Myller configurations in Finsler spaces.
Applications to the study of torse forming vector fields

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Abstract. In this article we define the Myller configurations in Finsler spaces and the new notion of torse forming vector fields in the sense of Myller in a Myller configuration, with respect to the Cartan Finsler connection. We apply the results to study vector fields tangent to a given Finsler submanifold, which are torse forming vector fields with respect to the induced Finsler connection.


Key words: torse forming vector fields, Myller configuration.

1 Preliminaries and notations

All the geometric objects used in this material are of class $C^\infty$.

Let $M$ be a real, differentiable manifold of dimension $n$, $(TM, \pi, M)$ the tangent bundle. We denote by $(\tilde{TM}, \tilde{\pi}, M)$ the vector bundle of non vanishing vectors, tangent to $M$, where $\tilde{\pi} = \pi/\tilde{T}M$. Let $(\pi^*TM, \pi^*, \tilde{T}M)$ be the pull-back bundle of the tangent bundle by $\tilde{\pi}$, and $\Gamma(\pi^*TM)$ its $\mathcal{F}(\tilde{T}M)$-module of sections. In this material the sections of the pull-back bundle will be called $\tilde{\pi}$-vector fields [9]. We consider local system of coordinates on $M$, $TM$ and $\pi^*TM$, denoted like usual by $(x^i)_{i \in 1,n}$, $(\tilde{x}, y^i)_{i \in 1,n}$ and respectively $(\tilde{x}^i)_{i \in 1,n}$.

Any local section of the tangent bundle determines a local section of the pullback bundle: for any $X \in \chi(U)$, let $\tilde{X} \in \tilde{\Gamma}(\tilde{T}^{-1}(U), \pi^*TM)$ be defined by: $\tilde{X}(\tilde{x}) = (\tilde{x}, X(\tilde{x}))$, $\forall \tilde{x} \in \tilde{T}^{-1}(U)$. $X$ is the lift of the vector field $X$ on $M$ to a local section of $\pi^*TM$ and is called a $\tilde{\pi}$ - vector field. Particularly, $(\frac{\partial}{\partial x^i})_{i \in 1,n}$ is a local basis in $\Gamma(\tilde{T}^{-1}(U), \pi^*TM)$.

We suppose that $F^n = (M, F(x, y))$ is a Finsler space:[1] It means a pair $F^n = (M, F(x, y))$, where $F : TM \rightarrow \mathbb{R}$ is a scalar function such that: 1) $F(x, y)$ is differentiable on $\tilde{T}M$ and continuous on the null section; 2) $F(x, y) > 0$ on $TM$; 3) $F$ is positively homogeneous of order 1 on the fibers of the tangent bundle: $F(x, \lambda y) = \lambda F(x, y)$, $\forall \lambda > 0$. 4) the distinguished tensor field $g_{ij}(x, y) = \frac{1}{2} \frac{\partial^2 F^2}{\partial y^i \partial y^j}(x, y)$ is positively defined $\Rightarrow \text{rank}[g_{ij}(x, y)] = n$ on $\tilde{T}M$. 

The fundamental tensor \( g_{ij}(x,y) \) determines a Riemannian natural metric on \( \pi^*TM : \tilde{g} = g_{ij}dx^idj \in \Gamma(\otimes^2 \pi^*TM) \), where \( \{dx^i\} \) is the basis in \( \pi^*TM \), dual to \( \{\frac{\partial}{\partial x^i}\} \). In this paper we will work with the Cartan Finsler connection on \( F^n \). We also use the next morphism of vector bundle: \( \rho : T(\tilde{T}M) \rightarrow \pi^*TM \), \( \rho(\tilde{X}) = (\pi_{\tilde{T}M}(\tilde{X}), d\tilde{\pi}(\tilde{X})) \), that induces a morphism between the \( \mathcal{F}(\tilde{T}M) \)-modules of sections, denoted also by \( \rho \) and called the lift of a vector field on \( TM \) to a \( \tilde{\pi} \)-vector field.

## 2 Myller configurations \( \mathcal{M}(\tilde{C}, \tilde{\xi}_1, \tilde{T}^m) \) and torse forming vector fields in the sense of Myller

In this section we generalize the notion of Myller configuration from Riemann spaces [6] to Finsler spaces. The first generalisations to Finsler spaces are made by Khu Quoc Anh [5]. Here, we introduce in a new way the Myller configurations in a Finsler space. For a more detailed study of Myller configurations in Finsler spaces please consult [3].

Let \( C \) be a regular curve on \( \tilde{T}M \), differentiable of class \( C^\infty \), locally given by \( x^i = x_i^s(s), y^r = y^r(s) \), with \( s \) the arc-length parameter of the projection \( C = \pi \circ C \).

We consider the orthogonal complement (with respect to \( \pi \)) of the linear subspace \( \tilde{T}^m(\tilde{C}(s)) \subset \pi^*_{\tilde{C}(s)}TM \), and we call it a \( \tilde{\pi} \)-versor field from the given distribution:

\[
(\tilde{C}, \tilde{\xi}_1) \text{ a } \tilde{\pi} \text{- versor field from the given distribution:} \quad \tilde{\xi}_1(\tilde{C}(s)) \in \tilde{T}^m(\tilde{C}(s)), \; \forall s \in I,
\]

\[
(2.1) \quad \tilde{T}^m : \tilde{C}(s) \rightarrow \tilde{T}^m(\tilde{C}(s)) \subset \pi^*_{\tilde{C}(s)}TM
\]

So, we obtain the triplet \( \mathcal{M}(\tilde{C}, \tilde{\xi}_1, \tilde{T}^m) \) and we call it a Myller configuration in the Finsler space \( F^n \).

The case \( m = n - 1 \) is treated in [4].

Let \( FC = (HTM, \nabla) \) be the Cartan Finsler connection of \( F^n \) and \( \frac{\nabla}{ds} \) the operator of covariant differentiation along \( \tilde{C} \).

For any \( s \in I \), we consider the orthogonal complement (with respect to \( \tilde{g}(\tilde{C}(s)) \)) of the linear subspace \( \tilde{T}^m(\tilde{C}(s)) \subset \pi^*_{\tilde{C}(s)}TM \), and we denote it by \( \tilde{T}^p(\tilde{C}(s)) \), \( p = n - m \).

We decompose \( \frac{\nabla}{ds}(\tilde{C}(s)) \) in \( \tilde{T}^m(\tilde{C}(s)) \oplus \tilde{T}^p(\tilde{C}(s)) \), putting in evidence the lengths and the versors of the sections:

\[
(2.2) \quad \tilde{\xi}_1 = \xi_1 \left( \frac{\partial}{\partial x^1} \right), \quad \tilde{g}(\tilde{\xi}_1, \tilde{\xi}_1) = g_{ij}\xi_1^j \xi_1^j = 1.
\]

We also consider a regular distribution of class \( C^\infty \) and dimension \( m \), \( 1 < m < n - 1 \), restricted to \( \tilde{C} \):

\[
(2.3) \quad \frac{\nabla}{ds}(\tilde{C}(s)) = G_1(\tilde{C}(s))\tilde{\eta}_2(\tilde{C}(s)) + N_{11}(\tilde{C}(s))\tilde{n}_1(\tilde{C}(s)), \; s \in I, \quad \tilde{g}(\tilde{\eta}_2, \tilde{\eta}_2) = \tilde{g}(\tilde{n}_1, \tilde{n}_1) = 1, \quad \tilde{g}(\tilde{\xi}_1, \tilde{\eta}_2) = \tilde{g}(\tilde{\xi}_1, \tilde{n}_1) = \tilde{g}(\tilde{\eta}_2, \tilde{n}_1) = 0, \quad G_1 \geq 0 \text{ for } m \geq 3 \text{ and } N_{11} \geq 0 \text{ for } p \geq 2.
\]
The fundamental formulae of the \( \tilde{\pi} \) versor field \((\tilde{C}, \xi_1)\) in the Myller configuration \( \mathcal{M}(\tilde{C}, \xi_1, \tilde{T}^m) \) of the Finsler space \( F^n \) are:

\[
\frac{\nabla \tilde{\eta}_a}{ds} = -G_{a-1} \tilde{\eta}_{a-1} + G_a \tilde{\eta}_a + \sum_{\beta=1}^{p} N_{a\beta} \tilde{\eta}_\beta, \quad a \in \overline{1,m}, \quad \tilde{\eta}_1 = \xi_1,
\]

\[
\frac{\nabla \tilde{\eta}_a}{ds} = -\sum_{b=1}^{m} N_{ba} \tilde{\eta}_b + \sum_{\beta=1}^{p} M_{a\beta} \tilde{\eta}_\beta, \quad \alpha \in \overline{1,p},
\]

and its invariants verify:

for \( p \leq m \):

\[
G_a > 0, \quad a \in \overline{1,m-2}, \quad G_0 = G_m = 0,
\]

\[
N_{a\alpha} > 0, \quad a \in \overline{1,p-1}, \quad N_{a\beta} = 0, \quad a < \beta,
\]

\[
M_{a\beta} + M_{\beta a} = 0,
\]

for \( p > m \):

\[
G_a > 0, \quad a \in \overline{1,m-1}, \quad G_0 = G_m = 0,
\]

\[
N_{a\alpha} > 0, \quad a \in \overline{1,m}, \quad N_{a\beta} = 0, \quad a < \beta,
\]

\[
M_{a\beta} + M_{\beta a} = 0, \quad M_{i,m+i} > 0, \quad i \in \overline{1,m-1}, \quad M_{i,m+j} = 0, \quad j > i.
\]

We obtained an orthonormal, positively orientated frame along \( \tilde{C} \):

\[
\mathcal{R}_M = \{ \tilde{\eta}_1, \cdots, \tilde{\eta}_m, \tilde{n}_1, \cdots, \tilde{n}_p \}, \quad \tilde{\eta}_1 = \xi_1,
\]

and \( G_a, N_{a\beta}, M_{a\beta} \), some functions that are invariant at changes of coordinates on \( \pi^* TM \) and at changes of natural parameter \( s \rightarrow s + a \) of \( C \). The sections of the frame \( \mathcal{R}_M \) are geometrically associated to \( \xi_1 \) in \( \mathcal{M}(\tilde{C}, \xi_1, \tilde{T}^m) \).

Another invariants are given by the coordinates of \( \rho(\frac{d\tilde{C}}{ds}) \) in \( \mathcal{R}_M \):

\[
\rho(\frac{d\tilde{C}}{ds}) = b^1 \tilde{\xi}_1 + \cdots + b^p \tilde{n}_p,
\]
We call $\bar{\eta}_a$ the geodesic $\bar{\pi}$-versor field of rank $a$ of $\xi_1$ in $\mathcal{M}(\bar{C}, \xi_1, \bar{T}^m)$, span{\bar{\eta}_1, \ldots, \bar{\eta}_a} - the geodesic space of rank $a$, $\bar{n}_a$ - the $\bar{\pi}$-normal versor field of rank $\alpha$, span{\bar{n}_1, \ldots, \bar{n}_a} - the normal space of rank $\alpha$.

The invariants of the Myller configurations are called: $G_a$ - the geodesic curvature of rank $a$, $N_{11}$ - the normal curvature of the $\bar{\pi}$-versor field $\xi_1$ in $\mathcal{M}(\bar{C}, \xi_1, \bar{T}^m)$.

**Theorem 2.2.** (fundamental) Let

$$ G_a, N_{\alpha \beta}, M_{\alpha \beta}, a \in \bar{1}, m, \alpha, \beta \in \bar{1}, p, b^1, \ldots, b^n, $$

be some continuous functions of parameter $s \in I$, satisfying the conditions $\sum_{i=1}^n (b^i)^2 = 1$, (2.7) or (2.8) and let

$$ R_0 = \{\bar{\eta}_0^1, \ldots, \bar{\eta}_0^m, \bar{n}_0^1, \ldots, \bar{n}_0^p\} $$

be an $\bar{\eta}$-orthonormal, positively orientated frame in $\pi_{\bar{\pi}}^* \bar{T} \bar{M}$, $\bar{x}_0 \in \bar{T} \bar{M}$. Then, there is in a neighborhood of $\bar{x}_0$ an unique curve $C: x^i = x^i(s)$ on $\bar{M}$, such that $s$ is its arc-length parameter, there is an unique horizontal curve $\bar{C}$ on $\bar{T} \bar{M}$ with $\pi \circ \bar{C} = C$, there is an unique regular distribution $\bar{T}^m$ of dimension $m$ restricted to $\bar{C}$ and an unique $\bar{\pi}$-versor field $\xi_1$ from this distribution, such that the invariants of $\xi_1$ in $\mathcal{M}(\bar{C}, \xi_1, \bar{T}^m)$ are exactly the given functions $G_a, N_{\alpha \beta}, M_{\alpha \beta}$ and the following initial conditions are satisfied:

$$ \bar{C}(s_0) = \bar{x}_0, \bar{\eta}_a(s_0) = \bar{\eta}_{0a}, \alpha \in \bar{1}, m, \bar{n}_a(s_0) = \bar{n}_{0\alpha}, \alpha \in \bar{1}, p. $$

**Remark 2.1.** We imposed the condition of horizontality to obtain the uniqueness of $\bar{C}$.

## 3 Torse forming versor / vector fields in the sense of Myller from $\bar{T}^m$

In this section we study a new notion, introduced by Al. Myller for 3 dimensional Euclidian case and extended by R. Miron to Riemannian spaces.

**Definition 3.2.**[8] 1) $\bar{X} \in \bar{T}^m$ is a torse forming $\bar{\pi}$-vector field in the sense of Myller if

$$ \nabla_{\bar{X}}(s) = \alpha(s)\rho \left( \frac{d\bar{C}}{ds} \right) + \beta(s)\bar{X}(s) + \gamma(s)\bar{n}(s), s \in I, $$

where $\alpha, \beta, \gamma \in \mathcal{F}(\bar{T} \bar{M})$, restricted to $\bar{C}$ and $\bar{n} \in \bar{T}^p$ is a $\bar{\pi}$-vector field normal to the Myller configuration $\mathcal{M}(\bar{C}, \xi_1, \bar{T}^m)$.

2) A $\bar{\pi}$-versor field $(\bar{C}, \xi_1)$ is called a torse forming versor field in the sense of Myller if there is a $\bar{\pi}$-vector field $\bar{X}(s) = \lambda(s)\xi_1(s)$, torse forming in the sense of Myller. $(\bar{C}, \xi_1)$ is named concurrent in the sense of Myller if $\alpha = \text{cst.} \neq 0$ and $\beta = 0$, recurrent in the sense of Myller if $\alpha = 0$ and parallel in the sense of Myller if $\alpha = 0$ and $\beta = 0$.  

\[(2.10) \sum_{i=1}^n (b^i)^2 = 1.\]
Theorem 3.3. \((\tilde{C}, \tilde{\xi}_1)\) with \(G_1 > 0\) is a torse forming versor field in the sense of Myller \((\alpha \neq 0)\) if and only if
\[
(3.1) \quad b^3 = \cdots = b^m = 0.
\]

Proof: We suppose that \(\alpha = 1\) and that there exists a vector field \(\overline{X}(s) = \lambda(s)\overline{\xi}_1(s)\), some real, differentiable functions \(\beta, \gamma\) defined on \(\tilde{C}\) and a versor field \(\overline{\pi} \in \tilde{T_p}\), such that
\[
(3.2) \quad \frac{d\lambda}{ds}(s) + \lambda(s)\nabla_\overline{\xi}_1(s) = \rho\left(\frac{d\tilde{C}}{ds}\right) + \beta(s)\lambda(s)\overline{\xi}_1(s) + \gamma(s)\overline{\pi}(s).
\]
Replacing \(\nabla \overline{\xi}_1\) and \(\rho\left(\frac{d\tilde{C}}{ds}\right)\) from the fundamental equations, we get the system:
\[
(3.3) \quad \left\{ \begin{array}{l}
\phantom{=} b^3 = b^4 = \cdots = b^m, \\
\phantom{=} \frac{d\lambda}{ds} = b^1 + \beta\lambda, \\
\phantom{=} G_1\lambda = b^2.
\end{array} \right.
\]
So, \(b^3 = b^4 = \cdots = b^m\) are necessary conditions for \(\tilde{\xi}_1\) to be a torse forming versor field in the sense of Myller. For the converse statement, \(G_1 > 0\) implies \(b^2 \neq 0\). We consider \(\overline{X} = \frac{\lambda}{b^2}\overline{\xi}_1\). Then, there exists the functions \(\alpha = 1, \beta = \frac{G_2}{b^2}(\frac{d\lambda}{ds} - b^1)\), such that the definition 3.2 is satisfied. We obtained the next result:

For concurrent \(\tilde{\pi}\)-versor fields we obtain a result similar with one of Professor’s Miron [7]. The proof is similar with the former one.

Theorem 3.4. \((\tilde{C}, \tilde{\xi}_1)\) with \(G_1 > 0\) is a concurrent versor field in the sense of Myller \(\iff\)
\[
(3.3) \quad \left\{ \begin{array}{l}
\phantom{=} b^3 = b^4 = \cdots = b^m = 0, \\
\phantom{=} \frac{d\lambda}{ds}(\frac{d\xi_1}{ds}) = b^1.
\end{array} \right.
\]
And for parallelism:

Theorem 3.5. The \(\overline{\pi}\)-versor field \((\tilde{C}, \tilde{\xi}_1)\) is parallel in the sense of Myller \(\iff G_1 = 0\).

Applying these characterizations and the fundamental theorem, we formulate theorems of existence and uniqueness. Next, we can study the case of vector fields:

Theorem 3.6. A \(\overline{\pi}\)-vector field
\[
\tilde{X} = \sum_{a=1}^{m} X^a \overline{\eta}_a
\]
is a torse forming vector field in the sense of Myller if and only if its components in \(\mathcal{R}_M\) verify the next system of differentiable equations:
\[
(3.4) \quad \frac{dX^a}{ds} = G_a X^{a+1} + G_{a-1} X^{a-1} = \alpha b^a + \beta X^a,
\]
with \(a \in \overline{1, m}\), \(G_0 = G_m = 0\) and \(\alpha, \beta\) functions of class \(C^\infty\) on \(\tilde{T\overline{M}}\), restricted to \(\tilde{C}\).
The next theorem of existence and uniqueness is a consequence of the existence and uniqueness of the solutions of the system above:

**Theorem 3.7.** Let \( M(\tilde{C}, \tilde{\xi}_1, T^m) \) be a Myller configuration, \( \alpha, \beta \in \mathcal{F}(TM) \) some differentiable functions restricted to \( \tilde{C} \) and \( X_0 \in T^m(\tilde{C}(s_0)) \). Then, there is an unique torse forming \( \tilde{\pi} \)-vector field \( \tilde{X} \) from \( T^m \) which satisfies \( \tilde{X}(s_0) = X_0 \).

All these results can be particularized for concurrent, recurrent and parallel vector fields. We also can introduce the parallel transport in the sense of Myller of vector fields from \( T^m \) and prove:

**Theorem 3.8.** The parallel transport in the sense of Myller of \( \tilde{\pi} \)-vector fields from \( T^m \) preserves the length of vectors and the angle between them.

**Proof** Multiplying the equations of the former system with \( X^a \), respectively, we find that \( \frac{d}{ds} \tilde{\pi}(X, X) = 0 \). Here, by "the angle" of vectors we understand not the usual angle used in Finsler geometry but the formal generalisation \( \cos \theta = \frac{\tilde{\pi}(X, Y)}{\tilde{\pi}(X, X)\tilde{\pi}(Y, Y)} \).

A similar study can be made for \( \tilde{\pi} \)-torse forming versor/vector fields from \( T^p \). The entire theory of torse forming vector fields in the sense of Myller is presented in an article in preparation.

## 4 Applications to the study of vector fields tangent or normal to a Finsler submanifold

We consider an differentiable immersion of class \( C^\infty \) \( j : \tilde{M} \to M \) of a \( m \) dimensional submanifold \( \tilde{M} \) in \( M \). Locally, \( j \) is an embedding. It is known [2] that the Finsler structure of \( M \) induces on the given submanifold a Finsler structure. Let \( CT = (\nabla, HTM) \) be the Cartan Finsler connection of \( F^m \). We remember that \( HTM \) is the Cartan nonlinear connection on \( TM \). We denote by \( CF = (\nabla, HTM) \) the induced Finsler connection on the given submanifold, and by \( CF^\perp = (\nabla^\perp, HTM) \) the Finsler connection induced on the normal bundle. \( HTM \) is the nonlinear connection induced on \( TM \) by the Cartan nonlinear connection \( HTM \).

Now, we’ll associate to the Finsler submanifold \( M \) a special Myller configuration. Let \( C \) be a regular curve of class \( C^\infty \) on \( \tilde{M} \), locally given by \( C : s \to u^a(s), s \in I, \) with \( s \) the arc-length parameter. We consider the next family of linear spaces along \( C : C(s) \to T_{C(s)}\tilde{M} := T^m(C(s)) \). Let \( \xi_1 \) a vector field along \( C \), tangent to \( \tilde{M} \), \( \xi_1(s) \in T^m(C(s)), s \in I \). The canonic lift \( \tilde{C} \) of the curve \( C \) to \( \tilde{TM} \) is: \( \tilde{C} : s \to (x^i(s), y^i(s) = \frac{dx^i}{ds}(s)), s \in I \) and \( \tilde{\xi}_1 \), the lift of \( \xi_1 \) to a section of \( \pi^*TM \) is: \( \tilde{\xi}_1(\tilde{C}(s)) = (\tilde{C}(s), \xi_1(C(s))) \). We consider the case when \( \tilde{\xi}_1 \) is a \( \tilde{\pi} \)-versor field along \( \tilde{C} \) \( \Leftrightarrow \tilde{g}(\tilde{\xi}_1, \tilde{\xi}_1) = 1 \).

We also consider \( T^m : \tilde{C}(s) \to \{ \tilde{C}(s) \} \times T_{C(s)}\tilde{M} = (\pi^*TM)_{C(s)}, s \in I \). So, we defined a Myller configuration \( M_{\tilde{C}}(\tilde{C}, \tilde{\xi}_1, T^m) \) geometrically associated to the Finsler subspaces \( F^m = (\tilde{M}, \tilde{F}) \). Its invariants will be called the invariants of the vector field \( \xi_1 \) along \( C \) in the Finsler subspace \( F^m \).
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Theorem 4.1. ([9, 2]) The next Gauss-Weingarten formulae hold good:

\[ \nabla_{\tilde{X}}\tilde{Y} = \nabla_{\tilde{X}}\tilde{Y} + \tilde{H}(\tilde{X}, \tilde{Y}), \quad \tilde{X} \in \chi(\tilde{TM}), \quad \tilde{Y} \in \Gamma(\pi^*TM), \]  
\[ (4.1) \]

\[ \nabla_{\tilde{n}}\tilde{\xi} = -\tilde{B}_{\tilde{n}}\tilde{X} + \nabla_{\tilde{X}}\nabla_{\tilde{n}}\tilde{\xi}, \quad \tilde{X} \in \chi(\tilde{TM}), \quad \tilde{\xi} \in \Gamma(\tilde{N}). \]

\( \nabla \) is the induced connection on \( \pi^*TM \) by the connection \( \nabla \) on \( \pi^*TM \) (we know that it is metrical but it is not the Cartan connection of the subspace \( F^m = (M, F(u, v)) \), 

\( \tilde{H} \) - the second fundamental form of the Finsler subspace \( \tilde{M} \), 

\( \tilde{B}_{\tilde{n}} \) - the Weingarten operator associated to the normal \( \tilde{n} \)-vector field \( \tilde{\xi} \), 

\( \nabla^\perp \) - the normal connection induced on the normal bundle \( N \).

Using the Gauss-Weingarten formulae and the results of the former section, the next theorem can be easily obtained:

Theorem 4.2. A \( \tilde{n} \)-vector field \( \tilde{X} \) along a curve on \( \tilde{TM} \), tangent to \( \tilde{M} \), is a torse forming (concurrent / parallel) \( \tilde{n} \)-vector field in the sense of Myller in \( M \) with respect to the Cartan connection \( FC \) if and only if it is a torse forming (concurrent / parallel) \( \tilde{n} \)-vector field with respect to the induced Finsler connection \( FC = (\nabla, HTM) \).

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