Lag anti-synchronization of delay coupled chaotic systems via a scalar signal

Mohammad Ali Khan

Abstract. In this letter, a chaotic anti-synchronization (AS) scheme is proposed based on combining a nonlinear with lag-in-time observer design via a scalar signal. The necessary and sufficient conditions for anti-synchronization of delay coupled systems are derived. Finally, numerical simulation results are presented to show the feasibility and effectiveness of the proposed anti-synchronization scheme, by taking the delay coupled chaotic unified chaotic system (UCS) and the Lorenz-Stenflo system (LSS) as examples.

M.S.C. 2010: 26A33, 53C63,58A05, 58A40.
Key words: Anti-synchronization (AS), Lag anti-synchronization(LAS), unified chaotic system (UCS), Lorenz-Stenflo system(LSS).

1 Introduction

Experimental observations have pointed out that chaotic systems are common in nature. It is found that in Chemistry (Belouov-Zhabotinski reaction), in Non-linear Optics (lasers), in Electronics (Chua-Matsumoto circuits), in Fluid Dynamics (Rayleigh-Benard convection), etc, chaotic systems exist. Chaos is found in meteorology, solar system, heart and brain of living organisms and so on. Synchronization and control of interacting chaotic oscillators is one of the fundamental phenomena of non-linear dynamics and chaos. Experimental realization of chaos synchronization and control have been achieved with a magneto-elastic ribbon, a heart, a thermal convection loop, a diode oscillator, an optimal multi-mode chaotic solid-state laser, a Belousov-Zhabotinski reaction diffusion chemical system and many other experiments.

One of the most striking discoveries in the study of chaos is that chaotic systems can be made to synchronize with each other. Synchronization of chaos is a phenomenon that may occur when two or more chaotic dynamical systems are coupled. This was discovered by Pecora and Carroll in 1990 [1]. Since Pecora and Carroll's [1] work many effective methods for chaos control and synchronization, namely, OGY method [2], adaptive control [3], active control [4], gradient based control [5], differential geometric method [6], inverse optimal control [7], lag synchronization [8], [9], projective synchronization [10], adaptive synchronization [11], hybrid synchronization

Differential Geometry - Dynamical Systems, Vol.17, 2015, pp. 72-80.

[12] etc, to mention a few. Usually two dynamical systems are called synchronized if the distance between their corresponding states converges to zero as time goes to infinity. This type of synchronization is known as identical synchronization [1]. A generalization of the concept for unidirectionally coupled dynamical systems was proposed by Rulkov *et.al.* [13], where two systems are called synchronized if a static functional relationship exists between the states of the systems. They called this kind of synchronization a generalized synchronization (GS) [14], [15]. Kocarev and Parlitz [16] formulated a condition for the occurrence of GS between two coupled continuous dynamical systems. Yang and Chua [17] proposed GS of continuous dynamical systems via linear transformations. Tarai *et.al.* [18] introduce synchronization between two generalized bidirectionally coupled chaotic system.

The anti-synchronization is a phenomenon that the state variables of synchronized systems have the same absolute values but opposite signs. We say that anti-synchronization of two systems S_1 and S_2 are achieved if the following holds:

$$\lim_{t\to\infty} |x_1(t) + x_2(t)| = 0,$$

where $x_1(t)$, $x_2(t)$ are the state vectors of the systems S_1 and S_2 respectively. It was well known that the first observation of synchronization of oscillators by Huygens in the seventeenth century was, in fact anti-synchronization(AS) between the pendulum clocks. Paradimetically speaking, in 2003, Kim *et.al.* [19] have observed anti-synchronization phenomena in coupled identical chaotic oscillators. In 2004, Y. Zhang and J. Sun [20] have studied anti-synchronization based on a suitable separation technique. Again, in 2005, Hu *et.al.* [21] have presented adaptive control for anti-synchronization of Chua's chaotic system. In 2008, Li *et.al.* [22] have had anti-synchronization of two different chaotic systems. Of late, M. Mossa Al-Sawalha and M.S.M. Noorani [23] have looked into anti-synchronization of two hyperchaotic systems via nonlinear control in 2009. Recently, in 2012, Khan *et.al.* [24] have studied anti-synchronization of different chaotic systems via linear transformation.

In this paper, We introduce the theory of anti-synchronization of two chaotic systems with lag-in-time observer design via a scalar signal. Firstly, we discuss the theory for three dimensional unified chaotic systems and secondly discuss for four-dimensional Lorenz-Stenflo systems. Finally numerical simulation results are presented to show the efficiency of our method.

2 LAS scheme for delay coupled chaotic systems

Any dynamical system can be decomposed into two parts in the following way:

$$\dot{X} = AX + B\Psi(X), \tag{2.1}$$

where $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times n}$ are constant matrices and $\Psi : \mathbb{R}^n \to \mathbb{R}^n$ is a non-linear function. We consider the following unidirectional delay coupled system

$$\begin{cases} \dot{X} = AX + B\Psi(X), \\ \dot{Y} = AY + B\Psi(Y) - g(Z(t-\tau) + S(Y)), \end{cases} \tag{2.2}$$

where $S(X)$ is an artificial output of the system (2.1) which can be properly designed to feed the non-linear and lag in time observer (2.2). Since the adoption of a scalar signal is suitable feature for secure communication applications, it is assumed that $Z = S(X) \in \mathbb{R}$. By choosing the anti-synchronization signal $S(X) = \Psi(X) + KX$; where $K = [k_1, k_2, ..., k_n] \in \mathbb{R}^{1\times n}$, the system (2.2) becomes

$$\dot{Y} = AY + B\Psi(Y) - g(\Psi(Y) + \Psi(X(t-\tau)) + K(Y + X(t-\tau))). \tag{2.3}$$

Theorem. *If $g(\Psi(Y) + \Psi(X(t-\tau)) + K(Y + X(t-\tau))) = B(\Psi(Y) + \Psi(X(t-\tau)) + K(Y + X(t-\tau)))$, then the two dynamical systems (2.1) and (2.3) are in a stable of LAS if and only if all eigenvalues of the matrix $A - BK$ have a negative real part.*

Proof. Let us define the anti-synchronization error as $e(t) = Y(t) + X(t-\tau)$ then from system (2.2) and (2.3), we obtain

$$\begin{aligned}\dot{e}(t) = \quad & AY + B\Psi(Y) - B(\Psi(Y) + \Psi(X(t-\tau)) \\ & + K(Y + X(t-\tau))) + AX(t-\tau) + B\Psi(X(t-\tau)),\end{aligned}$$

i.e.,

$$\dot{e}(t) = (A - BK)e(t). \tag{2.4}$$

Therefore $e(t) = 0$ is asymptotically stable if and only if all eigenvalues of the matrix $A - BK$ have negative real parts. □

3 The LAS of delay coupled unified chaotic systems

In this section, we study LAS of two unified chaotic systems. Lorenz [25] had found the first classical chaotic system in 1963. Chen and Ueta [26] have found a chaotic system which is similar to Lorenz system but not topologically equivalent to Lorenz system in 1999. Recently, a chaotic system is presented by Lu *et.al.* [27], which bridged the gap between Lorenz and Chen systems. A new unified chaotic system with continuous periodic switch between Lorenz and Chen system is presented by Lu and Wu [28] in 2004. The unified chaotic system can be described by the following system of differential equations.

$$\begin{cases} \dot{x} = (25a + 10)(y - x), \\ \dot{y} = (28 - 35a)x - xz + (29a - 1)y, \\ \dot{z} = xy - \frac{8+a}{3}z, \end{cases} \tag{3.1}$$

where $a \in [0, 1]$. When $a = 0, 0.8, 1$ this system represents Lorenz chaotic system, Lu chaotic system and Chen chaotic system respectively. Practically unified chaotic system is chaotic for any $a \in [0, 1]$. The unified chaotic system can be decomposed into two parts

$$\dot{X} = AX + B\Psi(X), \tag{3.2}$$

where

$$A = \begin{pmatrix} -25a - 10 & 25a + 10 & 0 \\ 28 - 35a & 29a - 1 & 0 \\ 0 & 0 & -\frac{8+a}{3} \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix},$$

and

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \Psi(X) = \begin{pmatrix} 0 \\ -xz \\ xy \end{pmatrix}.$$

Now according to our scheme the slave system can be expressed as the following system of equations,

$$\text{(3.3)} \qquad \begin{cases} \dot{x_1} = (25a+10)(y_1 - x_1) - u_1, \\ \dot{y_1} = (28-35a)x_1 + x(t-\tau)z(t-\tau) + (29a-1)y_2 - u_2, \\ \dot{z_1} = -x(t-\tau)y(t-\tau) - \frac{8+a}{3}z_1 - u_3, \end{cases}$$

where $K = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix}$ is a real constant matrix and

$$\begin{cases} u_1 = k_{11}(x_1 + x(t-\tau)) + k_{12}(y_1 + y(t-\tau)) + k_{13}(z_1 + z(t-\tau)), \\ u_2 = k_{21}(x_1 + x(t-\tau)) + k_{22}(y_1 + y(t-\tau)) + k_{23}(z_1 + z(t-\tau)), \\ u_3 = k_{31}(x_1 + x(t-\tau)) + k_{32}(y_1 + y(t-\tau)) + k_{33}(z_1 + z(t-\tau)). \end{cases}$$

As the result of of system (3.1) and (3.3), the error system becomes

$$\text{(3.4)} \qquad \begin{cases} \dot{e_1} = -(25a+10+k_{11})e_1 + (25a+10-k_{12})e_2 - k_{13}e_3, \\ \dot{e_2} = (28-35a-k_{21})e_1 + (29a-1-k_{22})e_2 - k_{23}e_3, \\ \dot{e_3} = -k_{31}e_1 - k_{32}e_2 - (\frac{8+a}{3} + k_{33})e_3. \end{cases}$$

One can show that there exist a large amount of choices for selecting $K = [k_1 \ \ k_2 \ \ k_3]$ where $k_i = [k_{1i} \ \ k_{2i} \ \ k_{3i}]^T$ for all $i = 1, 2, 3$ such that the system (3.4) is globally asymptotically stable, see Table 1.

The fourth order Runge-Kutta method is used to solve the differential equations with time step size equal to 0.001 in all numerical simulation in this letter. The initial values of the error systems for unified chaotic systems are chosen, respectively as $(e_1(0), e_2(0), e_3(0)) = (12, -3, -14)$. We choose $a = 1$ for the coefficient mtrix A and the feedback matrix $K = \begin{pmatrix} -27.4 & 35.3 & -1.2 \\ -15.3 & 28.8 & -2.9 \\ 2.8 & 10.0 & -2.6 \end{pmatrix}$, by which all eigenvalues of the matrix $(A - BK)$ are negative real parts and consequently the system (3.4) is exponentially stable.

K	$A - BK$	The eigenvalues of $A - BK$
$\begin{pmatrix} -27.4 & 35.3 & -1.2 \\ -15.3 & 28.8 & -2.9 \\ 2.8 & 10.0 & -2.6 \end{pmatrix}$	$\begin{pmatrix} -7.6 & -0.3 & 1.2 \\ 8.3 & -0.8 & 2.9 \\ -2.8 & -10.0 & -0.4 \end{pmatrix}$	$-8.2083, -.2958 \pm 6.2726i$
$\begin{pmatrix} -31.4 & 35.2 & -2.2 \\ -7.5 & 28.1 & -0.6 \\ 8.8 & 10.0 & -2.4 \end{pmatrix}$	$\begin{pmatrix} -3.6 & -0.2 & 2.2 \\ 0.5 & -0.1 & 0.6 \\ -8.8 & -10.0 & -0.6 \end{pmatrix}$	$-1.4458 \pm 4.6773i, -1.4084$
$\begin{pmatrix} -30.4 & 35.5 & -3.2 \\ -7.5 & 28.5 & -0.3 \\ 10.8 & 20.0 & -2.5 \end{pmatrix}$	$\begin{pmatrix} -4.6 & -0.5 & 3.2 \\ 0.5 & -0.5 & 0.3 \\ -10.8 & -20.0 & -0.5 \end{pmatrix}$	$-1.8055 \pm 5.9345i, -1.9891$

Table 1.

Remark 1. We can see that the system (2.4) is an ordinarily and linear homogeneous-differential equation with the coefficient matrix $A-BK$. Therefore, the system (2.4) is globally asymptotically stable at the unique equilibrium point $e = 0$ if and only if all the eigenvalues of matrix $A-BK$ have the negative real part. Hence, the choice of K is very important (see Table 1). As for example, if we choose $K = \begin{pmatrix} -70 & 10 & -3 \\ -9 & 0 & 7 \\ 8 & -6 & 0 \end{pmatrix}$, then the eigenvalues of the matrix $A - BK$ are $\lambda_1 = 40.6023, \lambda_2 = 1.0583, \lambda_3 = 18.3395$, which has no negative real part and consequently, the system (2.4) is not exponentially stable, i.e, anti-synchronization between delay coupled unified chaotic systems (3.1) and (3.3) does not occur.

4 The LAS of delay coupled Lorenz-Stenflo systems

In this section, we study the LAS of two chaotic Lorenz-Stenflo [29] systems. The master system of LSS can be described as the following system of equations:

$$\begin{cases} \dot{x} = \sigma(y-x) + sv, \\ \dot{y} = rx - y - xz, \\ \dot{z} = xy - bz, \\ \dot{v} = -x - \sigma v, \end{cases} \tag{4.1}$$

where $\sigma > 0$, $r > 0$, $b > 0$ and $s > 0$ are the four parameters. The LSS exhibits chaotic dynamics with $\sigma = 1.0, r = 26.0, b = 0.7$ and $s = 1.5$. According to our scheme the slave system is

$$\begin{cases} \dot{x_1} = -\sigma(x_1 - y_1) + sv_1 - u_1, \\ \dot{y_1} = -y_1 - rx_1(t-\tau) + x_1(t-\tau)z_1(t-\tau) - u_2, \\ \dot{z_1} = -bz_1 - x_1(t-\tau)y_1(t-\tau) - u_3, \\ \dot{v_1} = -\sigma v_1 + x_1(t-\tau) - u_4, \end{cases} \tag{4.2}$$

where $K = \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{pmatrix}$ is real constant matrix and

$$u_1 = k_{11}(x_1 + x(t-\tau)) + k_{12}(y_1 + y(t-\tau)) + k_{13}(z_1 + z(t-\tau)) + k_{14}(v_1 + v(t-\tau)),$$
$$u_2 = k_{21}(x_1 + x(t-\tau)) + k_{22}(y_1 + y(t-\tau)) + k_{23}(z_1 + z(t-\tau)) + k_{24}(v_1 + v(t-\tau)),$$
$$u_3 = k_{31}(x_1 + x(t-\tau)) + k_{32}(y_1 + y(t-\tau)) + k_{33}(z_1 + z(t-\tau)) + k_{34}(v_1 + v(t-\tau)),$$
$$u_4 = k_{41}(x_1 + x(t-\tau)) + k_{42}(y_1 + y(t-\tau)) + k_{43}(z_1 + z(t-\tau)) + k_{44}(v_1 + v(t-\tau),$$

As the result of the system (4.1) and (4.2), the error system becomes

$$\begin{cases} \dot{e_1} = -(\sigma + k_{11})e_1 + (\sigma - k_{12})e_2 - k_{13}e_3 + (s - k_{14})e_4, \\ \dot{e_2} = -k_{21}e_1 - (1 + k_{22})e_2 - k_{23}e_3 - k_{24}e_4, \\ \dot{e_3} = -k_{31}e_1 - k_{32}e_2 - (b + k_{33})e_3 - k_{34}e_4, \\ \dot{e_4} = -k_{41}e_1 - k_{42}e_2 - k_{43}e_3 - (\sigma + k_{44})e_4. \end{cases} \tag{4.3}$$

There exist a large amount of choices for selecting $K = [k_1 \quad k_2 \quad k_3 \quad k_4]$ where $k_i = [k_{1i} \quad k_{2i} \quad k_{3i} \quad k_{4i}]^T$ for all $i = 1, 2, 3, 4$ such that the system (4.3) is globally asymptotically stable (see Table 2).

Numerical simulation are done by fourth order Runge-Kutta method. We assume that the initial values of error systems for Lorenz-Stenflo systems are chosen respectively as $(e_1(0), e_2(0), e_3(0), e_4(0)) = (12, -3, -14, -9)$. We choose the feedback matrix $K = \begin{pmatrix} 0.6 & -2.0 & 5.0 & -0.5 \\ 3.0 & 0.6 & -1.0 & 2.0 \\ -7.5 & -1.2 & 0.3 & -9.0 \\ -5.0 & -0.6 & 9.0 & 1.0 \end{pmatrix}$, by which all eigenvalues of the matrix $A - BK$ are negative real parts, and consequently the system (4.3) is exponentially stable. Simulation results are reported in Fig.4-Fig.7.

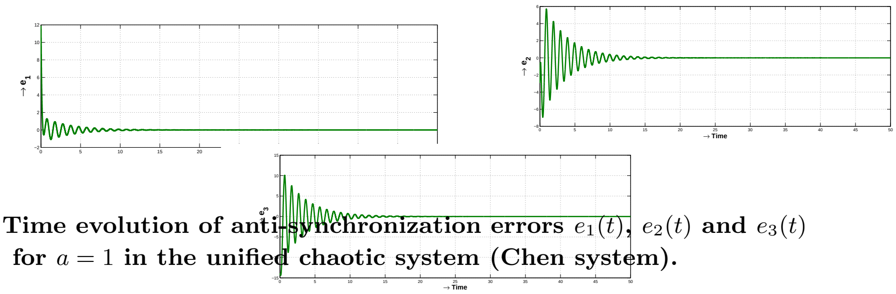

Fig. 1-3. Time evolution of anti-synchronization errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ for $a = 1$ in the unified chaotic system (Chen system).

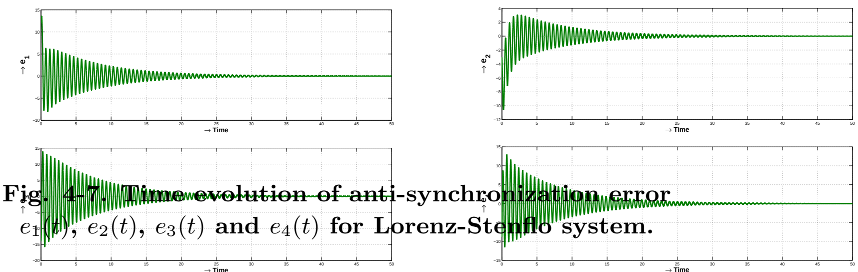

Fig. 4-7. Time evolution of anti-synchronization error $e_1(t)$, $e_2(t)$, $e_3(t)$ and $e_4(t)$ for Lorenz-Stenflo system.

The plots show that the error is going to zero asymptotically, i.e., the delay coupled Lorenz-Stenflo systems are anti-synchronized.

K	$A - BK$	The eigenvalues of $A - BK$
$\begin{pmatrix} 0.6 & -2.0 & 5.0 & -0.5 \\ 3.0 & 0.6 & -1.0 & 2.0 \\ -7.5 & -1.2 & 0.3 & -9.0 \\ -5.0 & -0.6 & 9.0 & 1.0 \end{pmatrix}$	$\begin{pmatrix} -1.6 & 3.0 & -5.0 & 2.0 \\ -3.0 & -1.6 & 1.0 & -2.0 \\ 7.5 & 1.2 & -1.0 & 9.0 \\ 5.0 & 0.6 & -9.0 & -2.0 \end{pmatrix}$	$-.129 \pm 11.1i, -4.364, -1.58$
$\begin{pmatrix} 6.6 & -2.0 & 0 & 1.5 \\ 3.0 & 0.6 & -1.0 & 2.0 \\ -0.5 & -2.2 & 0.3 & -9.0 \\ -1 & -0.6 & 9.0 & 1.0 \end{pmatrix}$	$\begin{pmatrix} -7.6 & 3.0 & 0.0 & 0.0 \\ -3.0 & -7.6 & 0 & 0 \\ 0 & 0 & -8.0 & 9.0 \\ 0 & 0 & -9.0 & -7.0 \end{pmatrix}$	$-7.6 \pm 3i. -7.5 \pm 8.9861i$
$\begin{pmatrix} 0.6 & -2.0 & 1.0 & 0.5 \\ 3.0 & 0.6 & -1.0 & 2.0 \\ -0.5 & -2.2 & 0.3 & -9.0 \\ -1.0 & -0.6 & 9.0 & 1.0 \end{pmatrix}$	$\begin{pmatrix} -1.6 & 3.0 & -1.0 & 1.0 \\ -3.0 & -1.6 & 1.0 & -2.0 \\ 0.5 & 2.2 & -1.0 & 9.0 \\ 1.0 & 0.6 & -9.0 & -2.0 \end{pmatrix}$	$-1.38 \pm 2.68i, -1.72 \pm 9.006i$

Table 2

Remark 2. The system (2.4) globally asymptotically stable at the unique equilibrium point $e = 0$ if and only if all the eigenvalues of matrix $A - BK$ have the negative real parts. There are large amount of choices for selecting K, such that the eigenvalues of some matrices of $A - BK$ have negative real parts and some are not. So choice of K is very important (see Table 2). As for example, if we choose $K = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ -2 & 0 & -1.7 & 0 \\ -2 & 0 & 0 & -2 \end{pmatrix}$, then the eigenvalues of the matrix $A - BK$ are

$$\lambda_1 = 1.0, \quad \lambda_2 = -4.244, \quad \lambda_3 = 6.244, \quad \lambda_4 = 1.0,$$

all of which are not in negative real parts and consequently, the system (4) is not exponentially stable, i.e, the anti-synchronization between delay coupled LSS (3.1) and (3.3) does not occur.

5 Conclusions

In this paper, a new scheme for LAS between delay coupled chaotic systems via a scalar signal is proposed. We have discussed our scheme taking coupled unified chaotic systems (UCS) as well as Lorenz-Stenflo systems (LSS). The novelty of this technique is the predictability. Knowing the behavior of the master system, we can predict the behavior of the slave system. Finally numerical simulation results show the effectiveness of our method. The potential application of LAS of delay coupled chaotic systems to channel independent chaotic secure communication.

References

[1] Pecora and Carroll, *Synchronization in chaotic systems*, Physical Review Letters 64 (1990), 821-824.

[2] E.Ott, C.Grebogi and J.A.York, *Controlling chaos*, Physical Review Letters 64 (1990), 1196-1199.

[3] S.Chen and J.Lu, *Synchronization of an uncertain unified chaotic system via adaptive control*, Chaos, Solitions and Fractals 14 (2002), 643-647.

[4] M.A. Khan, *Synchronization of different 3D chaotic systems by generalized active control*, Journal of Information and Computing Science, 7 (2012), 272-283.

[5] M.A. Khan, *Gradient based control and synchronization of Lorenz system with periodic parametric forcing*, Journal of Adv. Computer Science and Technology, 1 (2012), 222-231.

[6] Fuh CC, Tung PC, *Controlling chaos using differential geometric method*, Physical Review Letters, 75 (1995), 2952-2955.

[7] EN. Sanchez, JP. Perez, M.Martinez and G. Chen, *Chaos stabilization: an inverse optimal control approach*, Latin Am. Appl. Res:Int. J., 32 (2002), 111-114.

[8] S. Pal, B. Sahoo and S. Poria, *Generalized lag-synchronization of delay coupled chaotic systems via linear transformation*, Physica Scripta, 87 (2013), 045011-045018.

[9] A.Tarai(Poria), S.Poria and P.Chatterjee, *Synchronization of bidirectionally coupled chaotic Chen's system with delay*, Chaos, Solitions and Fractals, 41 (2009), 643-647.

[10] M.A. Khan and S.Poria, *Projective synchronization of bidirectionally coupled chaotic systems via linear transformations*, Int. Journal of Appl. Mathematical Research, 1 (2012), 541-548.

[11] M.A. Khan, *Adaptive synchronization of two coupled Newton-Leipnik system with uncertain parameter*, Int. Journal of Basic and Appl. Sci., 1 (2012), 439-447.

[12] M.A. Khan, *Hybrid synchronization of Lorenz-Stenflo system with delay*, Journal of Advanced Computer Science and Technology, 1 (2012), 240-249.

[13] N. Rulkov, M. Suschik and L. Tsimring, *Generalized synchronization of chaos directionally coupled chaotic system*, Phys. Rev. E. 51 (1995), 980-994.

[14] M.A. Khan and S. Poria, *Generalized synchronization of bidirectionally coupled chaotic systems*, Int. Journal of Appl. Mathematical Research, 1 (2012), 303-313.

[15] M.A. Khan, *Generalized synchronization of bidirectionally coupled chaotic systems via linear transformations*, Int. Journal of Basic and Applied Sciences, 1, (2012), 270-282.

[16] L. Kocarev and U. Parlitz, *Generalized synchronization, predictability and equivalence of unidirectionally coupled dynamical systems*, Physical Review Letters, 76, (1996), 1816-1819.

[17] T. Yang and LO. Chua, *Generalized synchronization of chaos via linear transformations*, Int. J. Bifurcation and Chaos, 9, (1999), 215-219.

[18] A. Tarai (Poria), S. Poria and P. Chatterjee, *Synchronization of generalized linearly bidirectionally coupled unified chaotic system*, Chaos, Solitions and Fractals, 40, (2009), 885-892.

[19] C. Kim, S.Rim, W. Kye, J. Rye and Y. Park, *Anti-synchronization of chaotic oscillator*, Physics Letters A, 320, (2003), 39-46.

[20] Y. Zhang and J. Sun, *Chaotic synchronization and anti-synchronization based on suitable separation*, Physics Letters A, 330 (2004), 442-447.

[21] J. Hu, S. Chen and L. Chen, *Adaptive control for anti-synchronization of Chua's chaotic system*, Physics Letters A, 339 (2005), 445-460.

[22] W.Li, X.chen and S.Zhiping, *Anti-synchronization of two different chaotic systems*, Physica A. 387 (2008), 3747-3750.

[23] M.Mossa Al-Sawalha and M.S.M.Noorani, *Anti-synchronization of two hyperchaotic systems via nonlinear control*, Commun Nonlinear Sci Numer Simulat. 14 (2009), 3402-3411.

[24] M.A. Khan, S.N.Pal and S. Poria, *Generalized anti-synchronization of different chaotic systems*, Int. J. of Applied Mechanics and Engineering. 17 (2012), 83-99.

[25] EN. Lorenz, *Deterministic non-periodic flows*, J. of Atm. Sci. 20 (1963), 130-141.

[26] GR. Chen and T. Ueta, *Yet another chaotic attractor*, Int. J. Bifurcat. Chaos. 9 (1999), 1465-1466.

[27] J. Lu, G. Chen, D. chen and S. Celikovsky, *Bridge the gap between the Lorenz system and the Chen system*, Int. J. Bifurcat. Chaos. 12 (2002), 2917C26.

[28] J.Lu and X.Lu, *A unified chaotic system with continuous periodic switch*, Chaos, Solitions and Fractals, 20 (2004), 245C251.

[29] L. Stenflo, *Generalized Lorenz equation for acoustic-gravity waves in the atmosphere*, Phys. Scr., 53 (1996), 83-86.

Author's address:

Mohammad Ali Khan
Department of Mathematics, Ramananda College,
Bishnupur, Bankura, West Bengal, India.
E-mail: mdmaths@gmail.com