TIME REVERSAL VIOLATION FROM THE INTERACTION OF A TIME SYMMETRIC SYSTEM WITH A TIME ASYMMETRIC ENVIRONMENT

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Abstract

The problem of the origin of time-reversal violation is studied assuming the fundamental premise that all time asymmetric effects occur because of the impossibility of separating a system from its time-asymmetric environment. Both small time variations of the time asymmetry parameter in particle theory and the possible cosmological variation of the time asymmetry parameter could provide us with signatures of such an underlying theory. Following a theoretical discussion we suggest how to look for these time-asymmetric effects in both particle accelerators and in astrophysical data.

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1 Introduction

The problem of time-asymmetry in particle theory carries with it a very deceptive disguise which discourages us from thinking of it in a fundamental way. This is because the C, P, T operators of field theory are defined to be quantum operators on quantum fields that conspire to generate a hamiltonian that describes the quantum evolution of a system and not the classical evolution [1]. We automatically lose our classical notions of time with the introduction of complex numbers. In this direction the CPT theorem states that a local relativistic invariant quantum field theory admitting Fermi-Bose symmetry conserves the combined operation of CPT [2]. Of course both the definitions of the operators C, P, T, as well as the exact Fermi-Bose correspondence depend on
the existence of an asymptotic set of states for the particles involved, which is not always the case [3]. Both the problems of quark-confinement [4] and instanton physics [5] forbid the notion of a free particle which might shed light on how both CP and CPT are violated [6]. An oft forgotten aspect of CP violation involves the fact that because of both complex amplitudes in particle interactions and coupled channels complex phases influences the total cross section for a physical process, because of this the time reversed or CP conjugate process does not proceed at the same rate as the initial process. If there were just a single term in the amplitude the phase would cancel out and the time reversal rate would be the same as the initial process. Thus both the principle of superposition (a fundamental tenent of Quantum Mechanics) and the existence of a weak phase conspire to create CP violation [7]. T. D. Lee [8] said it most simply in stating that CP conservation would require the remarkable fine tuning of all final state phases to reproduce the symmetric time reversal process. In a certain sense the violation of CP and T is an inevitable consequence of the existence of quantum theory and the principle of superposition with no direct connection with classical notions of time reversal invariance. In this sense CP or T violation is more in accord with the notions of environmental induced interactions and can link together the cosmological arrow of time with the elementary particle arrow of time [9].

In the last decade a new approach has infiltrated into the theoretical community which has its origin in string theory, the "Procrustean Principle" states that all systems are unavoidably open systems due the interaction of local string modes (particles) with the delocalized truncated modes of string theory [11, 12]. These ideas which utilize a density matrix approach to Quantum Mechanics lead to violation of CPT in the system, the variation of the fundamental constants of nature with cosmological evolution, the loss of quantum coherence in quantum systems [12, 13], and finally to dispersive variation of the light speed with frequency in the vacuum [14]. In addition to the above mentioned development there is motivation inspired by atomic physics to view CP or T violation as due to the interaction of a system with the environment. The precession of the electric dipole moment of an atom violates T conservation and it can be shown that it is due to the interaction of a T symmetric system (atom) with the external electric field [15, 16]. The phenomenological theory of CP or T violation in particle theory stems generically from the complex phases in Yukowa couplings of Higgs fields, pure Higgs couplings and complex vacuum expectation values of Higgs fields (soft CP violation) [17]. Though the Higgs field is certainly a local quantum field its origin and usefulness as a symmetry breaking field still carries with it a certain mysterious flavor [18]. Certainly the CP violation in the early universe that generates the baryon asymmetry would require Higgs fields [19], however the scale at which this CP violation occurs is so far removed from the weak scale that it is hard to imagine that there is any connection. If the Higgs particle is found it would verify that our theories are correct, however the origin of the Higgs field could still be due to cosmological dynamics that has an inherent arrow of time. This would relegate elementary particle CP or T violations as being due to cosmological T violation. Our attitude is that certain fields are phenomenological and although at low energy they manifest themselves as quantum fields, at higher energy, a more deterministic theory would replace their need. A knowledge of this theory is at the
heart of a true theory of CP or T violations. Inspired by the ideas embodied in the Procrustean principle [10, 11] and by previous studies on explicit T violations [20, 21] using a modified Schrodinger equation we in what follows discuss a submicroscopic mechanism of how the $\epsilon$ parameter is generated in CP violating processes [22, 23]. We also discuss how the parameter could vary with the cosmological evolution of the universe [24]. After this theoretical discussion we point to a number of signatures that can serve as tests in both accelerator experiments and in the radiation generated by black hole decay, supernova explosions, quasar emissions and other high-energy astrophysical processes.

2 A New Approach to Time Reversal Violations

To begin, we review briefly the present experimental data on CP violation phenomena, the following parameters [25]: $|\eta_-|$, $|\eta_{00}|$, $\delta$ measure certain asymmetries in $K_L$, $K_S$ decays, if $|\eta_-|$, $|\eta_{00}|$ are different, then their measurement are compatible with the CP violating parameters $\epsilon$, $\tilde{\epsilon}$ given by:

$$\eta_- = \epsilon + \epsilon'$$
$$\eta_{00} = \epsilon - 2\epsilon'.$$

A nonzero value of $\epsilon$ would imply direct CP violation ($\Delta S = 1$) while $\epsilon' = 0$ is compatible with ($\Delta S = 2$) box diagram effects. If CPT is conserved then the eigenstates of the $K^0$, $\bar{K}^0$ system are:

$$|K_S\rangle = \left(\begin{array}{c} 1 + \frac{\epsilon}{\sqrt{2+4\epsilon^2}} \\ \frac{\epsilon}{\sqrt{2+4\epsilon^2}} \end{array}\right),$$
$$|K_L\rangle = \left(\begin{array}{c} 1 - \frac{\epsilon}{\sqrt{2+4\epsilon^2}} \\ 1 + \frac{\epsilon}{\sqrt{2+4\epsilon^2}} \end{array}\right).$$

(1)

in the $K^0$, $\bar{K}^0$ basis.

We might remark that if T invariance holds, then even if CPT is broken we can write:

$$|K_S\rangle = \left(\begin{array}{c} 1 + \delta \\ 1 - \delta \end{array}\right) \frac{1}{\sqrt{2 + 2\delta^2}}$$
$$|K_L\rangle = \left(\begin{array}{c} 1 - \delta \\ -1 + \delta \end{array}\right) \frac{1}{\sqrt{2 + 2\delta^2}}.$$ 

(2)

Equation (2) might apply to situations that suggest that the requirements of CPT invariance are not fulfilled, these include the presence of non-local interactions, gravitational interactions and situations when special relativity is violated [26]. Kabir [27] has simplified the theory of the $K^0$, $\bar{K}^0$ system by stating that the hamiltonian of the $K^0$, $\bar{K}^0$ system can be written as a combination of these four matrices with complex coefficients:

$$\left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right), \left(\begin{array}{cc} 1 & 0 \\ 0 & -i \end{array}\right), \left(\begin{array}{cc} 0 & -i \\ i & 0 \end{array}\right), \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right).$$

(3)

The first two matrices in equation (3) respect CP, the third violates CP, while the fourth matrix violates CPT. For the first three matrices the coefficient of the third matrix determines $\epsilon$ in equation (1) which can also have a phase. The measured
phase of $\varepsilon$ is compatible with $\Phi = 44^\circ$. If CPT were violated the phase of $\varepsilon$ would be different than $44^\circ$ and there would be a measurable mass difference between $K^0$, $\bar{K}^0$ which is not found.

We now outline a theory that we have studied that allows for a calculation of the magnitude of $\varepsilon$ in terms of a sub-microscopic theory of $T$ violations [22, 23]. When $K^0$, $\bar{K}^0$ are created in a cosmological environment they have CP eigenvalues $\pm 1$ with states:

$$\left( \frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}} \right) (CP = 1), \left( \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}} \right) (CP = -1).$$  \hspace{1cm} (4)

Due to the interaction with the environment a fundamental preference for $K^0$ over $\bar{K}^0$ is generated. We consider a two step Markov process with transition matrix:

$$M = \frac{K^0}{\bar{K}^0} \begin{pmatrix} K^0 & \bar{K}^0 \\ 1 - q & q \\ p & 1 - p \end{pmatrix}$$  \hspace{1cm} (5)

The initial probabilities of $K^0$, $\bar{K}^0$ are $\frac{1}{2}$, $\frac{1}{2}$ respectively, after $n$ steps we have:

$$P_n(K^0) = \frac{p}{p - q} (1 - p - q)^n \left( \frac{1}{2} - \frac{p}{p + q} \right),$$  \hspace{1cm} (6)

$$P_n(\bar{K}^0) = \frac{q}{p - q} (1 - p - q)^n \left( \frac{1}{2} - \frac{q}{p + q} \right).$$

If $n \to \infty$, we have:

$$P_n(K^0) = \frac{p}{p + q},$$  \hspace{1cm} (7)

$$P_n(\bar{K}^0) = \frac{q}{p + q}.$$

After a long Markov time we set for $n \to \infty$:

$$\frac{p}{p + q} = \left( \frac{1 + \varepsilon}{\sqrt{2 + 2\varepsilon^2}} \right)^2, \frac{q}{p + q} = \left( \frac{1 - \varepsilon}{\sqrt{2 + 2\varepsilon^2}} \right)^2.$$

This gives $\frac{p}{q} = \left( \frac{1 + \varepsilon}{1 - \varepsilon} \right)^2$ which expresses the ratio of the Markov probabilities in terms of the $CP$ violating parameter $\varepsilon$. The wave function of $K_L$, $K_S$ in Markov time ($n$) and Minkowski time now reads:

$$\psi_S = \left( \frac{\sqrt{P_n(K^0)}}{P_n(\bar{K}^0)} \right) e^{-iE_{St}t}, \psi_L = \left( \frac{\sqrt{P_n(K^0)}}{-\sqrt{P_n(\bar{K}^0)}} \right) e^{-iE_{Lt}t}.\hspace{1cm} (8)$$
In equation (8) the eigenvalues $E_L$, $E_S$ are calculated from the CP conserving hamiltonian via the eigenvalue equation:

$$H_{CP} \left( \frac{1}{\sqrt{2}} \hat{\sigma}_1 \right) = E_{S,L} \left( \frac{1}{\sqrt{2}} \hat{\sigma}_1 \right).$$

It is well known that even in the usual theory of CP violation that CP violating terms do not contribute significantly to $E_S$, $E_L$. We also note that $E_S$, $E_L$ have a real and imaginary part. Here the imaginary part predicts the lifetime of $K_L$, $K_S$. We also note that $P_n (K^0)$, $P_n (\bar{K}^0)$ are the same for the states $\psi_L$ and $\psi_S$ since the coefficients of $K^0$, $\bar{K}^0$ have the same magnitude for both of these states. To test the above theory we might construct an initial beam of $K^0$ with $\psi = \psi_S(0) - \psi_L(0)$, where 0 refers to $t = 0$, $n = 0$. At Minkowski time $(t)$ and Markov time $(n)$ we have:

$$\psi = \left( \frac{\sqrt{P_n (K^0)}}{\sqrt{P_n (\bar{K}^0)}} \left( e^{iE_{S,L} t/\hbar} + e^{-iE_{S,L} t/\hbar} \right) \right).$$

The probability or number density in the beam at $t$ could be compared with the calculation of the probability based on equation (9). Since

$$K^0 \rightarrow \pi^- + e^+ + \nu_e, \quad \bar{K}^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e,$$

the electron-positron asymmetry as a function of $n$ and $t$ could be used to test equation (9) for Minkowski time $(t)$ and Markov time $(n)$. There are a number of refinements that can be made to the above theory, if we choose to relate the Markov time $(n)$ to the Minkowski time we might write:

$$m = \frac{n_0 t}{t + k_0},$$

where $n \rightarrow n_0$ as $t \rightarrow \infty$, or said differently Markov effects are unimportant after relaxation of the $K^0$, $\bar{K}^0$ system in the cosmological environment. A second refinement is to construct a forward and backward directed set of Markov processes to facilitate a theory of $T$ violation [22]. A third refinement of the above theory might include other basic states that $K^0$, $\bar{K}^0$ can oscillate to wherein the other states are not observed but do effect the Markov probabilities of $K^0$, $\bar{K}^0$, because the transition matrix will be enlarged. In another investigation [23] we considered calculating $\psi$ in terms of a random walk on a sub-microscopic lattice where $\psi$ was calculated as an averages over a binomial distribution. This approach might also be applied to calculating the phase of $\psi$ which we did not do in the theory discussed above. Though the above theory expressed in equation (9) is in a very embryonic stage, it still would allow us to look for fluctuations in the probabilities of $K^0$ and $\bar{K}^0$ in terms of both the Minkowski
time and the new Markov index \( n \) which expresses the interaction of the \( K^0, \bar{K}^0 \) with the time-asymmetric environment through relaxation effects. Actually Bernstein et. al. [28] proposed that CP violation was due to a long-range cosmological induced field shortly after the experimental discovery of CP violation by Christensen et. al [29]. Because of the development of the electroweak theory this idea did not gain any popularity. However the past 30 years have shown that CP violation still is a problem of complex phases and the present accepted theory has relegated the calculation of these phases to a technology involving many uncertain parameters (Yukawa couplings, vacuum expectation values, etc.).

As mentioned earlier in [20, 21], we developed a theory of explicit T violations by assuming a retarded version of the Schrodinger equation. Actually Caldirolo [30] had also discussed such a theory but the applications he studied were more directed to problems in atomic physics. The idea in this approach is that in addition to the usual CP violation expressed in the hamiltonian of the \( K^0, \bar{K}^0 \) system there is a cosmologically induced form of T violation that appears in the equation

\[
H \psi = i \hbar \left[ \frac{\psi (t + \frac{\pi}{2} - \epsilon)}{\tau} - \frac{\psi (t - \frac{\pi}{2} - \epsilon)}{\tau} \right],
\]

where \( \tau = \) discrete time parameter, \( \epsilon = \) explicit T violating parameter. When equation (10) is applied to the \( K^0, \bar{K}^0 \) system it yields the following states [20]:

\[
\psi_S = \left( \frac{1 + \epsilon}{\sqrt{1 + 2 \epsilon}} \right) e^{-\frac{2 \tau}{\hbar} \sin^{-1} \left( \frac{E_S \tau}{2 \hbar} \right)} \frac{2 r_S \sin \left( \frac{E_S \tau}{2 \hbar} \right)}{r_S \cos \left( \frac{E_S \tau}{2 \hbar} \right)}.
\]

\[
\psi_L = \left( -\frac{1 + \epsilon}{\sqrt{1 + 2 \epsilon}} \right) e^{-\frac{2 \tau}{\hbar} \sin^{-1} \left( \frac{E_L \tau}{2 \hbar} \right)} \frac{2 r_L \sin \left( \frac{E_L \tau}{2 \hbar} \right)}{r_L \cos \left( \frac{E_L \tau}{2 \hbar} \right)}.
\]

Here:

\[
r_S = -\frac{2}{\tau} \sin^{-1} \left( \frac{E_S \tau}{2 \hbar} \right), \quad r_L = -\frac{2}{\tau} \sin^{-1} \left( \frac{E_L \tau}{2 \hbar} \right).
\]

In equations (11) and (12) \( E_S, E_L \) contain the usual real and imaginary parts (one representing the energy, the other the decay time without explicit T violation). To test for explicit cosmological T violation we could measure corrections to the decay time and usual energies of \( E_L, E_S \) induced by \( \epsilon \) in equations (11) and (12). A second approach is to look for production of \( K^0, \bar{K}^0 \) in black hole decay or supernova explosions, studying the ratio of the electrons to positrons produced as a function of arrival time for \( K_L \) in the reactions:

\[
K^0 \rightarrow \pi^- + e^+ + \nu_e, \bar{K}^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e
\]

would be a probe to measuring \( \epsilon \) (explicit T violation parameter). Also by plotting \( \epsilon \) and \( \tau \) versus the distance at which this decay occurred could also provide us with a possible probe to the variation of \( \epsilon, \tau \) with the scale factor of the universe. In a
previous note [24] we have discussed the variation of \( \tau \) (discrete time interval parameter) with the scale factor through a modification of the Hubble law and a similar study could be made for \( \epsilon \). If any variation occurs it will definitely signal that \( \epsilon \) was coupled to the expansion of the universe and there is evidence for the connection of the cosmological arrow of time to the elementary particle arrow of time.

3 Conclusion

The above discussion of T violation in terms of both Markov effects and explicit cosmological induced effects points to a new way of looking at a problem whose field theoretic solution seems to depend both on the CPT theorem and a host of uncertain parameters. As shorter and shorter times are being probed in laser research [31] it seems possible that small fluctuations in the CP violating parameter might be probed through the local charge oscillations in the decay products of the \( K^0, \bar{K}^0 \) which in turn can be probed through the effect on a femtosecond laser pulse. Also the cosmological variation of the parameter \( \epsilon \) in equation (10) might be studied in the radiation coming from supernova [32] where the electron-positive asymmetry can be studied as a function of time in the decay of the \( K_L \) component of an initial beam of \( K_0 \) (here \( K_0 \) oscillates to \( K_L \) and \( K_S \)). The characteristics of the asymmetry will depend on \( \epsilon \) and \( \tau \) in equations (11) and (12) and comparison with other supernova black hole events at different red shifts can be sued as a probe to the cosmological variation of \( \epsilon \) and \( \tau \). Both of these parameters might depend on some power of the scale factor and might also depend on derivatives of the scale factor as in the case with the deceleration parameter. Recently Dzuba, et. al. [33] And Webb et. al. [34] have found evidence for the cosmological variation of the fine structure constant in quasar spectra which supports our premise that the fundamental constants should vary with cosmological evolution. Actually shortly after the discovery of CP violation, Ne’eman et. al. [35, 36, 37] in various papers tried to relate the cosmological arrow of time to microscopic CP violation which is somewhat related to our study. Also Agrawal et. al. [38] recently have discussed the possibility that symmetry breaking in particle theory might be a domain dependent effect which would suggest that the CP violating parameter is a function of both where you are and at what cosmological time you observe CP violation. These studies as well as the work of Nanopoulos and Nanopoulos et. al. [10, 11] suggesting a universal Procrustean principle imply that T violation is related to cosmological evolution independent of the details of the CPT theorem. The attractiveness of these ideas lie in the fact that they are more akin to our notions of time as an ordering parameter as opposed to the usual procedure of relegating CP or T violation to a number of interdependent and questionable properties of quantum field operators. It is hoped that these studies will encourage both the theoretical and experimental community to search for other examples of T violations and how they might be related to a cosmological arrow of time.
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