On quasi hemi-slant submanifolds of nearly Kaehler manifolds

R. Prasad, P. K. Singh and A. K. Rai

Abstract. In the present paper, we study the quasi hemi-slant submanifolds of nearly Kaehler manifolds. These submanifolds are generalization of hemi-slant submanifolds and semi-slant submanifolds. We also study the geometry of leaves of distributions which are involved in the definition of quasi hemi-slant submanifolds. Some results are worked out on quasi hemi-slant submanifolds of nearly Kaehler manifolds to be totally geodesic. Moreover, we also obtain the necessary and sufficient conditions for a quasi hemi-slant submanifold to be quasi hemi-slant product manifold. We provide examples of quasi hemi-slant submanifolds of nearly Kaehler manifolds.

M.S.C. 2010: 53C15, 53C40, 53B20, 53C55, 57R19.
Key words: Quasi hemi-slant submanifolds; nearly Kaehler manifold; Riemannian maps; totally geodesic.

1 Introduction

In differential geometry, there are numerous impostant applications of theory of submanifolds in mathematics and in physics. The properties of this theory became an interesting subject in differential geometry, both in complex geometry and in contact geometry.

In 1900, the theory of slant submanifolds was initiated by B. Y. Chen, as a generalization of holomorphic and totally real submanifolds in complex geometry ([6],[8]). This work is the generalization of CR-submanifolds introduced by Bejancu [1], in 1986. The theory of slant submanifolds became a very rich area of research for geometers and many mathematicians have studied these submanifolds. Slant submanifolds were also studied by different geometers in different ambient spaces [18]. In particular, N. Papaghiuc [16] introduced the notion of semi-slant submanifolds as a generalization of slant submanifolds in 1994. A. Lotta ([14],[15]) defined and studied slant submanifolds in contact geometry; J. L. Cabrerizo, A. Carriazo, L. M. Fernandez and M. Fernandez studied slant, semi-slant and bi-slant submanifold in contact geometry ([3],[4],[5]). The hemi-slant submanifolds are one of the classes of bi-slant submanifolds. Several geometers studied these submanifolds in different kind of structures and also other context related to it ([9],[11],[17],[19],[22],[23]).

Differential Geometry - Dynamical Systems, Vol.23, 2021, pp. 189-203.

In 1981, Chen [7] defined the canonical de Rham cohomology class for closed CR-submanifolds in a Kaehler manifolds and in 1982, Deshmukh [10] proved the same result for nearly Kaehler manifolds. After some time, in 2014, Sahin [21] studied the de Rham cohomology class of hemi-slant submanifolds of Kaehler manifolds. Motivated from above interesting and significant studies, we define quasi hemi-slant submanifolds of nearly Kahler manifolds which is generalization of anti-invariant, semi-invariant, slant, semi-slant and hemi-slant submanifolds.

The paper is organized as follows: In the second section, we collect some basic definitions and properties of some complex structures. In the third section, we define quasi hemi-slant submanifolds of nearly Kaehler manifolds. In the fourth section, we obtain necessary and sufficient conditions for such submanifolds to be integrable. In the fifth section, we study the geometry of leaves of distributions which are involved in the definition of quasi hemi-slant submanifolds and give the necessary and sufficient conditions for such submanifolds to be totally geodesic. In the last section, we provide some examples of such submanifolds.

2 Preliminaries

Let $\mathcal{N}$ be a Riemannian manifold with an almost complex structure J and g be the Riamannian metric. The manifold N with almost complex structure J and Riemannian metric g is said to be a nearly Kaehler manifold if,

$$(\overline{\nabla}_X J)Y + (\overline{\nabla}_Y J)X = 0, \tag{2.1}$$

for any vector field X, Y in $T\mathcal{N}$. The above condition is equivallent to the following equation

$$(\overline{\nabla}_X J)X = 0, \tag{2.2}$$

for X in $T\mathcal{N}$. For a nearly Kaehler manifold, we also have the following facts:

$$J^2 = -I \qquad and \qquad g(JX, JY) = g(X, Y), \tag{2.3}$$

for all vector fields $X, Y \in T\mathcal{N}$[24].

We know that any Kaehler manifold is always a nearly Kaehler manifold, but the converse is not always true.

Let $\gamma : \mathcal{M} \to \mathcal{N}$ be an isometrical immersion of a Riemannian manifolds $\mathcal{M}$ into $\mathcal{N}$. The Riemannian metric for $\mathcal{M}$ and $\mathcal{N}$ is denoted by the same symbol g. The Lie algebra of vector fields and the set of all normal fields on $\mathcal{M}$ are denoted by $T\mathcal{M}$ and $T^\perp\mathcal{M}$, respectively. The Levi-Civita connection on $T\mathcal{M}$ and on $T^\perp\mathcal{M}$ are denoted by ∇ and $\overline{\nabla}$, respectively. The Gauss and Weingarten equations are given as

$$\overline{\nabla}_X Y = \nabla_X Y + \Omega(X, Y), \tag{2.4}$$

$$\overline{\nabla}_X V = -S_V X + \nabla^\perp_X V, \tag{2.5}$$

for any X, Y in $T\mathcal{M}$ and V in $T^\perp\mathcal{M}$, where Ω is the second fundamental form and S is the shape operator. The shape operator and the second fundamental form are related by the following equation

$$g(S_V X, Y) = g(\Omega(X, Y), V). \tag{2.6}$$

The mean curvature vector $\mathcal{H}$ on a submanifold $\mathcal{M}$ is defined as

$$\mathcal{H} = \frac{1}{l} trace(\Omega) = \frac{1}{l}\sum_{i=1}^{l}\Omega(e_i, e_i), \tag{2.7}$$

where l denotes the dimension of submanifold $\mathcal{M}$ and $\{e_1, e_2, ..., e_l\}$ is the local orthonormal frame. The submanifold $\mathcal{M}$ is said to be a totally umbilical submanifold of a Riemannian manifold $\mathcal{N}$, if for the mean curvaure vector $\mathcal{H}$, the following relation holds

$$\Omega(X, Y) = g(X, Y)\mathcal{H}. \tag{2.8}$$

Using (2.8), the Gauss and Weingarten equation takes the form

$$\overline{\nabla}_X Y = \nabla_X Y + g(X, Y)\mathcal{H}, \tag{2.9}$$

and

$$\overline{\nabla}_X V = -Xg(\mathcal{H}, V) + \nabla^{\perp}_X V, \tag{2.10}$$

for X, $Y \in T\mathcal{M}$ and $V \in T^{\perp}\mathcal{M}$.

A submanifold $\mathcal{M}$ is said to be totally geodesic if $\Omega(X, Y) = 0$, for any X, $Y \in T\mathcal{M}$ and if $\mathcal{H}$ =0, the submanifold $\mathcal{M}$ is minimal.

Now, for a submanifold $\mathcal{M}$ of Riemannian manifold $\mathcal{N}$, we can write $X \in T\mathcal{M}$ as

$$JX = TX + NX, \tag{2.11}$$

where TX and NX are the tangential and normal parts of JX respectively. Similarily, for any $V \in T^{\perp}\mathcal{M}$, we have

$$JV = tV + nV, \tag{2.12}$$

where tV and nV are the tangential and normal components of JV respectively.

Now, denote by $\mathcal{P}_X Y$ and $\mathcal{Q}_X Y$ the tangential and normal parts of $(\overline{\nabla}_X J)Y$ respectively, ie.

$$(\overline{\nabla}_X J)Y = \mathcal{P}_X Y + \mathcal{Q}_X Y, \tag{2.13}$$

for X, $Y \in T\mathcal{M}$.

From equations (2.4), (2.5), (2.11) and (2.12), the following equations may easily be obtained

$$\mathcal{P}_X Y = (\overline{\nabla}_X T)Y - S_{NY}X - t\Omega(X, Y), \tag{2.14}$$

$$\mathcal{Q}_X Y = (\overline{\nabla}_X N)Y + \Omega(X, TY) - n\Omega(X, Y). \tag{2.15}$$

Similarily, for any $V \in T^{\perp}\mathcal{M}$, denote the tangential and normal parts of $(\overline{\nabla}_X J)Y$ by $P_X V$ and $Q_X V$ respectively, we obtain

$$\mathcal{P}_X V = (\overline{\nabla}_X t)V + TS_V X - S_{nV}X, \tag{2.16}$$

$$\mathcal{Q}_X V = (\overline{\nabla}_X n)V + \Omega(tV, X) + NS_V X, \tag{2.17}$$

The covariant derivative of tangential and normal parts of (2.11) and (2.12) is given as

$$(\overline{\nabla}_X T)Y = \nabla_X TY - T\nabla_X Y, \tag{2.18}$$
$$(\overline{\nabla}_X N)Y = \nabla^{\perp}_X NY - N\nabla_X Y, \tag{2.19}$$
$$(\overline{\nabla}_X t)V = \nabla_X tV - t\nabla^{\perp}_X V, \tag{2.20}$$

and

$$(\overline{\nabla}_X n)V = \nabla^{\perp}_X nV - n\nabla^{\perp}_X V, \tag{2.21}$$

for any $X, Y \in T\mathcal{M}$ and $V \in T^{\perp}\mathcal{M}$.

It is straightforward to verify the following properties of $\mathcal{P} - \mathcal{Q}$, which we enlist here for later use:

(a_1) (i) $g(\mathcal{P}_X Y, W) = -g(Y, \mathcal{P}_X W)$, (ii) $g(\mathcal{Q}_X Y, V) = -g(Y, \mathcal{Q}_X V)$,

(a_2) $\mathcal{P}_X JY + \mathcal{Q}_X JY = -J(\mathcal{P}_X Y + \mathcal{Q}_X Y)$,

for all $X, Y \in T\mathcal{M}$ and $V \in T^{\perp}\mathcal{M}$.

On a Riemannian submanifold $\mathcal{M}$ of a nearly Kaehler manifold $\mathcal{N}$, from equations (2.2) and (2.13), we have

$$(a)\ \mathcal{P}_X Y + \mathcal{P}_Y X = 0, \qquad (b)\ \mathcal{Q}_X Y + \mathcal{Q}_Y X = 0 \tag{2.22}$$

for any X, $Y \in T\mathcal{M}$.

The covariant derivative of the complex structure J is defined as

$$(\overline{\nabla}_X J)Y = \overline{\nabla}_X JY - J\overline{\nabla}_X Y, \tag{2.23}$$

or

$$\overline{\nabla}_X JX = J\overline{\nabla}_X X. \tag{2.24}$$

Let $\mathcal{N}$ be a nearly Kaehler manifold and $\mathcal{M}$ be a submanifold of $\mathcal{N}$. There are certain important classes of submanifolds viz totally real submanifolds, holomorphic submanifolds and CR-submanifolds as a generalization of totally real and holomorphic submanifolds. Chen generalized CR-submanifolds as slant submanifolds and Sahin introduced hemi-slant submanifolds from it. The quasi hemi-slant submanifolds is a generalization of hemi-slant submanifolds.

3 Quasi hemi-slant submanifolds of nearly Kaehler manifolds

In this section of the paper, we give the definition of quasi hemi-slant submanifolds of nearly Kaehler manifolds and obtain the necessary and sufficient conditions for the distribution to be integrable for such manifolds.

Definition 3.1. [2] Let $\mathcal{M}$ be a Riemannian manifold isometrically immersed in almost complex manifold $\mathcal{N}$. A submanifold $\mathcal{M}$ of an almost complex manifold $\mathcal{N}$ is said to be invariant, if $J\,(T_x\mathcal{M}) \subseteq T_x\mathcal{M}$, for every point $x \in \mathcal{M}$.

Definition 3.2. [12] A submanifold o$\mathcal{M}$ of an almost Hermitian manifold $\mathcal{N}$ is said to be anti-invariant, if $J\left(T_x\mathcal{M}\right) \subseteq T_x^{\perp}\mathcal{M}$, for every point $x \in \mathcal{M}$.

Definition 3.3. [3] A submanifold $\mathcal{M}$ of an almost Hermitian manifold $\mathcal{N}$ is said to be slant, if for each non-zero vector X tangent to $\mathcal{M}$ at $x \in \mathcal{M}$, the angle $\theta(X)$ between JX and $T_x\mathcal{M}$ is constant, i.e., it does not depend on the choice of the point $x \in \mathcal{M}$ and $X \in T_x\mathcal{M}$. In this case, the angle θ is called the slant angle of the submanifold. A slant submanifold $\mathcal{M}$ is called proper slant submanifold if neither $\theta = 0$ nor $\theta = \frac{\pi}{2}$.

We note that on a slant submanifold $\mathcal{M}$, if $\theta = 0$, then it is an invariant submanifold and if $\theta = \frac{\pi}{2}$, then it is an anti-invariant submanifold. This means slant submanifold is a generalization of invariant and anti-invariant submanifolds.

Definition 3.4. [13] Let $\mathcal{M}$ be a submanifold of nearly Kaehler manifold $\mathcal{N}$, then $\mathcal{M}$ is said to be a hemi-slant submanifold if there exist two orthogonal distributions D^θ and $D^\perp$ on $\mathcal{M}$ such that
(*i*) $T\mathcal{M} = D^\theta \oplus D^\perp$,
(*ii*) D^θ is a slant distribution with slant angle $\theta \neq \frac{\pi}{2}$,
(*iii*) $D^\perp$ is a totally real ie. $JD^\perp \subseteq T^\perp\mathcal{M}$,
it is clear from the above that CR-submanifolds and slant submanifolds are hemi-slant submanifolds with slant angle $\theta = \frac{\pi}{2}$ and $D^\theta = 0$, respectively.

Now, we introduce the notion of quasi hemi-slant submanifolds of nearly Kaehler manifolds which have great role in theory of submanifolds.

Definition 3.5. Let $\mathcal{M}$ be a submanifold of nearly Kaehler manifold $\mathcal{N}$, then $\mathcal{M}$ is said to be a quasi hemi-slant submanifold if there exist three orthogonal complementary distributions D, D^θ and $D^\perp$ such that
(*i*) $T\mathcal{M}$ admits the orthogonal direct decomposition

$$T\mathcal{M} = D \oplus D^\theta \oplus D^\perp, \tag{3.1}$$

(*ii*) The distribution D is invariant i.e. $JD = D$,
(*iii*) The distribution D^θ is slant and the angle θ between JX and space $(D^\theta)_q$ is constant and independent of choice of point $q \in \mathcal{M}$.
(*iv*) The distribution $D^\perp$ is J anti-invariant, i.e. $JD^\perp \subseteq T^\perp\mathcal{M}$,
in this case, we call θ the quasi hemi-slant angle of $\mathcal{M}$.

Suppose the dimension of distributions$D$, D^θ and $D^\perp$ are n_1, n_2 and n_3 respectively. Then we can easily see the following particular cases:
(*i*) If $n_1 = 0$, then $\mathcal{M}$ is hemi-slant submanifold.
(*ii*) If $n_2 = 0$, then $\mathcal{M}$ is semi invariant submanifold.
(*iii*) If $n_3 = 0$, then $\mathcal{M}$ is semi-slant submanifold.
We say that the quasi hemi-slant submanifold $\mathcal{M}$ is proper if $D \neq \{0\}$, $D^\perp \neq \{0\}$ and $\theta \neq 0, \frac{\pi}{2}$.

This means quasi hemi-slant submanifold is a generalization of invariant, anti-invariant, semi-invariant, slant, hemi-slant, semi-slant submanifolds and also they are the examples of the quasi hemi-slant submanifolds.

Let M be a quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. We denote the projections of $X \in T\mathcal{M}$ on the distributions D, D^θ and $D^\perp$ by P, Q and R respectively. Then we can write for any $X \in T\mathcal{M}$,

$$X = PX + QX + RX. \tag{3.2}$$

Now, from equation (2.11), we know that

$$JX = TX + NX,$$

where TX and NX are tangential and normal components of JX on $\mathcal{M}$, respectively. Using (2.11) and (3.2), we obtain

$$JX = TPX + NPX + TQX + NQX + TRX + NRX. \tag{3.3}$$

Since $JD = D$ and $JD^\perp \subseteq T^\perp\mathcal{M}$, we have $NPX = 0$ and $TRX = 0$. Therefore, we get

$$JX = TPX + TQX + NQX + NRX. \tag{3.4}$$

This means that

$$J(T\mathcal{M}) = D \oplus TD^\theta \oplus ND^\theta \oplus JD^\perp. \tag{3.5}$$

We also have,

$$T^\perp\mathcal{M} = ND^\theta \oplus JD^\perp \oplus \mu, \tag{3.6}$$

where μ is the invariant distribution of $T^\perp\mathcal{M}$ with respect to J.

For any non-zero vector field $V \in T^\perp\mathcal{M}$, we have from equation (2.12),

$$JV = tV + nV,$$

where tV and nV are tangential and normal components of V respectively.

Lemma 3.1. *Let $\mathcal{M}$ be the quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. Then we obtain*

$$TD = D,\ TD^\theta = D^\theta,\ TD^\perp = \{0\},\ tND^\theta = D^\theta,\ tND^\perp = D^\perp. \tag{3.7}$$

Now, using euations (2.3), (2.11) and (2.12) and (3.2), then on comparing the tangential and normal components, we have the following:

Lemma 3.2. *Let $\mathcal{M}$ be the quasi hemi-slant submanifolds of a nearly Kaehler manifold $\mathcal{N}$. Then the endomorphism T and projection morphisms N, t and n in the tangent bundle of $\mathcal{M}$, satisfy the following identities:*
(i) $T^2 + tN = -I$ *on* $T\mathcal{M}$,
(ii) $NT + nN = 0$ *on* $T\mathcal{M}$,
(iii) $Nt + n^2 = -I$ *on* $T^\perp\mathcal{M}$,
(iv) $Tt + tn = 0$ *on* $T^\perp\mathcal{M}$,
where I is the identity operator.

Lemma 3.3. *Let $\mathcal{M}$ be the quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. Then*
(*i*) $T^2X = -(\cos^2\theta)X$,
(*ii*) $g(TX, TY) = (\cos^2\theta)g(X, Y)$,
(*iii*) $g(NX, NY) = (\sin^2\theta)g(X, Y)$,
for any $X, Y \in D^\theta$.

Proof. The proof is same as one can found in [20]. □

Theorem 3.4. *Let $\mathcal{M}$ be the quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$, then for any $X, Y \in T\mathcal{M}$, we have the following*

$$\nabla_X TY - S_{NY}X - T\nabla_X Y - 2t\Omega(X, Y) + \nabla_Y TX - S_{NX}Y - T\nabla_Y X = 0, \tag{3.8}$$

and

$$\Omega(X, TY) + \nabla^\perp_X NY - N\nabla_X Y - 2n\Omega(X, Y) + \Omega(Y, TX) + \nabla^\perp_Y NX - N\nabla_Y X = 0. \tag{3.9}$$

Proof. For a nearly Kaehler manifold, we know that

$$(\overline{\nabla}_X J)Y + (\overline{\nabla}_Y J)X = 0,$$

which implies that

$$\overline{\nabla}_X JY - J\overline{\nabla}_X Y + \overline{\nabla}_Y JX - J\overline{\nabla}_Y X = 0.$$

Using equations (2.4) and (2.11), we have

$$\begin{aligned} & \overline{\nabla}_X TY + \overline{\nabla}_X NY - J\nabla_X Y - J\Omega(X, Y) + \overline{\nabla}_Y TX \\ +\overline{\nabla}_Y NX - J\nabla_Y X - J\Omega(Y, X) \quad = \quad & 0. \end{aligned}$$

Now, using equations (2.4), (2.5), (2.11) and (2.12), we get

$$\begin{aligned} & \nabla_X TY + \Omega(X, TY) - S_{NY}X + \nabla^\perp_X NY - T\nabla_X Y \\ & -N\nabla_X Y - t\Omega(X, Y) - n\Omega(X, Y) + \nabla_Y TX \\ & +\Omega(Y, TX) - S_{NX}Y + \nabla^\perp_Y NX - T\nabla_Y X \\ -N\nabla_Y X - t\Omega(Y, X) - n\Omega(Y, X) \quad = \quad & 0. \end{aligned}$$

On comparing the tangential and normal components, we have the required results.
□

---

4 Integrability of distributions

slant distribution (D^θ), invariant distribution (D) and anti-invariant distribution ($D^\perp$).

Theorem 4.1. *Let $\mathcal{M}$ be a proper quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. Then, the slant distribution D^θ is integrable if and only if*

$$g(S_{NW}Z - S_{NZ}W, TPX) = g(S_{NTW}Z - S_{NTZ}W, X) - g(\mathcal{Q}_W X, NZ) + g(\mathcal{Q}_Z X, NW). \tag{4.1}$$

for any $Z, W \in D^\theta$ and $X \in D \oplus D^\perp$.

Proof. We know that the slant distribution D^θ is integrable if and only if $[Z, W] \in D^\theta$ for any $Z, W \in D^\theta$.

Let us consider $Z, W \in D^\theta$ and $X = PX + RX \in D \oplus D^\perp$,we have

$$g([Z, W], X) = g(J\overline{\nabla}_Z W, JX) - g(J\overline{\nabla}_W Z, JX) \tag{4.2}$$

Now, we compute the both terms in right hand side of (4.2) as follows

$$g(J\overline{\nabla}_Z W, JX) = g(\overline{\nabla}_Z JW, JX) - g((\overline{\nabla}_Z J)W, JX).$$

Using equations (2.3), (2.13) and (3.4) and Lemma 3.3, we get

$$\begin{aligned}
g(J\overline{\nabla}_Z W, JX) &= g(\overline{\nabla}_Z TW, JX) + g(\overline{\nabla}_Z NW, JX) - g(\mathcal{P}_Z W, JX) \\
&= -g(\overline{\nabla}_Z JTW, X) + g((\overline{\nabla}_Z J)TW, X) - g(S_{NW}Z, JX) - g(\mathcal{P}_Z W, JX) \\
&= \cos^2\theta g(\overline{\nabla}_Z W, X) - g(\overline{\nabla}_Z NTW, X) + g(\mathcal{P}_Z TW, X) - g(S_{NW}Z, JX) \\
&\quad - g(\mathcal{P}_Z W, JX) \\
&= \cos^2\theta g(\overline{\nabla}_Z W, X) + g(S_{NTW}Z, X) + g(\mathcal{P}_Z TW, X) - g(S_{NW}Z, JX) \\
&\quad - g(\mathcal{P}_Z W, JX). \qquad (4.3)
\end{aligned}$$

Interchanging Z and W in (4.3), we get

$$\begin{aligned}
g(J\overline{\nabla}_W Z, JX) &= \cos^2\theta g(\overline{\nabla}_W Z, X) + g(S_{NTZ}W, X) + g(\mathcal{P}_W TZ, X) \\
&\quad - g(S_{NZ}W, JX) - g(\mathcal{P}_W Z, JX).
\end{aligned} \tag{4.4}$$

Then from (4.3) – (4.4) and using (4.2), we obtain

$$\begin{aligned}
\sin^2\theta g([Z, W], X) &= g(S_{NTW}Z - S_{NTZ}W, X) - g(S_{NW}Z - S_{NZ}W, JX) \\
&\quad + g(\mathcal{P}_Z TW - \mathcal{P}_W TZ, X) - g(\mathcal{P}_Z TW - \mathcal{P}_W TZ, JX).
\end{aligned} \tag{4.5}$$

Using $\mathcal{P} - \mathcal{Q}$ properties (a_2) and $(a_1)(ii)$,

$$\begin{aligned}
g(\mathcal{P}_W Z, JX) - g(\mathcal{P}_Z W, JX) &= g(\mathcal{P}_W TZ, X) - g(\mathcal{Q}_W X, NZ) \\
&\quad - g(\mathcal{P}_Z TW, X) + g(\mathcal{Q}_Z X, NW).
\end{aligned} \tag{4.6}$$

Using equations (4.5) and (4.6), we get

$$\begin{aligned}
\sin^2\theta g([Z, W], X) &= g(S_{NTW}Z - S_{NTZ}W, X) - g(S_{NW}Z - S_{NZ}W, TPX) \\
&\quad - g(\mathcal{Q}_W X, NZ) + g(\mathcal{Q}_Z X, NW),
\end{aligned} \tag{4.7}$$

from above equation, we obtain that $[Z, W] \in D^\theta$ if and only if equation (4.7) satisfies. This proves the theorem completely. □

In the following theorem, we give the sufficient condition for slant distribution D^θ to be intgrable with the help of above theorem:

Theorem 4.2. *Let $\mathcal{M}$ be a proper quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$, then the slant distribution D^θ is integrable, if*

$$\begin{aligned} g(\mathcal{Q}_Z X, NW) - g(\mathcal{Q}_W X, NZ) &\in ND^\theta \oplus \mu, \\ S_{NTW}Z - S_{NTZ}W &\in D^\theta \quad and \\ S_{NW}Z - S_{NZ}W &\in D^\perp \oplus D^\theta, \end{aligned} \tag{4.8}$$

for any $Z, W \in D^\theta$.

Theorem 4.3. *Let $\mathcal{M}$ be a proper quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. Then the anti-invariant distribution $D^\perp$ is integrable if and only if*

$$g(S_{JZ}W - S_{JW}Z, TX) = -g(\nabla^\perp_Z JW - \nabla^\perp_W JZ - 2\mathcal{Q}_W Z, NX) \tag{4.9}$$

for any $Z, W \in D^\perp$ and $X \in D \oplus D^\theta$.

Proof. We know that the anti-invariant distribution $D^\perp$ is integrable if and only if $[Z, W] \in D^\perp$ for any $Z, W \in D^\perp$.

Let us consider $Z, W \in D^\perp$ and $X \in D \oplus D^\theta$,we have

$$\begin{aligned} g([Z,W],X) &= g(J\overline{\nabla}_Z W, JX) - g(J\overline{\nabla}_W Z, JX) \\ &= g(\overline{\nabla}_Z JW, JX) - g((\overline{\nabla}_Z J)W, JX) - g(\overline{\nabla}_W JZ, JX) + g((\overline{\nabla}_W J)Z, JX), \end{aligned}$$

using equations (2.4), (2.5) and (3.4), we get

$$\begin{aligned} g([Z,W],X) &= g(S_{JZ}W - S_{JW}Z, TX) + g(\nabla^\perp_Z JW - \nabla^\perp_W JZ, NX) \\ &\quad - g(\mathcal{Q}_Z W - \mathcal{Q}_W Z, NX), \end{aligned}$$

using equation (2.22), we get

$$g([Z,W],X) = g(S_{JZ}W - S_{JW}Z, TX) + g(\nabla^\perp_Z JW - \nabla^\perp_W JZ - 2\mathcal{Q}_W Z, NX) \tag{4.10}$$

from above equation, we obtain that $[Z, W] \in D^\perp$ if and only if equation (4.10) satisfies. This proves the theorem completely. □

Similarily, for any $Z, W \in D$ and $X \in D^\theta \oplus D^\perp$,we have

$$\begin{aligned} g([Z,W],X) &= g(J\overline{\nabla}_Z W, JX) - g(J\overline{\nabla}_W Z, JX), \\ &= g(\overline{\nabla}_Z TW, JX) - g(\mathcal{P}_Z W, JX) - g(\overline{\nabla}_W TZ, JX) + g(\mathcal{P}_W Z, JX), \end{aligned}$$

using equations (2.4) and (3.4), we get

$$g([Z,W],X) = g(\nabla_Z TW - \nabla_W TZ, TQX) + g(\Omega(Z,TW) - \Omega(W,TZ) - 2\mathcal{P}_Z W, JX). \tag{4.11}$$

Hence, we have the following theorem:

Theorem 4.4. *Let $\mathcal{M}$ be a proper quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. Then the invariant distribution D is integrable if and only if*

$$g(\nabla_Z TW - \nabla_W TZ, TQX) = g(\Omega(Z,TW) - \Omega(W,TZ) - 2\mathcal{P}_Z W, JX),$$

for any $Z, W \in D$ and $X \in D^\theta \oplus D^\perp$.

5 Totally geodesic quasi hemi-slant submanifods and decomposition theorems

As we know that geodesicness and foliations are significant great notions. In the present section, we deal with the totally geodesic quasi hemi-slant submanifolds of nearly Kaehler manifolds. We investigate the geometry of leaves of slant, anti-invariant and invariant distributions and obtain necessary and sufficient conditions for quasi hemi-slant submanifolds $\mathcal{M}$ to be totally geodesic.

Theorem 5.1. *Let $\mathcal{M}$ be a proper quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. Then the slant distribution D^θ defines a totally geodesic foliation on $\mathcal{M}$ if and only if*

$$g(\nabla^\perp_X NY, NRZ) = g(S_{NY}X + \mathcal{P}_X Y, TPZ) - g(S_{NTY}X, Z) \tag{5.1}$$

and

$$g(S_{NY}X + \mathcal{P}_X Y - \nabla_X TY, t\xi) = g(\nabla^\perp_X NY + \Omega(X, TY), n\xi), \tag{5.2}$$

for any $X, Y \in D^\theta$, $Z \in D \oplus D^\perp$ and $\xi \in T\mathcal{M}^\perp$.

Proof. For any X, $Y \in D^\theta$, $Z = PZ + RZ \in D \oplus D^\perp$ and using equations (2.13), (2.23) and (3.2), we have

$$\begin{aligned} g(\overline{\nabla}_X Y, Z) &= g(\overline{\nabla}_X JY, JZ) - g((\overline{\nabla}_X J)Y, JZ) \\ &= g(\overline{\nabla}_X TY, JZ) + g(\overline{\nabla}_X NY, JZ) - g(\mathcal{P}_X Y, TPZ) \\ &= -g(\overline{\nabla}_X T^2 Y, Z) - g(\overline{\nabla}_X NTY, Z) \\ &\quad + g(\overline{\nabla}_X NY, TPZ + NRZ) - g(\mathcal{P}_X Y, TPZ). \end{aligned} \tag{5.3}$$

Using equations (2.5), (3.4) and Lemma 3.3, we get

$$\begin{aligned} g(\overline{\nabla}_X Y, Z) &= \cos^2\theta g(\overline{\nabla}_X Y, Z) + g(S_{NTY}X, Z) \\ &\quad - g(S_{NY}X, TPZ) + g(\nabla^\perp_X NY, NRZ) - g(\mathcal{P}_X Y, TPZ), \\ \sin^2\theta g(\overline{\nabla}_X Y, Z) &= g(S_{NTY}X, Z) - g(S_{NY}X, TPZ) + g(\nabla^\perp_X NY, NRZ) - g(\mathcal{P}_X Y, TPZ). \end{aligned}$$

which is the first required result (5.1) of theorem.

Now, for any X, $Y \in D^\theta$ and $\xi \in T^\perp\mathcal{M}$,

using equations (2.4), (2.5), (2.12) and (3.4), we have

$$\begin{aligned} g(\overline{\nabla}_X Y, \xi) &= g(\overline{\nabla}_X JY, J\xi) - g((\overline{\nabla}_X J)Y, J\xi) \\ &= g(\overline{\nabla}_X TY, J\xi) + g(\overline{\nabla}_X NY, J\xi) - g(\mathcal{P}_X Y, t\xi) \\ &= g(\nabla_X TY, t\xi) + g(\Omega(X, TY), n\xi) - g(S_{NY}X, t\xi) + \\ &\quad g(\overline{\nabla}^\perp_X NY, n\xi) - g(\mathcal{P}_X Y, t\xi). \end{aligned}$$

which is the second required result (5.2) of theorem.

Hence, the theorem is proved completely. ∎

Theorem 5.2. *Let $\mathcal{M}$ be a proper quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. Then the anti-invariant distribution $D^{\perp}$ defines a totally geodesic foliation on $\mathcal{M}$ if and only if*

$$g(\Omega(X,Y),NTQZ) = g(\nabla^{\perp}_X NY - \mathcal{Q}_X Y, NQZ) \tag{5.4}$$

and

$$g(S_{NY}X, t\xi) = g(\nabla^{\perp}_X NY, n\xi) - g(\mathcal{Q}_X Y, J\xi), \tag{5.5}$$

for any $X,Y \in D^{\perp}$, $Z \in D \oplus D^{\theta}$ and $\xi \in T\mathcal{M}^{\perp}$.

Proof. For any X, $Y \in D^{\perp}$, $Z = PZ + QZ \in D \oplus D^{\theta}$ and using equations (2.13), (2.23) and (3.2), we have

$$\begin{aligned} g(\overline{\nabla}_X Y, Z) &= g(\overline{\nabla}_X JY, JZ) - g((\overline{\nabla}_X J)Y, JZ) \\ &= g(\overline{\nabla}_X JY, JPZ) + g(\overline{\nabla}_X JY, TQZ) + g(\overline{\nabla}_X JY, NQZ) - g(\mathcal{Q}_X Y, JZ) \\ &= g(\overline{\nabla}_X Y, PZ) - g(\overline{\nabla}_X Y, T^2 QZ) - g(\overline{\nabla}_X Y, NTQZ) \\ &\quad + g(\overline{\nabla}_X NY, NQZ) - g(\mathcal{Q}_X Y, NQZ). \end{aligned} \tag{5.6}$$

As we know that

$$g(\overline{\nabla}_X Y, Z) = g(\overline{\nabla}_X Y, PZ) + g(\overline{\nabla}_X Y, QZ), \tag{5.7}$$

using equations (2.4), (2.5), (3.4) and Lemma 3.3, we get

$$g(\overline{\nabla}_X Y, \sin^2\theta QZ) = -g(\Omega(X,Y), NTQZ) + g(\nabla^{\perp}_X NY, NQZ) - g(\mathcal{Q}_X Y, NQZ),$$

which is the first required result (5.4) of theorem.
Now, for any X, $Y \in D^{\perp}$ and $\xi \in T^{\perp}\mathcal{M}$,
using equations (2.5), (2.12), (2.13) and (3.4), we have

$$\begin{aligned} g(\overline{\nabla}_X Y, \xi) &= g(\overline{\nabla}_X JY, J\xi) - g((\overline{\nabla}_X J)Y, J\xi) \\ &= g(\overline{\nabla}_X NY, t\xi) + g(\overline{\nabla}_X NY, n\xi) - g(\mathcal{Q}_X Y, J\xi) \\ &= -g(S_{NY}X, t\xi) + g(\nabla^{\perp}_X NY, n\xi) - g(\mathcal{Q}_X Y, J\xi), \end{aligned}$$

which is the second required result (5.5) of theorem.
Hence, the theorem is proved completely. □

Theorem 5.3. *Let $\mathcal{M}$ be a proper quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. Then the invariant distribution D defines a totally geodesic foliation on $\mathcal{M}$ if and only if*

$$g(\nabla_X TY, TQZ) = -g(\Omega(X,TY), NZ) + g(\mathcal{P}_X Y, TQZ) \tag{5.8}$$

and

$$g(\nabla_X TY - \mathcal{P}_X Y, t\xi) = -g(\Omega(X,TY), n\xi), \tag{5.9}$$

for any $X,Y \in D$, $Z \in D^{\theta} \oplus D^{\perp}$ and $\xi \in T\mathcal{M}^{\perp}$.

Proof. For any $X,\, Y \in D$, $Z = QZ + RZ \in D^\theta \oplus D^\perp$ and using equations (2.4), (2.23) and (3.4), we have

$$\begin{aligned} g(\overline{\nabla}_X Y, Z) &= g(\overline{\nabla}_X JY, JZ) - g((\overline{\nabla}_X J)Y, JZ) \\ &= g(\overline{\nabla}_X TY, JZ) - g(\mathcal{P}_X Y, JZ) \\ &= g(\nabla_X TY, TQZ) + g(\Omega(X, TY), JZ) - g(\mathcal{P}_X Y, TQZ). \end{aligned}$$

which is the first required result (5.8) of theorem.

Now, for any $X,\, Y \in D$ and $\xi \in T^\perp\mathcal{M}$,
using equations (2.4), (2.12) and (3.4), we have

$$\begin{aligned} g(\overline{\nabla}_X Y, \xi) &= g(\overline{\nabla}_X JY, J\xi) - g((\overline{\nabla}_X J)Y, J\xi) \\ &= g(\nabla_X TY, t\xi) + g(\Omega(X, TY), n\xi) - g(\mathcal{P}_X Y, t\xi) \end{aligned}$$

which is the second required result (5.9) of theorem.

Hence, the theorem is proved completely. □

From Theorem 5.1, 5.2 and 5.3, we have following decomposition theorem:

Theorem 5.4. *Let $\mathcal{M}$ be a proper quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. Then the fiber of $\mathcal{M}$ is a local product Riemannian manifold of the form $\mathcal{M}_{D^\theta} \times \mathcal{M}_{D^\perp}$, where $\mathcal{M}_{D^\theta}$ and $\mathcal{M}_{D^\perp}$ are leaves of D^θ and $D^\perp$ respectively, if and only if the conditions* (5.1), (5.2), (5.4), (5.5), (5.8) *and* (5.9) *holds.*

Theorem 5.5. *Let $\mathcal{M}$ be a proper quasi hemi-slant submanifold of a nearly Kaehler manifold $\mathcal{N}$. Then $\mathcal{M}$ is a totally geodesic submanifold of a nearly Kaehler manifold $\mathcal{N}$ if and only if*

$$\begin{aligned} g(\Omega(X, PY) + \cos^2\theta\,\Omega(X, QY), \xi) &= g(\nabla^\perp_X NTQY, \xi) + g(S_{NQY}X + S_{NRY}X + \mathcal{P}_X Y, t\xi) \\ &\quad - g(\nabla^\perp_X NY - \mathcal{Q}_X Y, n\xi), \end{aligned} \tag{5.10}$$

for any $X,\, Y \in T\mathcal{M}$ and $\xi \in T^\perp\mathcal{M}$.

Proof. For any $X,\, Y \in T\mathcal{M}$, and $\xi \in T^\perp\mathcal{M}$, using equations (2.2), (2.23) and (3.2), we have

$$\begin{aligned} g(\overline{\nabla}_X Y, \xi) &= g(\overline{\nabla}_X PY, \xi) + g(\overline{\nabla}_X QY, \xi) + g(\overline{\nabla}_X RY, \xi) \\ &= g(\overline{\nabla}_X JPY, J\xi) + g(\overline{\nabla}_X TQY, J\xi) + g(\overline{\nabla}_X NQY, J\xi) \\ &\quad + g(\overline{\nabla}_X JRY, J\xi) - g((\overline{\nabla}_X J)Y, J\xi). \end{aligned} \tag{5.11}$$

Using equations (2.4), (2.5) and Lemma 3.3, we have

$$\begin{aligned} g(\overline{\nabla}_X Y, \xi) &= g(\overline{\nabla}_X PY, \xi) - g(\overline{\nabla}_X T^2QY, \xi) - g(\overline{\nabla}_X NTQY, \xi) \\ &\quad + g(\overline{\nabla}_X NQY, J\xi) + g(\overline{\nabla}_X NRY, J\xi) - g((\overline{\nabla}_X J)Y, J\xi) \\ &= g(\Omega(X, PY), \xi) + \cos^2\theta\, g(\Omega(X, QY), \xi) - g(\overline{\nabla}^\perp_X NTQY, \xi) \\ &\quad + g(-S_{NQY}X + \overline{\nabla}^\perp_X NQY, J\xi) + g(-S_{NRY}X + \overline{\nabla}^\perp_X NRY, J\xi) \\ &\quad - g((\overline{\nabla}_X J)Y, J\xi), \end{aligned} \tag{5.12}$$

using equations (2.11), (2.12) and (2.13), we have

$$\begin{aligned} g(\overline{\nabla}_X Y, \xi) &= g(\Omega(X, PY) + \cos^2\theta\Omega(X, QY), \xi) - g(\overline{\nabla}^{\perp}_X NTQY, \xi) \\ &\quad - g(S_{NQY}X + S_{NRY}X + \mathcal{P}_X Y, t\xi) + g(\overline{\nabla}^{\perp}_X NY - \mathcal{Q}_X Y, n\xi). \end{aligned}$$

Hence the proof. □

6 Examples

Let (x_i, y_i) be cartesian coordinates on an Euclidean space R^{2n} for $i = 1, 2, ..., n$. We can canonically choose an almost complex structure J on R^{2n} as follows:

$$\begin{aligned} & J(a_1\frac{\partial}{\partial x_1} + a_2\frac{\partial}{\partial x_2} + a_3\frac{\partial}{\partial x_3} + a_4\frac{\partial}{\partial x_4} + ... + a_{2n-1}\frac{\partial}{\partial x_{2n-1}} + a_{2n}\frac{\partial}{\partial x_{2n}}) \\ = & (a_1\frac{\partial}{\partial x_2} - a_2\frac{\partial}{\partial x_1} + a_3\frac{\partial}{\partial x_4} - a_4\frac{\partial}{\partial x_3} + ... + a_{2n-1}\frac{\partial}{\partial x_{2n}} - a_{2n}\frac{\partial}{\partial x_{2n-1}}), \end{aligned}$$

where $a_1, a_2, a_3, ..., a_{2n-1}, a_{2n}$ are C^∞ functions defined on R^{2n}.

In this section, we will use this notation.

Example 6.1. Consider a submanifold $\mathcal{M}$ of R^{14} defined by

$$\begin{aligned} f(x_1, x_2, ..., x_8) &= (\frac{x_1 + \sqrt{3}x_2}{2}, \frac{x_1 - \sqrt{3}x_2}{2}, x_3\cos\theta, x_4, x_3\sin\theta, 0, x_5\cos\theta, \\ &\quad 0, -x_5\sin\theta, 0, \frac{\sqrt{5}x_7 + 2x_8}{2}, \frac{\sqrt{5}x_7 - 2x_8}{2}, x_6, 0), \end{aligned}$$

where θ is constant.

By direct computation, it is easy to check that the tangent bundle of $\mathcal{M}$ is spanned by the set $\{X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8\}$, where
$X_1 = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}, X_2 = \sqrt{3}(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}), X_3 = \frac{\partial}{\partial x_3}\cos\theta + \frac{\partial}{\partial x_5}\sin\theta, X_4 = \frac{\partial}{\partial x_4}$,
$X_5 = \frac{\partial}{\partial x_7}\cos\theta - \frac{\partial}{\partial x_9}\sin\theta, X_6 = \frac{\partial}{\partial x_{13}}, X_7 = \frac{\sqrt{5}}{2}(\frac{\partial}{\partial x_{11}} + \frac{\partial}{\partial x_{12}}), X_8 = \frac{\partial}{\partial x_{11}} - \frac{\partial}{\partial x_{12}}$.
Then using the canonical complex structure of R^{14}, we have
$JX_1 = \frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}$, $JX_2 = -\sqrt{3}(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2})$, $JX_3 = -(\frac{\partial}{\partial x_4}\cos\theta + \frac{\partial}{\partial x_6}\sin\theta)$,
$JX_4 = \frac{\partial}{\partial x_3}$, $JX_5 = -\frac{\partial}{\partial x_8}\cos\theta + \frac{\partial}{\partial x_{10}}\sin\theta$, $JX_6 = -\frac{\partial}{\partial x_{14}}$,
$JX_7 = \frac{\sqrt{5}}{2}(\frac{\partial}{\partial x_{11}} - \frac{\partial}{\partial x_{12}})$, $JX_8 = -(\frac{\partial}{\partial x_{11}} + \frac{\partial}{\partial x_{12}})$.
It is easy to see that $D = span\{X_1, X_2, X_7, X_8\}$, $D^\theta = span\{X_3, X_4\}$ and $D^\perp = span\{X_5, X_6\}$ are invariant, slant and anti-invariant distributions respectively.

Example 6.2. Consider a submanifold $\mathcal{M}$ of R^{12} defined by

$$\begin{aligned} f(x_1, x_2, ..., x_6) &= (\frac{x_1 + \sqrt{3}x_2}{2}, \frac{x_1 - \sqrt{3}x_2}{2}, \frac{x_3}{\sqrt{2}}, x_4, \frac{x_3}{\sqrt{2}}, 0, \frac{x_5}{\sqrt{2}}, \\ &\quad 0, -\frac{x_5}{\sqrt{2}}, 0, x_6, 0), \end{aligned}$$

where $\theta = \frac{\pi}{4}$.

By direct computation, it is easy to check that the tangent bundle of $\mathcal{M}$ is spanned by the set $\{X_1, X_2, X_3, X_4, X_5, X_6\}$, where
$X_1 = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}, X_2 = \sqrt{3}(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}), X_3 = \frac{1}{\sqrt{2}}(\frac{\partial}{\partial x_3} + \frac{\partial}{\partial x_5}),$
$X_4 = \frac{\partial}{\partial x_4}, X_5 = \frac{1}{\sqrt{2}}(\frac{\partial}{\partial x_7} - \frac{\partial}{\partial x_9}), X_6 = \frac{\partial}{\partial x_{11}}.$
Then using the canonical complex structure of R^{12}, we have
$JX_1 = \frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2},\ JX_2 = -\sqrt{3}(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}),\ JX_3 = -\frac{1}{\sqrt{2}}(\frac{\partial}{\partial x_4} + \frac{\partial}{\partial x_6}),$
$JX_4 = \frac{\partial}{\partial x_3},\ JX_5 = -\frac{1}{\sqrt{2}}(\frac{\partial}{\partial x_8} - \frac{\partial}{\partial x_{10}}),\ JX_6 = -\frac{\partial}{\partial x_{12}}.$
It is easy to see that $D = span\{X_1, X_2\}$, $D^\theta = span\{X_3, X_4\}$ and $D^\perp = span\{X_5, X_6\}$ are invariant, slant and anti-invariant distributions respectively.

References

[1] A. Benjancu, *Geometry of CR-submanifolds*, D. Reidel Pub. comp., Dordrecht, 1986.

[2] A. M. Blaga, *Invariant, anti-invariant and slant submanifolds of para-Kenmotsu manifold*, BSG Publ. 24, (2017), 125-138.

[3] J. L. Cabrerizo, A. Carriazo, L. M. Fernandez, A. Fernandez, *Slant submanifolds in Sasakian manifolds*, Glasg. Math. J. 42, 1 (2000), 125-138.

[4] J. L. Cabrerizo, A. Carriazo, L. M. Fernandez, A. Fernandez, *Semi-slant submanifolds of a Sasakian manifold*, Geom. Dedicata 78, 2 (1999), 183-199.

[5] A. Carriazo, *Bi-slant immesions*, Proc. ICRAMS , Kharagpur, India (2000), 88-97.

[6] B. Y. Chen, *Geometry of Slant Submanifolds*, Katholieke Universiteit, Leuven, 1990.

[7] B. Y. Chen, *Cohomology of CR-submanifolds*, Ann. Fac. Sci. Toulouse Math. 3, (1981), 167-172.

[8] B. Y. Chen, *Slant immersions*, Bull. Austral. Math. Soc. 41, 1 (1990), 135-147.

[9] U. C. De, A. A. Shaikh, *Complex Manifolds and Contact Manifolds*, Narosa Publ. House, 2009.

[10] S. Deshmukh, *Cohomology of CR-submanifolds of nearly Kaehler manifolds*, Math. chronicle 16, (1982), 47-55.

[11] V. A. Khan, M. A. Khan, *Semi-slant submanifolds of a nearly Kaehler manifold*, Turk J. Math. 31, (2007), 341-353.

[12] M. Kon, *Remarks on anti-invariant submanifolds of a Sasakian manifold*, Tensor (N.S.) 30, (1976), 239-245.

[13] M. A. Lone, M. H. Shahid, S. D. Sharma, *Hemi-slant submanifolds of nearly Kaehler manifolds*, Gulf Journal of Mathematics 4. 3(2016), 55-64.

[14] A. Lotta, *Slant submanifold in contact geometry*, Bull. Math. Soc. Romania 39, (1996), 183-198.

[15] A. Lotta, *Three-dimensional slant submanifolds of K-contact manifold*, Balkan J. Geom. Appl. 3, 1 (1998), 37-51.

[16] N. Papaghuic, *Semi-slant submanifolds of Kaehlerian manifold*, An. St. Univ. AI. I. Cuza. Iasi. Math. (N.S.) 40, 1 (1994), 55-61.

[17] K. S. Park, R. Prasad, *Semi-slant submersions*, Bull. Korean Math. Soc 50, 3 (2013), 951-962.

[18] B. Sahin, *Warped product submanifolds of a Kaehler manifold with a slant factor*, Ann. Pol. Math. 95, (2009), 107-126.

[19] B. Sahin, *Riemannian submersions, Riemannian maps in Hermitian geometry and their applications*, Elsevier, Academic Press (2017).

[20] B. Sahin, *Slant submersions from almost Hermitian manifolds*, Bull. Math. de la Soc. des Sciences Math. de Roumanie 54, (2011), 93-105.

[21] F. Sahin, *Cohomology of hemi-slant submanifolds of a Kaehler manifolds*, J. Adv. Studies Topology 5, (2014), 27-31.

[22] M. H. Shahid et al., *Slant submanifolds of cosymplectic manifold*, An. St. Univ. Al. I. Cuza Iasi, Matematica 50, 1 (2004), 33-50.

[23] H. M. Tastan, B. Sahin, S. Yanan, *Hemi-slant submersions*, Mediterr. J. Math, DOI 10, 4 (2016), 2171-2184.

[24] K. Yano, M. Kon, *Structures on Manifolds*, World Scientific Publishing Co., Singapore, 1984.

Authors' addresses:

Rajendra Prasad, Punit Kumar Singh
Department of Mathematics and Astronomy,
University of Lucknow, Lucknow, India.
E-mail: rp.manpur@rediffmail.com , singhpunit1993@gmail.com

Amit Kumar Rai
University School of Basic and Applied Sciences,
Guru Gobind Singh Indraprastha University, New Delhi, India.
E-mail: rai.amit08au@gmail.com