GEOMETRICAL OBJECTS ON SUBBUNDLES

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Abstract

The extended reduction of a vector bundle to a vector subbundle is defined by the authors in a previous paper. Using a splitting, defined in [4] and called a Finsler splitting,

a certain reduction of a vector bundle is also considered. The aim of this paper is to investigate the possibility that the result remains valid in the case when a restricted prolongation of a vector bundle, defined in this paper, is considered instead. As it is shown in the paper, the answer is negative in the general case.

AMS Subject Classification: 57R22, 53C99.
Key words: vector subbundle, reduction of a vector bundle, Finsler splitting, restricted prolongation.

1 Introduction

The reductions of vector bundles and the F-splitting are studied by the authors in [4], [5] and [6], where the extension of order r of a linear group is defined and the following result is proved:

Theorem 1.1 *[6, Theorem 2.2] Let ξ' be a vector subbundle of the vector bundle ξ and $r \geq 1$. Then:*

1)Every Finsler splitting of the inclusion $i : \xi' \to \xi$ defines a canonical reduction of the structural group $G^r_{0m,n}$ of $\mathcal{O}G_0\xi'(\xi)^r$ to $H^r_{0m,n}$, the reduced principal bundle being $\mathcal{O}H_0\xi'(\xi)^r$.

2)Every reduction of the structural group $G^r_{0m,n}$ of $\mathcal{O}G_0\xi'(\xi)^r$ to $H^r_{0m,n}$ is such that $H^r_{0m,n}$ is the structural group of $\mathcal{O}H_0\xi'(\xi)^r$ and it is induced by a Finsler splitting, as above.

The aim of this paper is to investigate the possibility that the result remains valid in the case when the reduction mentioned in the above theorem is replaced by a restricted prolongation, defined in the third section of the paper. We show that in the general case the answer is negative.

Editor Gr.Tsagas *Proceedings of the Workshop on Global Analysis, Differential Geometry and Lie Algebras, 1996,* 62-69

2 Reductions of vector bundles

This section contains the principal notions concerning reductions of vector bundles and the algebraic form of the reduction, as in [6].

Let $\xi = (E, \pi, M)$ be a vector bundle with the fibre $F \cong I\!R^n$, $G \subset GL_n(I\!R)$ be a Lie subgroup and $\xi' = (E', \pi', M)$ be a vector subbundle with the fibre type $F' \cong I\!R^k \subset I\!R^n$. Let us denote by $L(\xi) = (L(E), p, M)$ the principal bundle of the frames of the vector bundle ξ and by $L\xi'(\xi) = \pi'^* L(\xi) = (\pi'^* L(E) = LE'(E), p_1, E')$ the induced principal bundle, which is also the principal bundle of frames of the vector bundle $\pi'^* \xi \stackrel{not.}{=} \xi'(\xi) = (\pi'^*(E) = E'(E), \pi_1, E')$. Notice that a *Finsler splitting* of the inclusion $i : \xi' \to \xi$ is a left splitting of the inclusion morphism $(\pi')^* \xi' \stackrel{(\pi')^* i}{\to} (\pi')^* \xi$ (see [5].)

Let us suppose that the structural group $GL_n(I\!R)$ of the principal bundle $L(\xi)$ is reducible to the subgroup G, and denote as $L(\xi)_G$ the reduced principal bundle. There is a local trivial bundle ξ_G, associated with the principal bundle $L(\xi)_G$, defined by the left action of G on F.

Definition 2.1 *[6]* We say that the bundle ξ_G is the *G-reduced bundle of* ξ. If H is a subgroup of G and there is a reduction of the structural group G of $L(\xi)_G$ to H, we say in an analogous way that ξ_H is a *H-reduced bundle of* ξ_G.

Notice that a reduction of the structural group G of $L(\xi)_G$ to H is also a reduction of the structural group $GL_n(I\!R)$ of $L(\xi)$ to H.

Example 2.1 Let $G_0 \subset GL_n(I\!R)$ be the subgroup of automorphisms which leave invariant the vector subspace $F' \cong I\!R^k$ of $I\!R^n$. We have:

$$G_0 = \left\{ \begin{pmatrix} A & C \\ 0 & B \end{pmatrix} ; A \in GL_k(I\!R),\, B \in GL_{n-k}(I\!R),\, C \in \mathcal{M}_{k,n-k}(I\!R) \right\}. \qquad (1)$$

The principal bundle $L(\xi)_{G_0}$ always exists and it consists of all the frames of $L(\xi)$ which extend frames on ξ'; these frames are called frames on ξ *adapted* to ξ'.

For the same G_0 as above, we can consider the principal bundle $L\xi'(\xi)_{G_0}$, which also consists of frames on $L\xi'(\xi)$ which extend frames on $\xi'(\xi')$, called as frames on $\xi'(\xi)$, *adapted* to $\xi'(\xi')$.

Example 2.2 Let G_0 be as above, $F'' \cong I\!R^{n-k}$ a vector subspace of F, so that $F = F' \oplus F''$ and $H_0 \subset G_0$ the subgroup of the elements which leave invariant the vector subspaces F' and F''. We have:

$$H_0 = \left\{ \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} ;\, A \in GL_k(I\!R),\, B \in GL_{n-k}(I\!R) \right\}. \qquad (2)$$

The principal bundle $L(\xi)_{H_0}$ always exists and it is a reduction of $L(\xi)_{G_0}$, which corresponds to a reduction of the structural group G_0 of $L(\xi)_{G_0}$ to the structural group H_0 of $L(\xi)_{H_0}$. It consists of frames of $L(\xi)_{G_0}$ which are also adapted to another subbundle $\xi'' = (E'', \pi'', M)$ of ξ. It follows that in every point $x \in M$ we have the direct sum of the vector spaces $E_x = E'_x \oplus E''_x$. A such reduction is also called a Whitney sum of the vector bundles ξ' and ξ'' and it is denoted by $\xi' \oplus \xi''$. This is equivalent with a left splitting S of the inclusion morphism $i : \xi' \to \xi$, when $\xi'' = \ker S$.

In the case of the principal bundle $L\xi'(\xi)$, a reduction of the group G_0 of $L\xi'(\xi)_{G_0}$ to H_0 is equivalent with a left splitting S of the inclusion $i' = \pi'^* i : \pi'^*\xi' \overset{not.}{=} \xi'(\xi') \to \pi'^*\xi \overset{not.}{=} \xi'(\xi)$, called a Finsler splitting in [5]. In this case $\xi'(\xi)$ has an H_0-reduction as Whitney sum $\xi'(\xi) = \xi'(\xi') \oplus \ker S$.

It is well known that the reduction of the structural group G of a principal bundle P to a subgroup $H \subset G$ is equivalent with the existence of a global section in a fibre bundle associated to P, which has the fibre G/H, with the natural action of G on G/H [1, pg.57, Propzition 5.6].

Proposition 2.1 *[6, Proposition 1.1]There is a canonical identification*

$$G_0/H_0 \cong \mathrm{M}_k = \left\{ \begin{pmatrix} 0 & P \\ 0 & I_{n-k} \end{pmatrix}; P \in \mathcal{M}_{k,n-k}(I\!R) \right\}, \tag{3}$$

the classes being at left, such that the left action $\odot$ of the group G_0 on M_k is the adjunction:

$$\begin{pmatrix} E & G \\ 0 & F \end{pmatrix} \odot \begin{pmatrix} 0 & P \\ 0 & I_{n-k} \end{pmatrix} = \begin{pmatrix} E & G \\ 0 & F \end{pmatrix} \cdot \begin{pmatrix} 0 & P \\ 0 & I_{n-k} \end{pmatrix} \cdot \begin{pmatrix} E & G \\ 0 & F \end{pmatrix}^{-1} =$$

$$= \begin{pmatrix} 0 & (E \cdot P + G) \cdot F^{-1} \\ 0 & I_{n-k} \end{pmatrix}. \tag{4}$$

Since the vector subbundle ξ' is given, the reduction of the group $GL_n(I\!R)$ to G_0 implies that the G_0-reductions of the vector bundles $L(\xi)$ and $L\xi'(\xi)$ are uniquely defined. It follows that considering the bundles with the fibres $GL_n(I\!R)/G_0$, associated with the principal bundles of frames $L(\xi)$ and $L\xi'(\xi)$, the sections in these bundles, which correspond to the reductions of $GL_n(I\!R)$ to G_0, are uniquely defined by ξ'.

In the case of the Example 2.2, the reductions of the structural group G_0 to H_0 implies that the H_0-reductions of the principal bundles $L(\xi)_{G_0}$ and $L\xi'(\xi)_{G_0}$ are equivalent with sections in the bundles F_1 and F_2 which are associated with these principal bundles and have as fibres G_0/H_0.

3 The restricted prolongation of a subgroup of the linear group

Definition 3.1 If $G \subset GL_n(I\!R)$ is a Lie subgroup then we denote by $\mathcal{A}(G)$ the real subalgebra of the matrices of $\mathcal{M}_n(I\!R)$ generated by G.

It is obvious that $\mathcal{A}(G)$ is also a Lie subalgebra of $\mathcal{M}_n(I\!R)$.

Example 3.1 *1.* In the case of G_0 and H_0 given by formulas (1) and (2), the subalgebras $\mathcal{A}(G_0)$ and $\mathcal{A}(H_0)$ have the form:

$$\mathcal{A}(G_0) = \left\{ \begin{pmatrix} A & C \\ 0 & B \end{pmatrix}; A \in \mathcal{M}_k(I\!R),\ B \in \mathcal{M}_{n-k}(I\!R),\ C \in \mathcal{M}_{k,n-k}(I\!R) \right\}, \tag{5}$$

$$\mathcal{A}(H_0) = \left\{ \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}; A \in \mathcal{M}_k(I\!R),\ B \in \mathcal{M}_{n-k}(I\!R) \right\}. \tag{6}$$

2. In the case of orthogonal groups we have $\mathcal{A}(SO(n)) = \mathcal{A}(O(n)) = \mathcal{M}_n(I\!R)$ for $n \geq 3$. For $n = 2$ we have $\mathcal{A}(O(2)) = \mathcal{M}_2(I\!R)$ and

$$\mathcal{A}(SO(2)) = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix}; a, b \in I\!R \right\}.$$

3. In the case of the subgroup $Sp(n; I\!R) \cap O(2n)$ (according to [7, Example 10, Pg. 25] which is the real form of the unitary complex group $U(n)$):

$$Sp(n; I\!R) \cap O(2n) = \left\{ \begin{pmatrix} A & -B \\ B & A \end{pmatrix}; A, B \in \mathcal{M}_n(I\!R), A^t A + B^t B = I_n \right\},$$

we have, for $n \geq 1$:

$$\mathcal{A}(Sp(n; I\!R) \cap O(2n)) = \left\{ \begin{pmatrix} A & -B \\ B & A \end{pmatrix}; A, B \in \mathcal{M}_n(I\!R) \right\}.$$

4. In the case of the real form of the complex general linear group $GL_n(\mathbb{C}) \subset GL_{2n}(I\!R)$

$$GL_n(\mathbb{C}) = \left\{ \begin{pmatrix} A & -B \\ B & A \end{pmatrix} \in GL_{2n}(I\!R);\ A, B \in \mathcal{M}_n(I\!R) \right\},$$

we also have:

$$\mathcal{A}(GL_n(\mathbb{C})) = \left\{ \begin{pmatrix} A & -B \\ B & A \end{pmatrix}; A, B \in \mathcal{M}_n(I\!R) \right\}.$$

Let $G \subset GL_n(I\!R)$ be a subgroup. Let us denote by $G^1_{m,n}$ the Lie subgroup of $GL(m+n, I\!R)$ which consists of the matrices which have the form:

$$\begin{pmatrix} A^i_j & 0 \\ 0 & B^a_b \end{pmatrix},\ (A^i_j) \in GL_m(I\!R),\ (B^a_b) \in G.$$

We define now the *restricted prolongation* of order $r \in I\!N^*$ (or *r-prolongation*) of the group $G^1_{n,m}$, denoted by $\bar{G}^r_{n,m}$, as being the set of the elements which have the form:

$$a = (A^i_{j_1}, A^i_{j_1 j_2}, \ldots, A^i_{j_1 j_2 \cdots j_r};\ B^a_b, B^a_{bj_1}, \ldots, B^a_{bj_1 \cdots j_{r-1}}), \tag{7}$$

where the components are symmetric in the indices $\{j_k\} \subset \{1,\ldots,m\}$, $\left(A^i_j\right) \in GL_m(IR)$, $(B^a_b) \in G$ and, in addition as the extended prolongation defined in [6], the matrices B, considered fixing the j-indices, belong to the algebra $\mathcal{A}(G)$. In other words, considering the components in as components of linear maps in canonical bases $a : (A,B) = ((A_1(\cdot), A_2(\cdot,\cdot),\ldots, A_r(\cdot,\ldots,\cdot), B_0(\cdot), B_1(\cdot,\cdot),\ldots, B_{r-1}(\cdot,\cdot,\ldots,\cdot)))$ and fixing $v_1,\ldots,v_p \in IR^m$, then the map $u \to B_p(u,v_1,\ldots,v_p)$ defines an endomorphism in $\mathcal{A}(G)$.

The composition law for elements which have the form (7) is like a composition of linear maps. In order to make it explicitly, let us consider

$$a : (A,B) = (A_1(\cdot), A_2(\cdot,\cdot),\ldots, A_r(\cdot,\ldots,\cdot); B_0(\cdot),, B_1(\cdot,\cdot),\ldots, B_{r-1}(\cdot,\cdot,\ldots,\cdot))$$

$$B_0(\cdot),, B_1(\cdot,\cdot),\ldots, B_{r-1}(\cdot,\cdot,\ldots,\cdot))$$

and $b : (C,D)$. Then $b\cdot a : (A',B')$, where:

$$\begin{array}{l}
A'_1(u) = C_1A_1(u)\\
A'_2(u_1,u_2) = C_1A_2(u_1,u_2) + C_2(A_1(u_1), A_1(u_2))\\
A'_3(u_1,u_2,u_3) = C_1A_3(u_1,u_2,u_3) + C_2(A_2(u_1,u_2), A_1(u_3))+\\
+C_2(A_1(u_1), A_2(u_2,u_3) + C_2(A_2(u_1,u_3), A_1(u_2)) + C_3(A_1(u_1), A_1(u_2), A_1(u_3))\\
\cdots\cdots\cdots\\
B'_0(u) = D_0B_0(u)\\
B'_1(v,u_1) = D_0B_1(v,u_1) + D_1(B_0(v), A_1(u_1))\\
B'_2(v,u_1,u_2) = D_0B_2(v,u_1,u_2) + D_1(B_1(v,u_1), A_1(u_2))+\\
+D_1(B_0(v), A_2(u_1,u_2)) + D_1(B_1(v,u_2), A_1(u_1)) + D_2(B_0(v), A_1(u_1), A_1(u_2))\\
\cdots\cdots\cdots
\end{array} \tag{8}$$

Using the coordinates, the expressions are the same as in [2, pag. 70].

It is easy to see that $\bar{G}^r_{n,m}$ is a Lie group. In fact, $\bar{G}^r_{n,m}$ is a Lie subgroup of $G^r_{n,m}$, and the structural functions of the bundle $\mathcal{O}G\xi^r$, given by the formula:

$$\varphi_{UU'}(u) = (\frac{\partial x^{i'}}{\partial x^i}(x),\ldots,\frac{\partial^r x^{i'}}{\partial x^{i_1}\cdots\partial x^{i_r}}(x), g^{a'}_a(x),\ldots,\frac{\partial^{r-1}g^{a'}_a}{\partial x^{j_1}\cdots\partial x^{j_{r-1}}}(x)) \tag{9}$$

belong to the group $\bar{G}^r_{n,m}$, thus it is defined a reduction $\mathcal{O}\bar{G}\xi^r$ which correspond to a reduction of the structural group $G^r_{n,m}$ to the subgroup $\bar{G}^r_{n,m}$.

Considering a fibered manifold $\mu = (T,s,M)$, then a principal bundle $\mathcal{O}\bar{G}\mu(\xi)^r$ over the base T is induced as well. In the sequel we consider $\xi' = (E',\pi_1,M)$ a vector subbundle of ξ and consequently $\mathcal{O}\bar{G}\xi'(\xi)^r = i^*\mathcal{O}\bar{G}\xi^r$, where $i : E' \to E$ is the inclusion.

In the sequel we study the reductions of the structural group $\bar{G}^r_{0m,n}$ of the principal bundle $\mathcal{O}\bar{G}_0\xi'(\xi)^r$, with G_0 given by the formula (1), to the subgroup $\bar{H}^r_{0m,n}$, where H_0 is given by the formula (2). These reductions can be related to geometrical $\bar{G}_0$-objects, as done in [6] for extended reductions.

In the case of extended prolongations it can be proved that there is a canonical isomorphism $G^r_{0m,n}/H^r_{0m,n} \cong G_0/H_0$. According to [1, pg.57, Propzition 5.6] it is an essential fact that the conclusion of theorem 1.1 be valid. We shall see that in the case of r-prolongations there is not a equivalence of $\bar{G}^r_{0m,n}/\bar{H}^r_{0m,n}$ and G_0/H_0. It implies that in the general case there are reductions of the group $\bar{G}^r_{0m,n}$ of the principal bundle $\mathcal{O}\bar{G}_0\xi'(\xi)^r$ to the subgroup $\bar{H}^r_{0m,n}$ which do not come from a Finsler splitting. We study in detail only the case $r = 2$. The general case is analogous, but the calculus is more complicated.

Now we state the main theorem of this paper.

Theorem 3.1 *There is a one to one map between* $\bar{G}^2_{0m,n}/\bar{H}^2_{0m,n}$ *and the subgroup of* $\bar{G}^2_{01m,n} \subset \bar{G}^2_{0m,n}$ *which consists of elements which have the form:*

$$\left(I_m, 0, \begin{pmatrix} I_k & B^\alpha_u \\ 0 & I_{n-k} \end{pmatrix}, \begin{pmatrix} 0 & B^\alpha_{ui} \\ 0 & 0 \end{pmatrix}\right).$$

Proof. If $a, g \in \bar{G}^2_{0m,n}$ have the form:

$$a = \left(\bar{A}^i_j, \bar{A}^i_{jk}, \begin{pmatrix} \bar{B}^\alpha_\beta & \bar{B}^\alpha_u \\ 0 & \bar{B}^v_u \end{pmatrix}, \begin{pmatrix} \bar{B}^\alpha_{\beta i} & \bar{B}^\alpha_{ui} \\ 0 & \bar{B}^v_{ui} \end{pmatrix}\right),$$

$$g = \left(A^i_j, A^i_{jk}, \begin{pmatrix} B^\alpha_\beta & B^\alpha_u \\ 0 & B^v_u \end{pmatrix}, \begin{pmatrix} B^\alpha_{\beta i} & B^\alpha_{ui} \\ 0 & B^v_{ui} \end{pmatrix}\right), \tag{10}$$

then the product ag has the form:

$$ag = \left(\overline{\overline{A}}^i_j, \overline{\overline{A}}^i_{jk}, \begin{pmatrix} \overline{\overline{B}}^\alpha_\beta & \overline{\overline{B}}^\alpha_u \\ 0 & \overline{\overline{B}}^v_u \end{pmatrix}, \begin{pmatrix} \overline{\overline{B}}^\alpha_{\beta i} & \overline{\overline{B}}^\alpha_{ui} \\ 0 & \overline{\overline{B}}^v_{ui} \end{pmatrix}\right), \tag{11}$$

where

$$\overline{\overline{A}}^j_i = \bar{A}^k_i A^j_k,\ \overline{\overline{A}}^i_{jk} = \bar{A}^i_l A^l_{jk} + \bar{A}^i_{pq} A^p_j A^q_k, \tag{12}$$

$$\overline{\overline{B}}^\alpha_\beta = \bar{B}^\alpha_\gamma B^\gamma_\beta\,, \tag{13}$$

$$\overline{\overline{B}}^\alpha_u = \bar{B}^\alpha_\gamma B^\gamma_u + \bar{B}^\alpha_w B^w_u\,, \tag{14}$$

$$\overline{\overline{B}}^v_u = \bar{B}^v_w B^w_u\,, \tag{15}$$

$$\overline{\overline{B}}^\alpha_{\beta i} = \bar{B}^\alpha_\gamma B^\gamma_{\beta i} + \bar{B}^\alpha_{\gamma k} B^\gamma_\beta A^k_i\,, \tag{16}$$

$$\overline{\overline{B}}^\alpha_{ui} = \bar{B}^\alpha_\gamma B^\gamma_{ui} + \bar{B}^\alpha_v B^v_{ui} + \bar{B}^\alpha_{\gamma k} B^\gamma_u A^k_i + \bar{B}^\alpha_{vk} B^v_u A^k_i\,, \tag{17}$$

$$\overline{\overline{B}}^v_{ui} = \bar{B}^v_w B^w_{ui} + \bar{B}^v_{wk} B^w_u A^k_i. \tag{18}$$

It is easy to see that $\bar{G}^2_{01m,n}$ is a closed Lie subgroup of $\bar{G}^2_{0m,n}$.

If $a \in G_0$ and $g \in H_0$, then $B^\alpha_u = 0$, $B^\alpha_{ui} = 0$ and ag has the form (11), where the components are given by the formulas (12)-(18). The initial conditions have an influence only on relations (14) and (17), which become:

$$\overline{\overline{B}}^\alpha_u = \bar{B}^\alpha_w B^w_u\,,\ \overline{\overline{B}}^\alpha_{ui} = \bar{B}^\alpha_v B^v_{ui} + \bar{B}^\alpha_{vk} B^v_u A^k_i\,. \tag{19}$$

It can be shown that the matrix $(\bar{B}_u^\alpha)\cdot(\bar{B}_u^v)^{-1} = (\bar{B}_w^\alpha \widetilde{\bar{B}}{}^v_u)$ does not depend on the left class of g in $\bar{G}^2_{0m,n}/\bar{H}^2_{0m,n}$. We are going to find a matrix which consists of linear forms on IR^m (the components are obtained fixing j), namely $(\overline{\overline{B}}{}^\alpha_{ui})_{\alpha,u}$, which does not depend on the class of g. It can be obtained as follows:

Let us fix $g \in \bar{G}^2_{0m,n}$, which has the form (10). We are looking for $a \in \bar{H}^2_{0m,n}$ such that ag has the components $\left(\overline{\overline{B}}{}^a_{bi}\right)$ in a simplest form. If we impose the conditions:

$$\overline{\overline{A}}{}^k_i = \delta^k_i,\ \overline{\overline{B}}{}^\alpha_\beta = \delta^\alpha_\beta,\ \overline{\overline{B}}{}^u_v = \delta^u_v,\ \overline{\overline{B}}{}^\alpha_{\beta i} = 0,\ \overline{\overline{B}}{}^v_{ui} = 0,$$

we obtain the following expressions for the components of a:

$$A^k_i = \widetilde{\bar{A}}{}^k_i,\ B^\alpha_\beta = \widetilde{\bar{B}}{}^\alpha_\gamma,\ B^u_v = \widetilde{\bar{B}}{}^u_v,\ B^\gamma_{\beta i} = -\widetilde{\bar{B}}{}^\alpha_\gamma \bar{B}^\gamma_{\delta k} B^\delta_\beta A^k_i, \quad B^v_{ui} = -\widetilde{\bar{B}}{}^v_t \bar{B}^t_{wk} B^w_u A^k_i,$$

where $\left(\bar{A}^k_i\right)^{-1} = \left(\widetilde{\bar{A}}{}^k_i\right)$, $\left(\bar{B}^v_u\right)^{-1} = \left(\widetilde{\bar{B}}{}^v_u\right)$ and $\left(\bar{B}^\alpha_\beta\right)^{-1} = \left(\widetilde{\bar{B}}{}^\alpha_\beta\right)$. It implies that the component $\overline{\overline{B}}{}^\alpha_{ui}$ of ag, given by the second formula in relations (19)), has the form:

$$\overline{\overline{B}}{}^\alpha_{ui} = \left(-\bar{B}^\alpha_w \widetilde{\bar{B}}{}^w_t \bar{B}^t_{vk} + \bar{B}^\alpha_{vk}\right) \widetilde{\bar{B}}{}^v_u \widetilde{\bar{A}}{}^k_i .$$

A straightforward computation leads to the fact that $\overline{\overline{B}}{}^\alpha_{ui}$ is the same for every element in the left class $g\cdot\bar{H}^2_{0m,n} \in \bar{G}^2_{0m,n}/\bar{H}^2_{0m,n}$. It follows that the element

$$\left(I_m, 0, \begin{pmatrix} I_k & \overline{\overline{B}}{}^\alpha_u \\ 0 & I_{n-k} \end{pmatrix}, \begin{pmatrix} 0 & \overline{\overline{B}}{}^\alpha_{ui} \\ 0 & 0 \end{pmatrix}\right) \in \bar{G}^2_{01m,n}$$

is in the same class with g and it can be uniquely associated with the class $g\cdot\bar{H}^2_{0m,n}$.

Conversely, giving an element in $\bar{G}^2_{01m,n}$, it can be associated with its class in $\bar{G}^2_{0m,n}/\bar{H}^2_{01m,n}$. These two associations are inverse each to other, thus they define a bijection.□

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