Some topological properties of the solution set of a fractional integro-differential inclusion

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We study fractional integro-differential inclusions of the form

\[(1) \quad D^\alpha_c x(t) \in F(t, x(t), V(x(t))) \quad a.e. \, ([0, T]), \quad x(0) = x_0, \quad x'(0) = x_1,\]

where \( \alpha \in (1, 2], \) \( D^\alpha_c \) is the Caputo fractional derivative, \( F : [0, T] \times \mathbb{R} \times \mathbb{R} \to \mathcal{P}(\mathbb{R}) \) is a set-valued map and \( x_0, x_1 \in \mathbb{R} \). \( V : C([0, T], \mathbb{R}) \to C([0, T], \mathbb{R}) \) is a nonlinear Volterra integral operator \( V(x)(t) = \int_0^t k(t, s, x(s))ds \) with \( k(\ldots) : [0, T] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R} \) a given function.

We prove the arcwise connectedness of the solution set of problem (1) when the set-valued map is Lipschitz in the second and third variable. Moreover, under such type of hypotheses on the set-valued map, we establish a more general topological property of the solution set of problem (1). Namely, we prove that the set of selections of the set-valued map \( f \) that correspond to the solutions of problem (1) is a retract of \( L^1([0, T], \mathbb{R}) \). Both results are essentially based on Bressan and Colombo results concerning the existence of continuous selections of lower semicontinuous multifunctions with decomposable values.