STRUCTURE EQUATIONS IN INVARIANT FRAMES

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Abstract

The study of higher order Lagrange spaces founded on the notion of bundle of velocities of order k has been given by Radu Miron and Gheorghe Atanasiu in [2]. The bundle of accelerations correspond in this study to k=2.

The notion of invariant geometry of order 2 was introduced by the author in [4]. In this paper we shall give the structure equations of a N-linear connection in the bundle of accelerations in invariant frames.

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1 General Invariant Frames

Let us consider the bundle $E = Osc^2 M$, a nonlinear connection N with the coefficients $\left(\underset{(1)}{N^i{}_j} \ , \ \underset{(2)}{N^i{}_j} \right)$ and the duals $\left(\underset{(1)}{M^i{}_j} \ , \ \underset{(2)}{M^i{}_j} \right)$.

The invariant frames adapted to the direct decomposition:

$$T_u(\ Osc^2 M \ ) \ = \ N_0(u) \oplus N_1(u) \oplus V_2(u), \quad \forall u \in E,$$

will be $\Re \ = \ (\ e^{(0)i}{}_{\alpha} \, , e^{(1)i}{}_{\alpha} \, , e^{(2)i}{}_{\alpha} \ )$ and the dual $\Re^* \ = \ (\ f^{(0)\alpha}{}_{i} \, , f^{(1)\alpha}{}_{i} \, , f^{(2)\alpha}{}_{i} \ )$.

The duality conditions are:

$$< \ e^{(A)i}{}_{\alpha} \, , f^{(B)\alpha}{}_{j} \ > \ = \ \delta^i_j \, \delta^A_B, \quad (\, A, B = 0, 1, 2 \,).$$

In this frame the adapted basis has the representation:

$$\frac{\delta}{\delta x^i} \ = \ f^{(0)\alpha}{}_{i} \frac{\delta}{\delta s^{(0)\alpha}} \ , \quad \frac{\delta}{\delta y^{(1)i}} \ = \ f^{(1)\alpha}{}_{i} \frac{\delta}{\delta s^{(1)\alpha}} \ , \quad \frac{\delta}{\delta y^{(2)i}} \ = \ f^{(2)\alpha}{}_{i} \frac{\delta}{\delta s^{(2)\alpha}}$$

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and the cobasis are:

$$\delta x^i \;=\; e^{(0)i}_{\;\;\alpha}\delta s^{(0)\alpha};\quad \delta y^{(1)i} \;=\; e^{(1)i}_{\;\;\alpha}\delta s^{(1)\alpha};\quad \delta y^{(2)i} \;=\; e^{(2)i}_{\;\;\alpha}\delta s^{(2)\alpha}.$$

We have the relations:

$$\left\langle \frac{\delta}{\delta s^{(A)\alpha}}\,,\,\delta s^{(B)\beta}\right\rangle \;=\; \delta^{\beta}_{\alpha}\,\delta^{B}_{A}\quad,\quad (A,B\;=\;0,1,2\,).$$

This representation lead us to an invariant frames transformation group. The frame change according expressions:

$$\overline{e}^{(A)i}_{\;\;\alpha} \;=\; \overset{A}{C}{}^{\beta}_{\alpha}\;(x,y^{(1)},y^{(2)}),e^{(A)i}_{\;\;\beta}\quad;\quad f^{(B)\alpha}_{\;\;j} \;=\; \overline{\overset{B}{C}}{}^{\alpha}_{\beta}\,\overline{f}^{(B)\beta}_{\;\;j}\,,$$

and the group is isomorphic with the multiplicative nonsingular matrix group

$$\begin{pmatrix} \overset{0}{C}{}^{\alpha}_{\beta} & 0 & 0\\ 0 & \overset{1}{C}{}^{\alpha}_{\beta} & 0\\ 0 & 0 & \overset{2}{C}{}^{\alpha}_{\beta}\end{pmatrix}.$$

An invariant N-linear connection D has in the frame $\Re$ the coefficients:

$$\overset{0A}{L}{}^{\gamma}_{\beta\alpha} \;=\; f^{(A)\gamma}_{\;\;m}\left(\frac{\delta e^{(A)m}_{\;\;\beta}}{\delta s^{(0)\alpha}} \;+\; e^{(0)i}_{\;\;\alpha}e^{(A)j}_{\;\;\beta}L^{m}_{ij}\right),\quad (A=0,1,2),$$

$$\overset{BA}{C}{}^{\gamma}_{\beta\alpha} \;=\; f^{(A)\gamma}_{\;\;m}\left(\frac{\delta e^{(A)m}_{\;\;\beta}}{\delta s^{(B)\alpha}} \;+\; e^{(B)i}_{\;\;\alpha}e^{(A)j}_{\;\;\beta}\;\underset{(B)}{C^{m}_{ij}}\right),\quad (A=0,1,2\,;\,B=1,2\,).$$

Definition 1 If the vector field $X\;\in\;\chi\;(E)$ has the invariant components $X^{(A)\alpha}$ $(A=0,1,2)$, then we denote by $_{|}\,,\overset{(B)}{_{|}}$ the $h-$ and v_B covariant invariant derivative operators $(B=1,2)$, i.e.:

$$\begin{aligned} X^{(A)\alpha}_{\;\;\;\;|\beta} \;&=\; \frac{\delta X^{(A)\alpha}}{\delta s^{(0)\beta}} \;+\; \overset{0A}{L}{}^{\alpha}_{\varphi\beta}\,X^{(A)\varphi}, \\ X^{(A)\alpha}\overset{(B)}{|}_{\beta} \;&=\; \frac{\delta X^{(A)\alpha}}{\delta s^{(B)\beta}} \;+\; \overset{BA}{C}{}^{\alpha}_{\varphi\beta}\,X^{(A)\varphi}. \end{aligned}\tag{1}$$

The definition of the Lie bracket leads us to consider of the non-holonomy coefficients of Vranceanu:

$$\left[\frac{\delta}{\delta s^{(A)\alpha}}\,,\,\frac{\delta}{\delta s^{((B)\beta}}\right] \;=\; \underset{(AB)}{\overset{0}{W}{}^{\gamma}_{\alpha\beta}}\;\frac{\delta}{\delta s^{(0)\gamma}} \;+\; \underset{(AB)}{\overset{1}{W}{}^{\gamma}_{\alpha\beta}}\;\frac{\delta}{\delta s^{(1)\gamma}} \;+\; \underset{(AB)}{\overset{2}{W}{}^{\gamma}_{\alpha\beta}}\;\frac{\delta}{\delta s^{(2)\gamma}},$$

$(A,B=0,1,2;\quad A\leq B)$.

2 Torsion and Curvature d-tensor Fields

The torsion tensor of the N-linear connection D on E:

$$\mathcal{T}(\mathcal{X},\mathcal{Y}) \;=\; \mathcal{D}_{\mathcal{X}}\mathcal{Y} \;-\; \mathcal{D}_{\mathcal{Y}}\mathcal{X} \;-\; [\mathcal{X},\mathcal{Y}], \qquad \forall\, \mathcal{X},\mathcal{Y} \;\in\; \chi(\mathcal{E}),$$

in the invariant frame $\Re$, has a number of horizontal and vertical components corresponding to D^h , D^{v_1} and D^{v_2} respectively.

Theorem 2.1. *The torsion tensor of a N-linear connection D in the invariant frame $\Re$ is characterized by the d-tensor fields with local components*

$$\left\{\begin{aligned} \underset{(0)}{T^{\gamma}{}_{\beta\alpha}} &= \overset{(00)}{L^{\gamma}{}_{\beta\alpha}} - \overset{(00)}{L^{\gamma}{}_{\alpha\beta}} - \underset{(00)}{\overset{(0)}{W^{\gamma}{}_{\beta\alpha}}} \\ \underset{(0A)}{R^{\gamma}{}_{\beta\alpha}} &= \underset{(00)}{\overset{(A)}{W^{\gamma}{}_{\beta\alpha}}} \end{aligned}\right. ,$$

$$\left\{\begin{aligned} \underset{(1)}{K^{\gamma}{}_{\beta\alpha}} &= -\underset{(1)}{\overset{(10)}{C^{\gamma}{}_{\beta\alpha}}} - \underset{(01)}{\overset{(0)}{W^{\gamma}{}_{\beta\alpha}}} \\ \underset{(11)}{P^{\gamma}{}_{\beta\alpha}} &= \overset{(01)}{L^{\gamma}{}_{\beta\alpha}} + \underset{(01)}{\overset{(1)}{W^{\gamma}{}_{\beta\alpha}}} \\ \underset{(12)}{P^{\gamma}{}_{\beta\alpha}} &= \underset{(01)}{\overset{(2)}{W^{\gamma}{}_{\beta\alpha}}} \end{aligned}\right. ,$$

$$\left\{\begin{aligned} \underset{(2)}{K^{\gamma}{}_{\beta\alpha}} &= -\underset{(2)}{\overset{(20)}{C^{\gamma}{}_{\beta\alpha}}} - \underset{(02)}{\overset{(0)}{W^{\gamma}{}_{\beta\alpha}}} \\ \underset{(21)}{P^{\gamma}{}_{\beta\alpha}} &= \underset{(02)}{\overset{(1)}{W^{\gamma}{}_{\alpha\beta}}} - \underset{(01)}{\overset{(1)}{W^{\gamma}{}_{\beta\alpha}}} \\ \underset{(22)}{P^{\gamma}{}_{\beta\alpha}} &= \overset{(02)}{L^{\gamma}{}_{\beta\alpha}} - \underset{(02)}{\overset{(2)}{W^{\gamma}{}_{\beta\alpha}}} \end{aligned}\right. ,$$

$$\left\{\begin{aligned} \underset{(11)}{Q^{\gamma}{}_{\beta\alpha}} &= \underset{(1)}{\overset{(11)}{C^{\gamma}{}_{\beta\alpha}}} - \underset{(1)}{\overset{(11)}{C^{\gamma}{}_{\alpha\beta}}} - \underset{(11)}{\overset{(1)}{W^{\gamma}{}_{\beta\alpha}}} \\ \underset{(21)}{\overset{(2)}{Q^{\gamma}{}_{\beta\alpha}}} &= \underset{(11)}{\overset{(2)}{W^{\gamma}{}_{\beta\alpha}}} \end{aligned}\right. ,$$

$$\begin{cases} \underset{(12)}{Q}{}^{\gamma}{}_{\beta\alpha} = \overset{(21)}{\underset{(2)}{C}}{}^{\gamma}{}_{\beta\alpha} - \overset{(1)}{\underset{(12)}{W}}{}^{\gamma}{}_{\beta\alpha} \\ \underset{(22)}{Q}{}^{\gamma}{}_{\beta\alpha} = -\overset{(1)}{\underset{(12)}{C}}{}^{\gamma}{}_{\alpha\beta} - \overset{(2)}{\underset{(12)}{W}}{}^{\gamma}{}_{\beta\alpha} \end{cases},$$

$$\begin{cases} \underset{(2)}{S}{}^{\gamma}{}_{\beta\alpha} = \overset{(22)}{\underset{(2)}{C}}{}^{\gamma}{}_{\beta\alpha} - \overset{(22)}{\underset{(2)}{C}}{}^{\gamma}{}_{\alpha\beta} - \overset{(2)}{\underset{(22)}{W}}{}^{\gamma}{}_{\beta\alpha} \end{cases}.$$

Theorem 2.2. *The components given by Theorem 2.1 are the invariant components of the d-tensor fields of torsion of the N-linear connection D*

The curvature tensor field $\mathcal{R}$ of the N-linear connection D on $Osc^2(M)$ has the expression

$$\mathcal{R}(\mathcal{X},\mathcal{Y}) = [\,\mathcal{D}_{\mathcal{X}}\,,\,\mathcal{D}_{\mathcal{Y}}\,]\,\mathcal{Z} - \mathcal{D}_{[\mathcal{X},\mathcal{Y}]}\mathcal{Z}.$$

Theorem 2.3. *The curvature tensor field $\mathcal{R}$ of a N-linear connection D in the invariant frame $\Re$ is characterized by the following d-tensor fields on $Osc^2(M)$:*

$$\begin{aligned} R_{\gamma}{}^{\varphi}{}_{\beta\alpha} = {} & \frac{\delta \overset{(00)}{L}{}^{\varphi}_{\gamma\beta}}{\delta s^{(0)\alpha}} - \frac{\delta \overset{(00)}{L}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(0)\beta}} + \overset{(00)}{L}{}^{\eta}_{\gamma\beta}\overset{(00)}{L}{}^{\varphi}_{\eta\alpha} - \overset{(00)}{L}{}^{\eta}_{\gamma\alpha}\overset{(00)}{L}{}^{\varphi}_{\eta\beta} - \\ & - \overset{(0)}{\underset{(00)}{W}}{}^{\psi}_{\beta\alpha}\,\overset{(00)}{L}{}^{\varphi}_{\gamma\psi} + \overset{(1)}{\underset{(00)}{W}}{}^{\psi}_{\beta\alpha}\,\overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\psi} + \overset{(2)}{\underset{(00)}{W}}{}^{\psi}_{\beta\alpha}\,\overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\psi}\,, \end{aligned}$$

$$\begin{aligned} \underset{(1)}{P}{}_{\gamma}{}^{\varphi}{}_{\beta\alpha} = {} & \frac{\delta\ \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\beta}}{\delta s^{(0)\alpha}} - \frac{\delta \overset{(00)}{L}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(1)\beta}} + \overset{(10)}{\underset{(1)}{C}}{}^{\eta}_{\gamma\beta}\,\overset{(00)}{L}{}^{\varphi}_{\eta\alpha} - \overset{(00)}{L}{}^{\eta}_{\gamma\alpha}\,\overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\eta\beta} - \\ & - \overset{(0)}{\underset{(01)}{W}}{}^{\psi}_{\beta\alpha}\,\overset{(00)}{L}{}^{\varphi}_{\gamma\psi} + \overset{(1)}{\underset{(01)}{W}}{}^{\psi}_{\beta\alpha}\,\overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\psi} + \overset{(2)}{\underset{(01)}{W}}{}^{\psi}_{\beta\alpha}\,\overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\psi}\,, \end{aligned}$$

$$\begin{aligned} \underset{(2)}{P}{}_{\gamma}{}^{\varphi}{}_{\beta\alpha} = {} & \frac{\delta\ \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\beta}}{\delta s^{(0)\alpha}} - \frac{\delta \overset{(00)}{L}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(2)\beta}} + \overset{(20)}{\underset{(2)}{C}}{}^{\eta}_{\gamma\beta}\,\overset{(00)}{L}{}^{\varphi}_{\eta\alpha} - \overset{(00)}{L}{}^{\eta}_{\gamma\alpha}\,\overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\eta\beta} - \\ & - \overset{(0)}{\underset{(02)}{W}}{}^{\psi}_{\beta\alpha}\,\overset{(00)}{L}{}^{\varphi}_{\gamma\psi} + \overset{(1)}{\underset{(02)}{W}}{}^{\psi}_{\beta\alpha}\,\overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\psi} + \overset{(2)}{\underset{(02)}{W}}{}^{\psi}_{\beta\alpha}\,\overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\psi}\,, \end{aligned}$$

$$\underset{(11)}{S_{\gamma}{}^{\varphi}{}_{\beta\alpha}} = \frac{\delta \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\beta}}{\delta s^{(1)\alpha}} - \frac{\delta \overset{(01)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(1)\beta}} + \overset{(10)}{\underset{(1)}{C}}{}^{\eta}_{\gamma\beta}\ \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\eta\alpha} - \overset{(10)}{\underset{(1)}{C}}{}^{\eta}_{\gamma\alpha}\ \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\eta\beta} - \overset{(1)}{\underset{(11)}{W}}{}^{\psi}_{\beta\alpha}\ \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\psi} - \overset{(2)}{\underset{(11)}{W}}{}^{\psi}_{\beta\alpha}\ \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\psi},$$

$$\underset{(21)}{S_{\gamma}{}^{\varphi}{}_{\beta\alpha}} = \frac{\delta \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\beta}}{\delta s^{(1)\alpha}} - \frac{\delta \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(2)\beta}} + \overset{(20)}{\underset{(2)}{C}}{}^{\eta}_{\gamma\beta}\ \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\eta\alpha} - \overset{(10)}{\underset{(1)}{C}}{}^{\eta}_{\gamma\alpha}\ \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\eta\beta} - \overset{(1)}{\underset{(12)}{W}}{}^{\psi}_{\beta\alpha}\ \overset{(10)}{\underset{(1)}{C}}{}^{\varphi}_{\gamma\psi} - \overset{(2)}{\underset{(12)}{W}}{}^{\psi}_{\beta\alpha}\ \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\psi},$$

$$\underset{(22)}{S_{\gamma}{}^{\varphi}{}_{\beta\alpha}} = \frac{\delta \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\beta}}{\delta s^{(2)\alpha}} - \frac{\delta \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\alpha}}{\delta s^{(2)\beta}} + \overset{(20)}{\underset{(2)}{C}}{}^{\eta}_{\gamma\beta}\ \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\eta\alpha} - \overset{(20)}{\underset{(2)}{C}}{}^{\eta}_{\gamma\alpha}\ \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\eta\beta} - \overset{(2)}{\underset{(22)}{W}}{}^{\psi}_{\beta\alpha}\ \overset{(20)}{\underset{(2)}{C}}{}^{\varphi}_{\gamma\psi}.$$

Theorem 2.4. *The components given by Theorem 2.3 are the invariant components of the d-tensor fields of curvature of the N-linear connection D.*

Theorem 2.5. *The essential components in the frame $\Re$ of the curvature tensor field $\mathcal{R}$ are those given by Theorem 2.3.*

3 Structure Equations

Let (C, c), $c : I \longrightarrow Osc^2 M$, $C = Im\ c$ be a smooth curve parametrically given on $Osc^2 M$ and let $\dot{c}$ be the tangent field.

Proposition 3.1. *The covariant differential of the vector field X in the frame $\Re$ is:*

$$DX = \left\{ \left(\frac{\delta X^{(A)\alpha}}{\delta s^{(0)\beta}} \delta s^{(0)\beta} + \frac{\delta X^{(A)\alpha}}{\delta s^{(1)\beta}} \delta s^{(1)\beta} + \frac{\delta X^{(A)\alpha}}{\delta s^{(2)\beta}} \delta s^{(2)\beta} \right) + X^{(A)\gamma}\ \overset{(A)}{\omega}{}^{\alpha}{}_{\gamma} \right\} \frac{\delta}{\delta s^{(A)\alpha}},$$

($A = 0, 1, 2$, summation on A), where:

$$\overset{(A)}{\omega}{}^{\alpha}{}_{\gamma} = \overset{(0A)}{L}{}^{\alpha}_{\gamma\beta}\ \delta s^{(0)\beta} + \overset{(1A)}{\underset{(1)}{C}}{}^{\alpha}_{\gamma\beta}\ \delta s^{(1)\beta} + \overset{(2A)}{\underset{(2)}{C}}{}^{\alpha}_{\gamma\beta}\ \delta s^{(2)\beta}.$$

The 1-forms $\overset{(A)}{\omega}{}^{(\alpha}_{\gamma}$ will be called *invariant 1-forms of connection* for the N-linear connection D. They depend only on D.

Theorem 3.1. *The exterior differentials of the 1-forms $\delta s^{(A)\alpha}$ are given by:*

$$\begin{aligned} d(\delta s^{(0)\gamma}) &= \frac{1}{2} \underset{(00)}{\overset{(0)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(0)\alpha} \wedge \delta s^{(0)\beta} + \underset{(01)}{\overset{(0)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(0)\alpha} \wedge \delta s^{(1)\beta} + \\ &+ \underset{(02)}{\overset{(0)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(0)\alpha} \wedge \delta s^{(2)\beta}, \end{aligned}$$

$$\begin{aligned} d(\delta s^{(1)\gamma}) &= \frac{1}{2} \underset{(00)}{\overset{(1)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(0)\alpha} \wedge \delta s^{(0)\beta} + \underset{(01)}{\overset{(1)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(0)\alpha} \wedge \delta s^{(1)\beta} + \\ &+ \underset{(02)}{\overset{(1)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(0)\alpha} \wedge \delta s^{(2)\beta} + \frac{1}{2} \underset{(11)}{\overset{(1)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(1)\alpha} \wedge \delta s^{(1)\beta} + \\ &+ \underset{(12)}{\overset{(1)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(1)\alpha} \wedge \delta s^{(2)\beta}, \end{aligned} \tag{2}$$

$$\begin{aligned} d(\delta s^{(2)\gamma}) &= \frac{1}{2} \underset{(00)}{\overset{(2)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(0)\alpha} \wedge \delta s^{(0)\beta} + \underset{(01)}{\overset{(2)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(0)\alpha} \wedge \delta s^{(1)\beta} + \\ &+ \underset{(02)}{\overset{(2)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(0)\alpha} \wedge \delta s^{(2)\beta} + \frac{1}{2} \underset{(11)}{\overset{(2)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(1)\alpha} \wedge \delta s^{(1)\beta} + \\ &+ \underset{(12)}{\overset{(2)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(1)\alpha} \wedge \delta s^{(2)\beta} + \frac{1}{2} \underset{(22)}{\overset{(2)}{W}}{}^{\gamma}_{\beta\alpha} \delta s^{(2)\alpha} \wedge \delta s^{(2)\beta}. \end{aligned} \tag{3}$$

Using the invariant 1-forms of connection for the N-linear connection D we prove the following fundamental theorem:

Theorem 3.2. *The structure equations of the N-linear connection D on the total space E in invariant frames are*

$$d(\delta s^{(A)\alpha}) - \delta s^{(A)\beta} \wedge \overset{(A)}{\omega}{}^{\alpha}_{\beta} = - \overset{(A)}{\Omega}{}^{\alpha}, \tag{4}$$

$$d(\overset{(A)}{\omega}{}^{\alpha}_{\beta}) - \overset{(A)}{\omega}{}^{\gamma}_{\beta} \wedge \overset{(A)}{\omega}{}^{\alpha}_{\gamma} = - \overset{(A)}{\Omega}{}^{\alpha}_{\beta}, \tag{5}$$

$(A=0,1,2)$, where the 2-forms of torsion $\overset{(A)}{\Omega^{\alpha}}$ are given by:

$$\overset{(0)}{\Omega^{\alpha}} = \frac{1}{2} \underset{(0)}{T^{\alpha}_{\beta\gamma}} \delta s^{(0)\beta} \wedge \delta s^{(0)\gamma} + \underset{(1)}{K^{\alpha}_{\beta\gamma}} \delta s^{(0)\beta} \wedge \delta s^{(1)\gamma} + \underset{(2)}{K^{\alpha}_{\beta\gamma}} \delta s^{(0)\beta} \wedge \delta s^{(2)\gamma},$$

$$\overset{(1)}{\Omega^{\alpha}} = \frac{1}{2} \underset{(01)}{R^{\alpha}_{\beta\gamma}} \delta s^{(0)\beta} \wedge \delta s^{(0)\gamma} + \underset{(11)}{P^{\alpha}_{\beta\gamma}} \delta s^{(0)\beta} \wedge \delta s^{(1)\gamma} + \underset{(21)}{P^{\alpha}_{\beta\gamma}} \delta s^{(0)\beta} \wedge \delta s^{(2)\gamma},$$

$$\frac{1}{2} \underset{(1)}{S^{\alpha}_{\beta\gamma}} \delta s^{(1)\beta} \wedge \delta s^{(1)\gamma} + \underset{(2)}{K^{\alpha}_{\beta\gamma}} \delta s^{(1)\beta} \wedge \delta s^{(2)\gamma},$$

$$\overset{(2)}{\Omega^{\alpha}} = \frac{1}{2} \underset{(02)}{R^{\alpha}_{\beta\gamma}} \delta s^{(0)\beta} \wedge \delta s^{(0)\gamma} + \underset{(12)}{P^{\alpha}_{\beta\gamma}} \delta s^{(0)\beta} \wedge \delta s^{(1)\gamma} + \underset{(22)}{P^{\alpha}_{\beta\gamma}} \delta s^{(0)\beta} \wedge \delta s^{(2)\gamma},$$

$$\frac{1}{2} \underset{(21)}{Q^{\alpha}_{\beta\gamma}} \delta s^{(1)\beta} \wedge \delta s^{(1)\gamma} + \underset{(22)}{Q^{\alpha}_{\beta\gamma}} \delta s^{(1)\beta} \wedge \delta s^{(2)\gamma} +$$

$$\frac{1}{2} \underset{(2)}{S^{\alpha}_{\beta\gamma}} \delta s^{(2)\beta} \wedge \delta s^{(2)\gamma}.$$

References

[1] Gh. Atanasiu, *The equations of structure of a N-linear connection in the bundle of accelerations*, Balkan J. of Geom. and its Appl., I, 1, 1996, 11-19.

[2] R. Miron and Gh. Atanasiu, *Higher order Lagrange spaces*, Rev. Roum. Math. Pures Appl.,41, 3-4, 1996, 251-262.

[3] R. Miron, *The Geometry of higher order Lagrange spaces. Applications to Mechanics and Physics*, Kluwer Acad. Pub.FTPH.

[4] M. Păun, *The concept of invariant geometry of second order*, First Conf. of Balkan Soc. of Geom., Bucuresti, 23-27 sept. 1996 (to appear).

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