{2.1}$$

when $q = 1$ these are definig equations for $\exp(z)$. When $q \neq 1$, the $E(q,z) \neq \exp(z)$ and a solution analytic in some nieghborhood of $z = 0$ may be assumed to be given by:

$$E(q,z) = \sum_{n=0}^{\infty} c_n(q) z^n \tag{2.2}$$

Equation (2.1) produces the recursion relation:

$$c_{n+1}(q) = \frac{q^n}{[n+1]} c_n(q), c_0 = 1, n = 1, 2, \ldots \tag{2.3}$$

with solution:

$$c_n(q) = q^{\frac{n(n-1)}{2}} . \frac{z^n}{[n]!} \tag{2.4}$$

and:

$$E(q,z) = \sum_{n=0}^{\infty} q^{\frac{n(n-1)}{2}} . \frac{z^n}{[n]!} \tag{2.5}$$

which convergent for all z, when $0 \leq q \leq 1$. In the sequel the remarks by the authors[1)] are valid for the function $E(q,z)$; that interpolates between $E(0,z) = 1 + z$ and $E(1,z) = \exp z$. The function $E(q,z)$ has a infinitely cauntable number of roots, of which non lies on the positive real axis.

Now is simple to define a new familly of physical states

$$\left| q,z \right> = N(q,|z|^2)^{-\frac{1}{2}} E(q, z a_q^+) E(q, -z^* a_q) \left|0\right> \tag{2.6}$$

and the above staetes are coherent in the general sense according to ref. 1.

From the relation $< q,z \left| q,z \right> = 1$ we obtain the normalization:

$$N(q,|z|^2) = \sum_{n=0}^{\infty} \frac{q^{n(n-1)}}{[n]!} |z|^{2n} = E(q,|z|^2) \tag{2.7}$$

and then the normalized state is:

$$|q,z> = (E(q,|z|^2)) \sum_{n=0}^{\infty} \frac{q^{\frac{n(n-1)}{2}} z^n}{\sqrt{[n]!}} |n| > \tag{2.8}$$

For a given q calculate the overlap:

$$< q,z \left| q,z^{'} \right. >= \left\{ (E(q,|z|^2))E(q,\left|z^{'}\right|^2) \right\}^{-\frac{1}{2}} E(q,z^{'}z^*) \tag{2.9}$$

From the new coherent states (2.8),is now simple to calculate the probability amplifude, i.e.

$$|< q,z\,|n>|^2 = (E(q,|z|^2))^{-1} \frac{q^{n(n-1)}\,|z|^{2n}}{[n]!} \tag{2.10}$$

and:

$$\sum_{n=1}^{\infty} |< q,z\,|n>|^2 = (E(q,|z|^2))^{-1} \sum_{n=0}^{\infty} \frac{q^{n(n-1)}\,|z|^{2n}}{[n]!} = 1 \tag{2.11}$$

also the distribution (2.10) is a q-deformed Poisson distribution ,which for $q = 1$ coincides with the classical one.

Finally the normalized states (2.8) takes the form:

$$\begin{aligned} |q,z> &= (E(q,|z|^2))^{-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{q^{\frac{n(n-1)}{2}} z^n}{\sqrt{[n]!}} \Bigg| n >= \\ &= (E(q,|z|^2))^{-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(za_q^+)^n}{[n]_{\frac{1}{q}}!} \Bigg| 0 >= \\ &= E(q,|z|^2)^{-\frac{1}{2}} e_{\frac{1}{q}}(za_q^+)\,|0>= \\ &= E(q,|z|^2)^{-\frac{1}{2}} e_{\frac{1}{q}}(za_q^+) e_{\frac{1}{q}}(-z^* a_q)\,|0> \end{aligned} \tag{2.12}$$

where $e_{\frac{1}{q}}(za^+)$ is $\frac{1}{q}$-exponential function.

3 Coherent States in the Lie-Admissible Formulation

Before we start to deal with the coherent states in the Lie-admissible formulation we consider the most general commutation relation for the operators A, A^+, i.e.

$$AT(\widehat{n}+1)A^+ - A^+R(\widehat{n}+1)A = F(\widehat{n}+1) \tag{3.1}$$

where the functions $T(\widehat{n}+1), R(\widehat{n}+1)$ and $F(\widehat{n}+1)$ are deppendent from the number operator $\widehat{n} = a^+a, a, a^+$ are the are the usual boson operators and $\widehat{n}\,|n>= n\,|n>$ .By using the bosonization method[2] the operators A^+, A obtain the following expressions:

$$A = f(\widehat{n}+1)a \ , \ A^+ = a^+ f(n+1) \tag{3.2}$$

and the structure function $f(\widehat{n}+1)$ satisfies the following equation:

$$T(n+2)(n+1)f^2(n+1) - R(n)nf^2(n) = F(n+1) \tag{3.3}$$

For $nf^2(n) = L_n$ the above equation yields:

$$T(n+2)L_{n+1} - R(n)L_n = F(n+1) \tag{3.4}$$

or:

$$L_{n+1} - \sigma(n+1)L_n = g(n+1) \tag{3.5}$$

where $\sigma(n+1) = \dfrac{R(n)}{T(n+2)}$, $g(n+1) = \dfrac{F(n+1)}{T(n+2)}$ and $T(n+2) <> 0$.

For

$$L_n = \sigma(1)\sigma(2)\dots\sigma(n)S_n = \sigma(n)!S_n \tag{3.6}$$

equation (3.5) takes the form:

$$S_{n+1} - S_n = \frac{g(n+1)}{\sigma(n+1)!} \tag{3.7}$$

where $\sigma(n)! = \sigma(1)\dots\sigma(n)$. Because for

$$n = 0,\ L_0 = 0 \text{ and } S_0 = 0, \tag{3.8}$$

then the solution of the above equation has the form:

$$S_{n+1} = \sum_{n=0}^{\infty} \frac{g(l+1)}{\sigma(l+1)!} \tag{3.9}$$

and the relation (3.6) we obtain the solution:

$$L_{n+1} = \sigma(n+1)! \sum_{l=0}^{\infty} \frac{g(l+1)}{\sigma(l+1)!} \tag{3.10}$$

Finally we obtain the the structure function:

$$f(n+1) = \sqrt{\frac{L_{n+1}}{(n+1)}} \tag{3.11}$$

The expressions (3.2) take now the forms:

$$A = \sqrt{\frac{L_{\widehat{n+1}}}{(\widehat{n}+1)}}a,\ A^+ = a^+\sqrt{\frac{L_{\widehat{n+1}}}{\widehat{n}+1}} \tag{3.12}$$

and satisfay the following relations

$$A\,|n>=\sqrt{L_n}\,|n-1>,\ A^+\,|n>=\sqrt{L_{n+1}}\,|n+1> \tag{3.13}$$

For $n=0$ is

$$A\,|0>=0 \text{ and } A^+\,|0>=\sqrt{L_1}\,|1> \tag{3.14}$$

The operators A and A^+ are annihilation and creation operators.

The coherent states of the annihilation operator are defined:

$$A\,|\alpha>=\alpha\,|\alpha> \tag{3.15}$$

and after some algebras we obtain:

$$|\alpha>=(\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{L_n!})^{-\frac{1}{2}}\sum_{n=0}^{\infty}\frac{(\alpha A^+)^n}{L_n!}\sum_{n=0}^{\infty}\frac{(-\alpha^* A)^n}{L_n!}\Bigg| > \tag{3.16}$$

In the following we will start to find the coherent states for the isobosonic and genobosonic annihilation operators in the Lie-admissible formulation:

a) Coherent states for the General Iisotopic Case.

According to Jannussis[9] the annnihilation and creation operators A, A^+ satisfy the commutation relation:

$$AT(n+1)A^+ - A^+T(n+1)A = \frac{1}{T(n+1)} \tag{3.17}$$

and take the following expressions:

$$A = \frac{1}{\sqrt{T(n+1)T(n+2)}}a,\ A^+ = a^+\frac{1}{\sqrt{T(n+1)T(n+2)}} \tag{3.18}$$

where the element $T(n+1)$ is real and n is the convensional occupation number.

According to Santilli [8] we have:

$$A^*\,|\alpha_T>=AT(n+1)\,|\alpha_T> \tag{3.19}$$

and $|\alpha_T$ are the isobosonic coherent states , with the isonormalized coherent isobasis:

$$|\alpha_T> \frac{1}{\sqrt{T(n+1)}}\,|\alpha>, <\alpha\,|\alpha>=1, <\alpha_T\,|T(n+1)|\,\alpha_T>=1 \tag{3.20}$$

From (3.18) and (3.20) formula (3.19) takes the form:

$$\begin{aligned} A^*\,|\alpha_T &> = AT(n+1)\,|\alpha_T>=\frac{1}{\sqrt{T(n+1)T(n+2)}}\alpha\sqrt{T(n+1)}\,|\alpha>= \\ &= \frac{1}{\sqrt{T(n+1)}}\alpha\,|\alpha>=\alpha\frac{1}{\sqrt{T(n+1)}}\,|\alpha>=\alpha\,|\alpha_T> \end{aligned} \tag{3.21}$$

In the same way we obtain:

$$< \alpha_T|^* A^+ = \alpha^* < \alpha_T| \tag{3.22}$$

with:

$$< \alpha_T| =< \alpha| \sqrt{T^{-1}(n+1)}. \tag{3.23}$$

b) General Lie-Admissible Case with Real Elements T and R.

According to ref. 9 the corresponding annihilation and creation operators A, A^+ and B, B^+ are given by the following expressions:

$$A = f(n+1)a,\ A^+ = a^+ f(n+1) \tag{3.24}$$

$$B = g(n+1)a,\ B^+ = a^+ g(n+1) \tag{3.25}$$

where the structure functions $f(n+1)$ and $g(n+1)$ are real and have the following forms:

$$f(n+1) = \frac{1}{\sqrt{(n+1)T(n+1)T(n+2)}}\{1 + \frac{R(n)}{T(n)} + \ldots + \frac{R(n)R(n-1)\ldots R(1)}{T(n)T(n-1)\ldots T(1)}\}^{\frac{1}{2}} \tag{3.26}$$

$$\begin{aligned} g(n+1) \quad = \quad & \frac{1}{\sqrt{(n+1)R(n+1)T(n+2)}}\{1 + \frac{R(n+1)}{T(n+1)} + \ldots + \\ & + \frac{R(n+1)R(n)\ldots R(2)}{T(n+1)T(n)\ldots T(2)}\}^{\frac{1}{2}} \end{aligned} \tag{3.27}$$

In the following formula we can construct the Lie-admissible common normalized coherent basis:

$$|\alpha_{TR} >= \frac{1}{\sqrt[4]{T(n+1)R(n+1)}}\,|\alpha >, < \alpha\,|\alpha >= 1 \tag{3.28}$$

$$< \alpha_{TR} \left| \sqrt{T(n+1)R(n+1)} \right| \alpha_{TR} >= 1$$

and after some algebra we obtain:

$$\begin{aligned} A * |\alpha_{TR} \quad > \quad & = AT(n+1)\,|\alpha_{TR} >= f(n+1)aT(n+1)\,|\alpha_{TR} > \\ = \quad & f(n+1)T(n+2)a\,|\alpha_{TR} > \\ = \quad & f(n+1)T(n+2)a\frac{1}{\sqrt[4]{T(n+1)R(n+1)}}\,|\alpha > \end{aligned} \tag{3.29}$$

$$= \quad \alpha\sqrt[4]{\frac{R(n+1)T(n+2)}{T(n+1)R(n+2)}}\left\{\frac{1}{n+1}(1 + \frac{R(n)}{T(n)} + \ldots + \frac{R(n)R(n-1)\ldots R(1)}{T(n)T(n-1)\ldots T(1)})\right\}^{\frac{1}{2}}$$

$$\cdot\,|\alpha_{TR} \quad >$$

$$B * |\alpha_{TR} > \; = BT(n+1)\,|\alpha_{TR} >= g(n+1)T(n+2)a\,|\alpha_{TR} > \tag{3.30}$$

$$= \alpha \sqrt[4]{\frac{T(n+1)T(n+2)}{R(n+1)R(n+2)}} \left\{ \frac{1}{n+1} \left(1 + \frac{R(n+1)}{T(n+1)} + \ldots + \frac{R(n+1)R(n)\ldots R(2)}{T(n+1)T(n)\ldots T(2)} \right) \right\}^{\frac{1}{2}}$$

$$|\alpha_{TR} >$$

and:

$$< \alpha_{TR}| * A^{+} = \alpha^{*} < \alpha_{TR}| \tag{3.31}$$

$$\cdot \sqrt[4]{\frac{R(n+1)T(n+2)}{T(n+1)R(n+2)}} \left\{ \frac{1}{n+1} \left(1 + \frac{R(n)}{T(n)} + \ldots + \frac{R(n)R(n-1)\ldots R(1)}{T(n)T(n-1)\ldots T(1)} \right) \right\}^{\frac{1}{2}}$$

$$< \alpha_{TR}| * B^{+} = \alpha^{*} < \alpha_{TR}| \sqrt[4]{\frac{T(n+1)T(n+2)}{R(n+1)R(n+2)}} \cdot \tag{3.32}$$

$$\left\{ \frac{1}{n+1} \left(1 + \frac{R(n+1)}{T(n+1)} + \ldots + \frac{R(n+1)R(n)\ldots R(2)}{T(n+1)T(n)\ldots(2)} \right) \right\}^{\frac{1}{2}}$$

The above results for $T(n+1) = R(n+1)$ coincides exactly with the formulae (3.21) and (3.22).

c) General Coherent States with Complex Elements T and R.

When the elements $T(n+1)$and $R(n+1)$ are complex functions of the conventional occupation number n we have two pairs of the operators A, B and A', B' and according to ref. 9 we have:

$$A = f(n+1)a, B = a^{+}f(n+1) \tag{3.33}$$

$$A^{+} = a^{+}f^{+}(n+1), B^{+} = f^{+}(n+1)a \tag{3.34}$$

$$\acute{A} = g(n+1)a, \acute{B} = a^{+}g(n+1) \tag{3.35}$$

$$\acute{A}^{+} = a^{+}g^{+}(n+1), \acute{B}^{+} = g^{+}(n+1)a \tag{3.36}$$

where the structure functions $f(n+1)$ and $g(n+1)$ are complex functions and have the following forms:

$f(n+1) =$

$$\frac{1}{\sqrt{(n+1)T(n+1)T(n+2)}} \left\{ 1 + \frac{R(n)}{T(n)} + \ldots + \frac{R(n)R(n-1)\ldots R(1)}{T(n)T(n-1)\ldots T(1)} \right\}^{\frac{1}{2}} \tag{3.37}$$

$g(n+1) =$

$$\frac{1}{\sqrt{(n+1)R(n+1)T(n+2)}} \left\{ 1 + \frac{R(n+1)}{T(n+1)} + \ldots + \frac{R(n+1)R(n)\ldots R(2)}{T(n+1)T(n)\ldots T(2)} \right\}^{\frac{1}{2}} \tag{3.38}$$

and in the same way with (3.28)we can construct the general Lie-admissible common normalized coherent basis, i.e.

$$\left|\alpha_{|T||R|}\right.>=\frac{1}{\sqrt[4]{|T(n+1)|\,|R(n+1)|}}\,|\alpha>, <\alpha\,|\alpha>=1, \tag{3.39}$$
$$<\alpha_{|T||R|}\left|\sqrt{|T(n+1)|\,|R(n+1)|}\right|\alpha_{|T||R|}>=1$$

After some algebra we obtain:

$$\begin{aligned} A*\left|\alpha_{|T||R|}\right. &> = AT(n+1)\frac{1}{\sqrt[4]{|T(n+1)|\,|R(n+1)|}}\,|\alpha>= \\ &= \sqrt[4]{\frac{|T(n+1)|\,|R(n+1)|}{|T(n+2)|\,|R(n+2)|}}\sqrt{\frac{T(n+2)}{T(n+1)}}\{\frac{1}{n+1}(1+ \\ &\quad \frac{R(n)}{T(n)}+\ldots+\frac{R(n)R(n-1)\ldots R(1)}{T(n)T(n-1)\ldots T(1)})\}^{\frac{1}{2}} \\ \left|\alpha_{|T||R|}\right. &> \end{aligned} \tag{3.40}$$

$$\begin{aligned} < \quad & \alpha_{|T||R|}\right| * A^{+} = \alpha^{*} < \alpha_{|T||R|}| \\ & \sqrt[4]{\frac{|T(n+1)|\,|R(n+1)|}{|T(n+2)|\,|R(n+2)|}}\sqrt{\frac{T^{+}(n+2)}{T^{+}(n+1)}}\{\frac{1}{n+1}(1+\frac{R^{+}(n)}{T^{+}(n)}+\ldots \\ & +\frac{R^{+}(n)R^{+}(n-1)\ldots R^{+}(1)}{T^{+}(n)T^{+}(n-1)\ldots T^{+}(1)})\}^{\frac{1}{2}} \end{aligned} \tag{3.41}$$

$$\begin{aligned} \acute{A}*\left|\alpha_{|T||R|}\right. &> = \alpha\sqrt[4]{\frac{|T(n+1)|\,|R(n+1)|}{|T(n+2)|\,|R(n+2)|}}\,\sqrt{\frac{T(n+2)}{R(n+1)}} \\ &\quad \{\frac{1}{n+1}(1+\frac{R(n+1)}{T(n+1)}+\ldots+\frac{R(n+1)R(n)\ldots R(2)}{T(n+1)T(n)\ldots T(2)})\}^{\frac{1}{2}} \\ \left|\alpha_{|T||R|}\right. &> \end{aligned} \tag{3.42}$$

$$\begin{aligned} < \quad & \alpha_{|T||R|}|* \acute{A}^{+} = \alpha^{*} < \alpha_{|T||R|}| \\ & \sqrt[4]{\frac{|T(n+1)|\,|R(n+1)|}{|T(n+2)|\,|R(n+2)|}}\,\sqrt{\frac{T^{+}(n+2)}{R^{+}(n+1)}} \\ & \left\{\frac{1}{n+1}(1+\frac{R^{+}(n+1)}{T^{+}(n+1)}+\ldots+\frac{R^{+}(n+1)R^{+}(n)\ldots R^{+}(2)}{T^{+}(n+1)T^{+}(n)\ldots T^{+}(2)}\right\}^{\frac{1}{2}} \end{aligned} \tag{3.43}$$

$$B^{+}*|\,\alpha_{|T||R|} \quad > \quad = \alpha\sqrt[4]{\frac{|T(n+1)|\,|R(n+1)|}{|T(n+2)|\,|R(n+2)|}}\sqrt{\frac{T^{+}(n+2)}{T^{+}(n+1)}} \tag{3.44}$$

$$\left\{\frac{1}{n+1}\left(1+\frac{R^+(n)}{T^+(n)}+\ldots+\frac{R^+(n)R^+(n-1)\ldots R^+(1)}{T^+(n)T^+(n-1)\ldots T^+(1)}\right)\right\}^{\frac{1}{2}}$$

$$\Big|\alpha_{|T||R|}\quad>$$

$$<\quad \alpha_{|T||R|}\,|*B\,=\alpha^*<\,\alpha_{|T||R|}\big|\;\sqrt[4]{\frac{|T(n+1)|\,|R(n+1)|}{|T(n+2)|\,|R(n+2)|}}\sqrt{\frac{T(n+2)}{T(n+1)}}\quad (3.45)$$

$$\left\{\frac{1}{n+1}\left(1+\frac{R(n)}{T(n)}+\ldots+\frac{R(n)R(n-1)\ldots R(1)}{T(n)T(n-1)\ldots T(1)}\right)\right\}^{\frac{1}{2}}\quad (1)$$

$$\acute{B}^+*\Big|\,\alpha_{|T||R|}\quad>\quad=\alpha\sqrt[4]{\frac{|T(n+1)|\,|R(n+1)|}{|T(n+2)|\,|R(n+2)|}}\sqrt{\frac{T^+(n+2)}{R^+(n+1)}}\,\cdot \quad (3.46)$$

$$\left\{\frac{1}{n+1}\left(1+\frac{R^+(n+1)}{T^+(n+1)}+\ldots+\frac{R^+(n+1)R^+(n)\ldots R^+(2)}{T^+(n+1)T^+(n)\ldots T^+(2)}\right)\right\}^{\frac{1}{2}}$$

$$\Big|\alpha_{|T||R|}\quad>$$

$$<\quad \alpha_{|T||R|}\,\Big|*\acute{B}\,=\alpha^*<\,\alpha_{|T||R|}\Big| \quad (3.47)$$

$$\sqrt[4]{\frac{|T(n+1)|\,|R(n+1)|}{|T(n+2)|\,|R(n+2)|}}\sqrt{\frac{T(n+2)}{R(n+1)}}\,\cdot$$

$$\left\{\frac{1}{n+1}\left(1+\frac{R(n+1)}{T(n+1)}+\ldots+\frac{R(n+1)R(n)\ldots R(2)}{T(n+1)T(n)\ldots T(2)}\right)\right\}\frac{1}{2}$$

For the general genobosonic case, i.e., $R(n+1)=T^+(n+1)$ we obtain:

$$A=f(n+1)a,\;\; B=a^+f(n+1) \quad (3.48)$$

$$A^+=a^+f^+(n+1)\quad,\quad B^+=f^+(n+1)a \quad (3.49)$$

and

$$A*|\,\alpha_{TT^+}\quad>\quad=\alpha\,\sqrt{\frac{|T(n+1)|\,T(n+2)}{|T(n+2)|\,T(n+1)}}\,\cdot \quad (3.50)$$

$$\cdot\{\frac{1}{n+1}\left(1+\frac{T^+(n)}{T(n)}+\ldots+\frac{T^+(n)T^+(n-1)\ldots T^+(1)}{T(n)T(n-1)\ldots T(1)}\right)^{\frac{1}{2}}$$

$$|\alpha_{TT^+}\quad>$$

$$<\quad \alpha_{TT^+}\,|*A^+\,=\alpha^*<\,\alpha_{TT^+}|\;\sqrt{\frac{|T(n+1)|\,T^+(n+2)}{|T(n+2)|\,T^+(n+1)}}\,\cdot \quad (3.51)$$

$$\cdot\left\{\frac{1}{n+1}\left(1+\frac{T(n)}{T^+(n)}+\ldots+\frac{T(n)T(n-1)\ldots T(1)}{T^+(n)T^+(n-1)\ldots T^+(1)}\right)\right\}^{\frac{1}{2}}$$

$$B^{+} * |\alpha_{TT^{+}} \quad > \quad = \alpha \sqrt{\frac{|T(n+1)|\, T^{+}(n+2)}{|T(n+2)|\, T^{+}(n+1)}} \tag{3.52}$$

$$\{ \frac{1}{n+1} \left(1 + \frac{T(n)}{T^{+}(n)} + \ldots + \frac{T(n)T(n-1)\ldots T(1)}{T^{+}(n)T^{+}(n-1)\ldots T^{+}(1)} \right)^{\frac{1}{2}}$$

$$|\alpha_{TT^{+}} \quad >$$

$$< \quad \alpha_{TT^{+}} |*B = \alpha^{*} < \alpha_{TT^{+}}| \sqrt{\frac{|T(n+1)|\, T(n+2)}{|T(n+2)|\, T(n+1)}} \tag{3.53}$$

$$\left\{ \frac{1}{n+1} \left(1 + \frac{T^{+}(n)}{T(n)} + \ldots + \frac{T^{+}(n)T^{+}(n-1)\ldots T^{+}(1)}{T(n)T(n-1)\ldots T(1)} \right) \right\}^{\frac{1}{2}}$$

4 Conclusion

In the present paper we have studied a new functional equation similar with the equation of ref.1 and we obtained new coherent states $|q, z>$ which are more convinient for the q-deformed bosons.Also we have found the coherent states of the corresponding annihilation operator which satisface the general commutation relations (3.1).Finally we have determined the coherent states in the isobosonic and genobosonic formulation in Hadronic Mechanics.

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