Weak $M-$projective symmetries for a Sasakian manifold

Hülya Bağdatlı Yılmaz

Abstract. In this study, we deal with the $M-$projective curvature tensor on a Sasakian manifold. After defining both weakly $M-$projective symmetric Sasakian manifolds and weakly $M-$projective Ricci symmetric Sasakian manifolds, we obtained all the associated 1-forms and the relation among them. Moreover, we investigated a weakly $M-$projective Ricci symmetric Sasakian manifold having a cyclic parallel $M-$projective Ricci tensor.

M.S.C. 2010: 53C21, 53D15.
Key words: Sasakian manifold, $M-$projective curvature tensor, $M-$projective Ricci tensor.

1 Introduction

In the 19th century, the notion of contact transformation was introduced by Lie [12] to study systems of differential equations from a geometrical point of view. This notion can be seen as the beginning of contact geometry. The subject has manifold connections with other fields of pure mathematics, and a significant place in applied areas such as optics, control theory or mechanics, etc... until Chern [5] and Gray [9] contributed to the development of this geometry, contact geometry has not been receiving attention. Then the notion of a Sasakian manifold was introduced by Sasaki [16]. This manifold is a contact manifold with a Sasakian metric which is a special kind of Riemannian metric g. It forms an important class of a contact manifold and has been studied by many authors (see, [1, 7, 10, 11, 14, 21, 22]).

Pokhariyal and Mishra [15] have introduced the $M-$ *projective curvature tensor* of type $(0,4)$ on a Riemannian manifold (M^n, g) in the form:

$$\begin{aligned} W(Y,Z,U,V) &= R(Y,Z,U,V) - \frac{1}{2(n-1)}\left(S(Z,U)g(Y,V) - S(Y,U)g(Z,V)\right. \\ &\quad \left. + g(Z,U)S(Y,V) - g(Y,U)S(Z,V)\right), \end{aligned} \tag{1.1}$$

so that

$$W(Y,Z,U,V) = g(W(Y,Z)U,V),\ R(Y,Z,U,V) = g(R(Y,Z)U,V)$$

where $W(Y,Z)U$, $R(X,Y)Z$ and $S(X,Y)$ are the $M-$ projective curvature tensor of type $(1,3)$, the curvature tensor and the Ricci tensor of (M^n,g), respectively. The Ricci tensor S is defined by $S(X,Y)=g(LX,Y)$, where L is the Ricci operator. The $M-$ projective curvature tensor is important in the terms of relativity [15]. Further, the other properties of this tensor have been extensively studied on the various manifolds by several authors (see, [4, 13, 14, 24, 25]).

In 1992, Tamássy and Binh [19] introduced the notion of weakly symmetric manifolds. A non-flat Riemannian manifold (M^n,g) is called a *weakly-symmetric manifold* if there exist $1-$forms α, β, γ, δ and σ satisfying the condition

$$\begin{aligned}(\nabla_X R)(Y,Z,U,V) &= \alpha(X)R(Y,Z,U,V)+\beta(Y)R(X,Z,U,V)\\ &\quad+\gamma(Z)R(Y,X,U,V)+\delta(U)R(Y,Z,X,V)\\ &\quad+\sigma(V)R(Y,Z,U,X)\end{aligned} \tag{1.2}$$

for all vector fields $X,Y,Z,U,V\in\chi(M)$, where ∇ is the covariant derivative with respect to the Levi-Civita connection. Then, De and Bandyopadhyay [6] proved that in such a manifold $1-$forms β and δ are equal to γ and σ, respectively. Thus, the number of different 1-forms in the above relation is reduced from five to three.

In 1993, again Tamássy and Binh [20] introduced the notion of weakly Ricci symmetric manifolds. A Riemannian manifold (M^n,g) is called a *weakly Ricci symmetric manifold* if its Ricci tensor S of type $(0,2)$ is not identically equal to zero and satisfies the condition

$$(\nabla_X S)(Z,U)=\rho(X)S(Z,U)+\mu(Z)S(X,U)+\nu(U)S(Z,X), \tag{1.3}$$

where $1-$forms ρ, μ and ν are not zero.

In 1998, Gray [8] introduced two classes of Riemannian manifolds determined by the covariant differentiation of Ricci tensor S. One of these classes consisting of all Riemannian manifolds whose Ricci tensor S is cyclic parallel, i.e.

$$(\nabla_X S)(Y,Z)+(\nabla_Y S)(Z,X)+(\nabla_Z S)(X,Y)=0. \tag{1.4}$$

Recently, Chaubey [3] introduced the notion of weakly $M-$projectively symmetric manifolds. A non-$M-$projectively flat Riemannian manifold (M^n,g) $(n>2)$ is called a *weakly $M-$projectively symmetric manifold* if the $M-$ projective curvature tensor W of type $(0,4)$ satisfies the relation

$$\begin{aligned}(\nabla_X W)(Y,Z,U,V) &= \alpha(X)W(Y,Z,U,V)+\beta(Y)W(X,Z,U,V)\\ &\quad+\gamma(Z)W(Y,X,U,V)+\delta(U)W(Y,Z,X,V)\\ &\quad+\sigma(V)W(Y,Z,U,X)\end{aligned} \tag{1.5}$$

for all vector fields $X,Y,Z,U,V\in\chi(M)$, where α, β, γ, δ and σ are $1-$forms. Chaubey proved in the same study that equation (1.5) reduces to the form, [3]

$$\begin{aligned}(\nabla_X W)(Y,Z,U,V) &= \alpha(X)W(Y,Z,U,V)+\beta(Y)W(X,Z,U,V)\\ &\quad+\beta(Z)W(Y,X,U,V)+\delta(U)W(Y,Z,X,V)\\ &\quad+\delta(V)W(Y,Z,U,X)\end{aligned} \tag{1.6}$$

for all $X,Y,Z,U,V\in\chi(M)$ and non-zero $1-$forms α, β, δ called associated 1-forms.

In this study, we deal with both weakly $M-$projective symmetric Sasakian manifolds and weakly $M-$projective Ricci symmetric Sasakian manifolds. In Section 2 some preliminary results are reviewed. In Section 3 weakly $M-$projectively symmetric manifolds are defined and all the $1-$forms of such a manifold are obtained. Moreover, weakly $M-$projective Ricci symmetric Sasakian manifolds are defined and all the $1-$forms of such a manifold are determined. Then, weakly $M-$projective Ricci symmetric Sasakian manifolds having a cyclic parallel $M-$projective Ricci tensor W^* are investigated.

2 Sasakian manifolds

In this section, some basic concepts of Sasakian manifolds which will be used through the paper are considered.

Let (M^n, g) $(n = 2m+1)$ be an almost contact metric manifold equipped with an almost contact metric structure (ϕ, ξ, η, g). We have

$$\phi^2(X) = -X + \eta(X)\xi, \qquad \phi\xi = 0, \tag{2.1}$$

$$\eta(\xi) = 1, \quad g(X, \xi) = \eta(X), \quad \eta(\phi X) = 0\ , \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.3}$$

$$g(\phi X, Y) = -g(X, \phi Y), \tag{2.4}$$

$$(\nabla_X \eta)(Y) = g(\nabla_X \xi, Y), \tag{2.5}$$

where ϕ is a $(1,1)$ tensor field, η is a $1-$form, ξ is the corresponding vector field to the $1-$form η and g is Riemannian metric.

An almost contact metric manifold (M^n, g) is called *a contact metric manifold* if

$$d\eta(X, Y) = g(X, \phi Y), \tag{2.6}$$

for all $X, Y \in \chi(M)$. A contact metric manifold is said to be *a $K-$contact manifold* if the vector field ξ is a Killing vector field. The basic property of a K-contact manifold is that,

$$\nabla_X \xi = -\phi X. \tag{2.7}$$

It is said to be *a Sasakian manifold* if

$$(\nabla_X \phi)(Y) = g(X, Y)\xi - \eta(Y)X \tag{2.8}$$

for all $X, Y \in \chi(M)$. It is well-known that Sasakian manifold is always $K-$contact Riemannian manifold.

The following relations also can be stated in a Sasakian manifold:

$$(\nabla_X \eta)(Y) = g(X, \phi Y), \tag{2.9}$$

$$R(\xi, X)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.10}$$

$$S(X,\xi) = (n-1)\eta(X), \tag{2.11}$$

$$R(X,\xi)Y = \eta(Y)X - g(X,Y)\xi, \tag{2.12}$$

for all $X, Y \in \chi(M)$ (see, [2, 17, 18]).

3 Main Theorems

In this section, Weakly $M-$projective symmetric Sasakian manifolds and weakly $M-$projective Ricci symmetric Sasakian manifolds are examined as two subsections. Furthermore, weakly $M-$projective Ricci symmetric Sasakian manifolds having a cyclic parallel $M-$projective Ricci tensor W^* is considered. The main theorems and corollaries about them are also specified.

3.1 Weakly $M-$projective symmetric Sasakian manifolds

Let us now define weakly $M-$projective symmetric Sasakian manifolds and obtain both all the associated 1-forms and the relation among them.

Definition 3.1. A Sasakian manifold (M^n, g) $(n = 2m+1)$ is said to be weakly $M-$projective symmetric if its $M-$projective curvature tensor of type $(0,4)$ is not identically equal to zero and satisfies equation (1.6).

Let e_i $(1 \le i \le n)$ be an orthonormal basis of the tangent space at each point of the manifold. Then, equation (1.1) becomes

$$W^*(Z,U) = \sum_{i=1}^{n} W(e_i, Z, U, e_i) = \frac{1}{2(n-1)}\left(nS(Z,U) - rg(Z,U)\right), \tag{3.1}$$

where r is a scalar curvature of (M^n, g). The tensor W^* is called the $M-$projective Ricci tensor, which is a symmetric tensor of type $(0,2)$.

Considering $Y = V = e_i$ into equation (1.6), using equations (1.1) and (3.1), and summing over $i, (1 \le i \le n)$, we obtain

$$\begin{aligned} n\left(\nabla_X S\right)(Z,U) - \left(\nabla_X r\right)g(Z,U) \;\; = \;\; & \alpha(X)\left\{nS(Z,U) - rg(Z,U)\right\} \\ & + \beta(Z)\left\{nS(X,U) - rg(X,U)\right\} \\ & + \delta(U)\left\{nS(Z,X) - rg(Z,X)\right\} \\ & + 2(n-1)\left\{\beta(R(X,Z)U + \delta(R(X,U)Z\right\} \\ & - \{(\beta(X) + \delta(X))S(Z,U) - S(X,U)\beta(Z) \\ & - \delta(LU)g(Z,X) + (\beta(LX) + \delta(LX))g(Z,U) \\ & - g(X,U)\beta(LZ) - \delta(U)S(Z,X)\}. \end{aligned} \tag{3.2}$$

Substituting ξ for X, Z and U into equation (3.2) and then using properties $(2.1)_2$, (2.2), (2.7), (2.11) and (2.12), we achieve

$$\left[r - n(n-1)\right]\left[\alpha(\xi) + \beta(\xi) + \delta(\xi)\right] = dr(\xi). \tag{3.3}$$

Firstly, assumed that $r \neq n(n-1)$. Then, we can rewrite equation (3.3) as follows

$$\alpha(\xi)+\beta(\xi)+\delta(\xi)=\frac{dr(\xi)}{r-n(n-1)}. \tag{3.4}$$

Hence this result proved above is put in the following lemma.

Lemma 3.1. *In a weakly $M-$projective symmetric Sasakian manifold (M^n, g), $r \neq n(n-1)$, the associated $1-$forms α, β and δ satisfy equation (3.4).*

Now, considering $X = Z = \xi$ into equation (3.2) and using properties (2.1), (2.2), (2.4), (2.7), (2.9), (2.10), (2.11) and (2.12), we obtain

$$\delta(U)=\left(-(\alpha(\xi)+\beta(\xi))+\frac{dr(\xi)}{r-n(n-1)}\right)\eta(U). \tag{3.5}$$

By using equations (3.4) and (3.5), we achieve

$$\delta(U)=\delta(\xi)\eta(U). \tag{3.6}$$

Considering $X = U = \xi$ into equation (3.2) and using equations (2.1), (2.2), (2.4), (2.7), (2.9),(2.10), (2.11) and (2.12) provide

$$\beta(Z)=\left(-(\alpha(\xi)+\delta(\xi))+\frac{dr(\xi)}{r-n(n-1)}\right)\eta(Z). \tag{3.7}$$

Hence, using equation (3.4) and the above equation, we obtain

$$\beta(Z)=\beta(\xi)\eta(Z). \tag{3.8}$$

Similarly, substituting ξ for Z and U into equation (3.2), we achieve

$$\alpha(X)=-(\beta(\xi)+\delta(\xi))\eta(X)+\frac{dr(X)}{r-n(n-1)}. \tag{3.9}$$

From equations (3.4) and (3.9), we thus obtain

$$\alpha(X)=\left(\alpha(\xi)-\frac{dr(\xi)}{r-n(n-1)}\right)\eta(X)+\frac{dr(X)}{r-n(n-1)}. \tag{3.10}$$

From equations (3.6), (3.8) and (3.10) the following expression is yielded

$$\alpha(X)+\beta(X)+\delta(X)=\frac{dr(X)}{r-n(n-1)} \tag{3.11}$$

for all $X \in \chi(M)$.

We can then state the following theorem.

Theorem 3.2. *Let (M^n, g) be a weakly $M-$projective symmetric Sasakian manifold, $r \neq n(n-1)$. Then,*

1. *The associated $1-$forms α, β and δ are expressed as equations (3.10), (3.8) and (3.6), respectively.*

2. The sum of the associated $1-$forms α, β and δ is expressed as (3.11).

Again, considering $X = Z = \xi$, $X = U = \xi$, $Z = U = \xi$ into equation (3.2), respectively, and using properties (2.1), (2.2), (2.4), (2.7), (2.9),(2.10), (2.11) and (2.12), we obtain the following relations:

$$[r-n(n-1)]\,\delta(U) = (-\,[\alpha(\xi)+\beta(\xi)]\,[r-n(n-1)] + dr(\xi))\,\eta(U), \tag{3.12}$$

$$[r-n(n-1)]\,\beta(Z) = (-\,[\alpha(\xi)+\delta(\xi)]\,[r-n(n-1)] + dr(\xi))\,\eta(Z) \tag{3.13}$$

and

$$[r-n(n-1)]\,\alpha(X) = -\,[r-n(n-1)]\,[\beta(\xi)+\delta(\xi)]\,\eta(X) + dr(X). \tag{3.14}$$

Summing (3.12), (3.13) and (3.14) and using equation (3.3), we achieve

$$[r-n(n-1)]\,[\alpha(X)+\beta(X)+\delta(X)] = dr(X). \tag{3.15}$$

If the scalar curvature r equals $n(n-1)$ in equation (3.15), then the above equation is automatically satisfied.

We have thus proved the following

Theorem 3.3. *There always exists a weakly $M-$ projective symmetric Sasakian manifold whose scalar curvature r equals $n(n-1)$.*

3.2 Weakly $M-$projective Ricci symmetric Sasakian manifolds

Let us define weakly M-projective Ricci symmetric Sasakian manifolds and give both all the associated 1-forms and the relation among them. Moreover, weakly $M-$projective Ricci symmetric Sasakian manifolds having a cyclic parallel $M-$projective Ricci tensor W^* are investigated.

Definition 3.2. A Sasakian manifold (M^n, g) $(n = 2m+1)$ is said to be weakly $M-$projective Ricci symmetric if its $M-$projective Ricci tensor W^* of type $(0,2)$ is not identically equal to zero and satisfies the following condition

$$(\nabla_X W^*)(Z,U) = \rho(X)W^*(Z,U) + \mu(Z)W^*(X,U) + \nu(U)W^*(Z,X). \tag{3.16}$$

Using equations (3.1) and (3.16), we obtain

$$\begin{aligned} n\,(\nabla_X S)(Z,U) - (\nabla_X r)\,g(Z,U) \quad = \quad & \rho(X)\,\{nS(Z,U) - rg(Z,U)\} \\ & +\mu(Z)\,\{nS(X,U) - rg(X,U)\} \\ & +\nu(U)\,\{nS(Z,X) - rg(Z,X)\}\,. \end{aligned} \tag{3.17}$$

Substituting ξ for X, Z and U into equation (3.17) and then using properties $(2.1)_2$, (2.2), (2.7) and (2.11) yield

$$[r-n(n-1)]\,[\rho(\xi)+\mu(\xi)+\nu(\xi)] = dr(\xi). \tag{3.18}$$

Firstly, assumed that $r \neq n(n-1)$. From equation (3.18), it follows that

$$\rho(\xi)+\mu(\xi)+\nu(\xi) = \frac{dr(\xi)}{r-n(n-1)}. \tag{3.19}$$

Then, the following lemma holds true:

Lemma 3.4. *In a weakly $M-$projective Ricci symmetric Sasakian manifold (M^n, g), $r \neq n(n-1)$, the associated $1-$forms α, β and δ satisfy equation (3.19).*

Now, substituting ξ for X and Z into equation (3.17), using properties (2.1), (2.2), (2.4), (2.7), (2.9),(2.10) and (2.11), we achieve

$$\nu(U) = \left(-(\rho(\xi) + \mu(\xi)) + \frac{dr(\xi)}{r - n(n-1)}\right)\eta(U). \tag{3.20}$$

From equations (3.19) and (3.20), it follows that

$$\nu(U) = \nu(\xi)\eta(U). \tag{3.21}$$

Similarly, substituting ξ for X and U into equation (3.17), we obtain

$$\mu(Z) = \left(-(\rho(\xi) + \nu(\xi)) + \frac{dr(\xi)}{r - n(n-1)}\right)\eta(Z). \tag{3.22}$$

Using equation (3.19) and the above equation, we achieve

$$\mu(Z) = \mu(\xi)\eta(Z). \tag{3.23}$$

Finally, substituting ξ for Z and U into equation (3.17) yields

$$\rho(X) = -(\mu(\xi) + \nu(\xi))\eta(X) + \frac{dr(X)}{r - n(n-1)}. \tag{3.24}$$

It follows from equation (3.19) that

$$\rho(X) = \left(\rho(\xi) - \frac{dr(\xi)}{r - n(n-1)}\right)\eta(X) + \frac{dr(X)}{r - n(n-1)}. \tag{3.25}$$

From equations (3.19), (3.21), (3.23) and (3.25), we thus obtain

$$\rho(X) + \mu(X) + \nu(X) = \frac{dr(X)}{r - n(n-1)}. \tag{3.26}$$

Hence the following theorem is attained:

Theorem 3.5. *If a Sasakian manifold is weakly $M-$ projective Ricci symmetric, $r \neq n(n-1)$, then*

1. *The associated 1-forms ρ, μ and ν are expressed as equations (3.25), (3.23) and (3.21), respectively.*
2. *The sum of the associated 1-forms is expressed as equation (3.26).*

By proceeding in a similar manner as previous subsection, we achieve

$$[r - n(n-1)]\,[\rho(X) + \mu(X) + \nu(X)] = dr(X). \tag{3.27}$$

In equation (3.27), if the scalar curvature r equals $r = n(n-1)$, then it is evident that equation (3.27) holds.

Hence we can state the following:

Theorem 3.6. *There always exists a weakly $M-$ projective Ricci symmetric Sasakian manifold whose scalar curvature r equals $n(n-1)$.*

Let us now assume that the $M-$projective Ricci tensor W^* is cyclic parallel. Then, considering equation (1.4), we obtain

$$(\nabla_X W^*)(Z,U) + (\nabla_Z W^*)(U,X) + (\nabla_U W^*)(X,Z) = 0. \tag{3.28}$$

From equations (3.16) and (3.28), it follows that

$$w(X)W^*(Z,U) + w(Z)W^*(X,U) + w(U)W^*(Z,X) = 0, \tag{3.29}$$

where

$$w(X) = \rho(X) + \mu(X) + \nu(X). \tag{3.30}$$

Based on Walker's Lemma [23], it must be either $w(X) = 0$ or $W^*(Z,U) = 0$. By virtue of Definition 3.2, since $W^*(Z,U) \neq 0$, it must be $w(X) = 0$. From equation (3.30), we achieve

$$\rho(X) + \mu(X) + \nu(X) = 0, \tag{3.31}$$

for all $X \in \chi(M)$. Considering these results, we can specify the following corollary.

Corollary 3.7. *There is no weakly $M-$projective Ricci symmetric Sasakian manifold (M^n, g) whose $M-$projective Ricci tensor W^* is cyclic parallel unless $\rho+\mu+\nu$ equals zero everywhere.*

Further, it is the result of equation (3.26) that a weakly $M-$projective Ricci symmetric Sasakian manifold (M^n, g) whose $M-$projective Ricci tensor W^* is cyclic parallel, is of constant scalar curvature. Therefore, we can state this result in the following theorem.

Theorem 3.8. *If a weakly $M-$projective Ricci symmetric Sasakian manifold (M^n, g) has a cyclic parallel $M-$projective Ricci tensor W^*, then such a manifold is of constant scalar curvature.*

References

[1] C.L. Bejan, M. Crasmareanu, *Weakly-symmetry of Sasakian lifts on tangent bundles*, Publicationes Mathematicae Debrecen 83, (1-2)(2013), 63-69.

[2] B.D. Blair, *Contact manifolds in Riemannian geometry*, Lecture notes in Mathematics Spinger-Verlag. Berlin 1976.

[3] S.K. Chaubey, *On weakly $M-$projectively symmetric manifolds*, Novi Sad Journal of Mathematics 42, 1 (2012), 67-79.

[4] S.K. Chaubey, R.H. Ojha, *On $M-$projective curvature tensor of a Kenmotsu manifold*, Differentiel Geometry and Dynamical Systems 12 (2010), 52-60.

[5] S.S. Chern, *Pseudo-groupes continus infinis*, Colloques Internationaux du C.N.R.S., Strasbourg (1953), 119-136.

[6] U.C. De, S. Bandyopadhyay, *On weakly symmetric spaces*, Publicationes Mathematicae Debrecen 54 (1999), 377-381.

[7] U.C. De, J.B. Jun and A.K. Gazi, *Sasakian manifolds with quasi-conformal curvature tensor*, Bulletin of the Korean Mathematical Society 45, 2 (2008), 313-319.

[8] A. Gray, *Einstein-like manifolds which are not Einstein*, Geom. Dedicata 7 (1998), 259-280.

[9] J.W. Gray, *Some global properties of contact structures*, Annals of Mathematics 69 (1959), 421-450.

[10] J.P. Jaiswal, *The existence of weakly symmetric and weakly Ricci-Symmetric Sasakian manifolds admitting a quarter symmetric metric connection*, Acta Mathematica Hungarica 132, 4 (2011), 358-366.

[11] A. Kushwaha, D. Narain, *Some curvature properties on Sasakian manifolds*, Journal of International Academy of Physical Sciences 20, 4 (2016), 293-302.

[12] S. Lie, *Zur theorie partieller Differentialgleichungen*, GSttinger Nachrichten 1872.

[13] R.H. Ojha, *A note on the $M-$projective curvature tensor*, Indian Journal of Pure and Applied Mathematics 12, 8 (1975), 1531-1534.

[14] R.H. Ojha, *$M-$projectively flat Sasakian manifolds*, Indian Journal of Pure and Applied Mathematics 4, 17 (1986), 481-484.

[15] G.P. Pokhariyal, R.S. Mishra, *Curvature tensor and their relativistic significance II*, Yokohama Mathematical Journal 19 (1971), 97-103.

[16] S. Sasaki, *On differentiable manifolds with certain structures which are closely related to almost contact structure*, Tohoku Mathematical Journal 12 (1960), 459-476.

[17] S. Sasaki, *Lecture note on almost contact manifolds, Part I*, Tohoku University 1965.

[18] S. Sasaki, *Lecture note on almost contact manifolds, Part II*, Tohoku University 1967.

[19] L. Tamássy, T.Q. Binh, *On weakly symmetric and weakly projective symmetric Riemannian manifolds*, Coll. Math. Soc., J. Bolyai 56 (1992), 663-670.

[20] L. Tamássy, T.Q. Binh, *On weak symmetries of Einstein and Sasakian manifolds*, Tensor N.S. 53 (1993), 140-148.

[21] M. Tarafdar, *On pseudo-symmetric and pseudo-Ricci-symmetric Sasakian manifolds*, Periodica Mathematica Hungarica 22, 2 (1991), 125-129.

[22] V. Venkatesha, H.A. Kumara and D.M. Naik, *On a class of generalized $\phi-$recurrent Sasakian manifold*, Journal of the Egyptian Mathematical Society 27, 1 (2019), 1-14.

[23] A.G. Walker, *On Ruse's spaces of recurrent curvature*, Proceedings of London Mathematical Society 52 (1950), 36-64.

[24] V. Yadav, D. Narain and S.K. Dubey, *Some properties of $M-$projective curvature tensor in a $P-$Sasakian manifold*, Journal of Rajasthan Academy of Physical Sciences 15 (2016), 97-105.

[25] F. Özen Zengin, *On $M-$projectively flat $LP-$Sasakian manifolds*, Ukrainian Mathematical Journal 65, 11 (2013), 1725-1732.

Authors' addresses:

Hülya Bağdatlı Yılmaz
Department of Mathematics, Faculty of Sciences and Letters,
University of Marmara, 34722, Istanbul, Turkey.
E-mail: hbagdatli@marmara.edu.tr