Spin 1/2 particle with two mass parameters in external Coulomb field

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We consider the known equation [1-4] for a spin 1/2 particle with two mass parameters in presence of external Coulomb field

$$\left[\gamma^0\left(i\partial_t - \frac{\alpha}{r}\right) + i\gamma^3\partial_r + \frac{1}{r}\Sigma_{\theta\phi} - M_1 + i\frac{\beta_1}{r^2}\gamma^0\gamma^3\right]\Psi_1 - i\frac{\alpha_1}{r^2}\gamma^0\gamma^3\Psi_2 = 0,$$

$$\left[\gamma^0\left(i\partial_t - \frac{\alpha}{r}\right) + i\gamma^3\partial_r + \frac{1}{r}\Sigma_{\theta\phi} - M_2 - i\frac{\alpha_2}{r^2}\gamma^0\gamma^3\right]\Psi_2 + i\frac{\beta_2}{r^2}\gamma^0\gamma^3\Psi_1 = 0,$$

$$\Sigma_{\theta,\phi} = i\gamma^1\partial_\theta + \gamma^2\frac{i\partial_\phi + i\sigma^{12}\cos\theta}{\sin\theta},$$

where masses are parametrized as (θ belongs to interval (0, pi /2))

$$M_1 = \frac{2M}{1+\cos\theta}, \quad M_2 = \frac{2M}{1-\cos\theta}.$$

Other involved parameters are parametrized as follows:

$$\alpha_1 = -e^2 \frac{1}{3} \frac{(1-\cos\gamma)(-\cos\gamma\sqrt{12-3\cos^2\gamma} + \cos^2\gamma + 2)}{M\cos\gamma(1+\cos\gamma)},$$

$$\alpha_2 = e^2 \frac{2}{3} \frac{\sin^2\gamma}{M\cos\gamma},$$

$$\beta_1 = -e^2 \frac{2}{3} \frac{\sin^2\gamma}{M\cos\gamma} < 0,$$

$$\beta_2 = -\frac{1}{3} \frac{e^2(1+\cos\gamma)(\cos\gamma\sqrt{12-3\cos^2\gamma} + \cos^2\gamma + 2)}{M\cos\gamma(\cos\gamma - 1)} > 0;$$

we note the identities $\alpha_2 = -\beta_1$, $\alpha_1\beta_2 = -\beta_1^{\ 2}$.

We search for solutions with quantum numbers ε, j, m (assuming the diagonalization of the operator $i\partial_t, \vec{J}^2, J_3$; $D^{j}_{-m,\sigma}(\phi,\theta,0)$) - which stand for the Wigner functions; note that $j = 1/2, 3/2, ...; m = -j, ..., +j$

$$\Psi_1(x) = \frac{e^{-i\varepsilon t}}{r}\begin{vmatrix} f_1(r)D_{-1/2} \\ f_2(r)D_{+1/2} \\ f_3(r)D_{-1/2} \\ f_4(r)D_{+1/2} \end{vmatrix}, \quad \Psi_2(x) = \frac{e^{-i\varepsilon t}}{r}\begin{vmatrix} g_1(r)D_{-1/2} \\ g_2(r)D_{+1/2} \\ g_3(r)D_{-1/2} \\ g_4(r)D_{+1/2} \end{vmatrix}.$$

Using the Dirac matrices in spinor basis, we derive 8 radial equations. This system is consistent with the constraints (they follow from the diagonalization of the spatial reflection operator)

$$f_3 = \delta f_2, \quad f_4 = \delta f_1, \quad \delta = \pm 1, \quad g_3 = \delta g_2, \quad g_4 = \delta g_1, \quad \delta = \pm 1,$$

so we derive two systems of 4 equations each (depending on $\delta = +1, -1$).

We will use the new combinations of functions

$$f = (f_2 + f_1), \quad F = i(f_2 - f_1); \quad g = (g_2 + g_1), \quad G = i(g_2 - g_1).$$

It is convenient to study the cases of different parities, separately:

$\delta = +1$,

$$\left(\frac{d}{dr} - \frac{\nu}{r} + \frac{\beta_1}{r^2}\right)F - \left(\varepsilon + \frac{\alpha}{r} - M_1\right)f - \frac{\alpha_1}{r^2}G = 0,$$
$$\left(\frac{d}{dr} + \frac{\nu}{r} - \frac{\beta_1}{r^2}\right)f + \left(\varepsilon + \frac{\alpha}{r} + M_1\right)F + \frac{\alpha_1}{r^2}g = 0,$$
$$\left(\frac{d}{dr} - \frac{\nu}{r} - \frac{\alpha_2}{r^2}\right)G - \left(\varepsilon + \frac{\alpha}{r} - M_2\right)g + \frac{\beta_2}{r^2}F = 0,$$
$$\left(\frac{d}{dr} + \frac{\nu}{r} + \frac{\alpha_2}{r^2}\right)g + \left(\varepsilon + \frac{\alpha}{r} + M_2\right)G - \frac{\beta_2}{r^2}f = 0; \qquad (1)$$

The systems for states with opposite parities relate by symmetry:

$$M_1, M_2, \alpha_1, \alpha_2, \beta_1, \beta_2 \quad \Rightarrow \quad -M_1, -M_2, -\alpha_1, -\alpha_2, -\beta_1, -\beta_2.$$

It suffices to follow in detail only the case of parity $\delta = +1$. With the help of the first two equations in (1), we can exclude the variables $G(r)$ and $g(r)$; in this way we derive the system of 2-nd order for functions f, G. Because their explicit form is rather complicated, we write down only their general structure (we temporarily use the notation $M_1 - M_2 = M$):

$$\left[\frac{d^2}{dr^2} + \left(\frac{a_1}{r} + \frac{a_2}{r^2} \right) \frac{d}{dr} + b + \frac{b_1}{r} + \ldots \frac{b_4}{r^4} \right] f + \left(M \frac{d}{dr} + \frac{C_1}{r} + \frac{C_2}{r^2} + \frac{C_3}{r^3} \right) F = 0 , \quad (2)$$

$$\left[\frac{d^2}{dr^2} + \left(\frac{A_1}{r} + \frac{A_2}{r^2} \right) \frac{d}{dr} + B + \frac{B_1}{r} + \ldots \frac{B_4}{r^4} \right] F + \left(M \frac{d}{dr} f + \frac{c_1}{r} + \frac{c_2}{r^2} + \frac{c_3}{r^3} \right) f = 0. \quad (3)$$

The remaining functions $g(r), G(r)$ can be expressed through $f(r)$ and $F(r)$.

Let us describe the method of deriving 4-th order equations from equations (2)–(3). First, in order to simplify the structure of equations, we introduce special multipliers of F and f :

$$F = \bar{F}, \quad \left(M \frac{d}{dr} + \frac{C_1}{r} + \frac{C_2}{r^2} + \frac{C_3}{r^3} \right) \bar{F} = M \frac{d}{dr} \bar{F}, \quad = x^{\frac{-C_1}{M}} e^{\frac{C_2}{Mx}} e^{\frac{C_3}{2Mx^2}},$$

$$f = \varphi \bar{f}, \quad \left(M \frac{d}{dr} + \frac{c_1}{r} + \frac{c_2}{r^2} + \frac{c_3}{r^3} \right) \varphi \bar{F} = \varphi M \frac{d}{dr} \bar{F}, \quad \varphi = x^{\frac{-c_1}{M}} e^{\frac{c_2}{Mx}} e^{\frac{c_3}{2Mx^2}} .$$

Correspondingly, equations (2)–(3) take the form:

$$\frac{1}{M} \left[\frac{d^2}{dr^2} + \left(\frac{a_1}{r} + \frac{a_2}{r^2} \right) \frac{d}{dr} + b + \frac{b_1}{r} + \ldots \frac{b_4}{r^4} \right] \varphi \bar{f} + \frac{d}{dr} \bar{F} = 0,$$

$$\frac{1}{\varphi M} \left[\frac{d^2}{dr^2} + \left(\frac{A_1}{r} + \frac{A_2}{r^2} \right) \frac{d}{dr} + B + \frac{B_1}{r} + \ldots \frac{B_4}{r^4} \right] \bar{F} + \frac{d}{dr} \bar{f} = 0 .$$

With the use of the temporary notations $\bar{f}(r) = f_1(r), \bar{F}(r) = -f_2(r)$, we can transform our equations to the more symmetric form:

$$\left[K_2(x)\frac{d^2}{dx^2}+K_1(x)\frac{d}{dx}+K_0(x)\right]f_1=\frac{df_2}{dx},$$

$$\left[L_2(x)\frac{d^2}{dx^2}+L_1(x)\frac{d}{dx}+L_0(x)\right]f_2=\frac{df_1}{dx}.$$

Let us exclude the variable f_2:

$$f_2(x)=\int\left[K_2(x)\frac{d^2}{dx^2}+K_1(x)\frac{d}{dx}+K_0(x)\right]f_1,$$

$$\left(L_2\frac{d}{dx}+L_1\right)\left(K_2\frac{d^2}{dx^2}+K_1\frac{d}{dx}+K_0\right)f_1+L_0\int dx\left(K_2\frac{d^2}{dx^2}+K_1\frac{d}{dx}+K_0\right)f_1=0.$$

The second equation should by divided by $L_0(x)$, and then differentiation provides us with the following 4-th order equation for $f_1(x)$:

$$\left\{ \frac{d}{dx}\left(\frac{L_2}{L_0}\frac{d}{dx} + \frac{L_1}{L_0} \right)\left(K_2\frac{d^2}{dx^2} + K_1\frac{d}{dx} + K_0 \right) + \left(K_2\frac{d^2}{dx^2} + K_1\frac{d}{dx} + K_0 \right) \right\} f_1(x) = 0.$$

Similarly, we derive the 4-th order equation for f_2:

$$\left\{ \frac{d}{dx}\left(\frac{K_2}{K_0}\frac{d}{dx} + \frac{K_1}{K_0} \right)\left(L_2\frac{d^2}{dx^2} + L_1\frac{d}{dx} + L_0 \right) + \left(L_2\frac{d^2}{dx^2} + L_1\frac{d}{dx} + L_0 \right) \right\} f_2(x) = 0.$$

Omitting all the details of calculation, we write down the general structure of the 4-th order equation for $F(r)$ (we turn back to the initial function *F* from (2)–(3)):

$$\frac{d^4F}{dr^4}+\left(\frac{m_1}{r}+\frac{m_2}{r^2}+\frac{m_3 r^5+m_4 r^4+m_5 r^3+m_6 r^2+m_7 r+m_8}{P}\right)\frac{d^3F}{dr^3}+$$
$$+\left(n_0+\frac{n_1}{r}+\frac{n_2}{r^2}+\frac{n_3}{r^3}+\frac{n_4}{r^4}+\frac{n_5 r^5+n_6 r^4+n_7 r^3+n_8 r^2+n_9 r+n_{10}}{P}\right)\frac{d^2F}{dr^2}+$$
$$+\left(\frac{p_1}{r}+\frac{p_2}{r^2}+\frac{p_3}{r^3}+\frac{p_4}{r^4}+\frac{p_5}{r^5}+\frac{p_6 r^5+p_7 r^4+p_8 r^3+p_9 r^2+p_{10} r+p_{11}}{P}\right)\frac{dF}{dr}+$$
$$+\left(q_0+\frac{q_1}{r}+\frac{q_2}{r^2}+\frac{q_3}{r^3}+\frac{q_4}{r^4}+\frac{q_5}{r^5}+\frac{q_6}{r^6}+\frac{q_7 r^5+q_8 r^4+q_9 r^3+q_{10} r^2+q_{11} r+q_{12}}{P}\right)F=0, \tag{4}$$

where P is a 6-th order polynomial; all the remaining coefficients in the above equation are complicated and, for this reason, they are omitted.

Now, let us apply the substitution $F=e^{Kr}r^{H}e^{L/r}\tilde{F}$; we omit the explicit form of the resulting equation. As usually, we should determine the indices of singular points.

We impose a restriction on *K*: $K^4 + n_0 K^2 + q_0 = 0$. Allowing for the identities:

$$n_0 = -{M_1}^2 - {M_2}^2 + 2\varepsilon^2, \quad q_0 = (\varepsilon^2 - M_1^2)(\varepsilon^2 - M_2^2),$$

we get $\left(K^2 - {M_1}^2 + \varepsilon^2\right)\left(K^2 - {M_2}^2 + \varepsilon^2\right) = 0$, which infer four different possible values for K:

$$K_1 = \pm\sqrt{{M_1}^2 - \varepsilon^2} < 0, \quad K_2 = \pm\sqrt{{M_2}^2 - \varepsilon^2} < 0.$$

We shall further follow only the case of negative K, since exactly such solutions may correspond to bound states.

The next constraint is:

$$\frac{L^2(L^2 - m_2 L + n_4)}{r^8} = 0, m_2 = 0, n_4 = -4\beta_1^2 \quad \Rightarrow$$

$$L = +2\beta_1, \quad -2\beta_1, \quad 0, \quad 0.$$

For *H,* we find four possibilities:

$$H = \nu, \; H = 1 - \nu, \; H = +\sqrt{\nu^2 - \alpha^2}, \; H = -\sqrt{\nu^2 - \alpha^2},$$

which correspond to the following four different variants:

$$I. \qquad L = 2\beta_1 < 0, \quad H = \nu > 0;$$

$$II. \qquad L = -2\beta_1 > 0, \quad H = 1 - \nu \leq 0;$$

$$III. \qquad L = 0, \quad H = +\sqrt{\nu^2 - \alpha^2} > 0;$$

$$IV. \qquad L = 0, \quad H = -\sqrt{\nu^2 - \alpha^2} < 0.$$

Only the variants I and III are appropriate to describe the bound states.

The 4-th order equation in the case I, after multiplying by $r^6 P$, gives (we write down only the general structure):

$$\left(P_{12}r^{12}+P_{11}r^{11}+P_{10}r^{10}+P_9r^9+P_8r^8+P_7r^7+P_6r^6\right)\frac{d^4\tilde{F}}{dr^4}+$$

$$+\left(Q_{12}r^{12}+Q_{11}r^{11}+Q_{10}r^{10}+Q_9r^9+Q_8r^8+Q_7r^7+Q_6r^6+Q_5r^5+Q_4r^4\right)\frac{d^3\tilde{F}}{dr^3}+$$

$$+\left(M_{12}r^{12}+M_{11}r^{11}+M_{10}r^{10}+M_9r^9+M_8r^8+M_7r^7+M_6r^6+M_5r^5+M_4r^4+M_3r^3+M_2r^2\right)\frac{d^2\tilde{F}}{dr^2}+$$

$$+\Big(N_{12}r^{12}+N_{11}r^{11}+N_{10}r^{10}+N_9r^9+N_8r^8+N_7r^7+$$

$$+N_6r^6+N_5r^5+N_4r^4+N_3r^3+N_2r^2+N_1r+N_0\Big)\frac{d\tilde{F}}{dr}+$$

$$+\left(L_{11}r^{11}+L_{10}r^{10}+L_9r^9+L_8r^8+L_7r^7+L_6r^6+L_5r^5+L_4r^4+L_3r^3+L_2r^2+L_1r+L_0\right)\tilde{F}=0.$$

The solutions may be constructed as power series $\tilde{F}=\sum_{l=0}^{\infty}d_l r^l$.

We derive the 13-term recurrent relations with the general structure:

$$Q_{k-11}d_{k-11}+Q_{k-10}d_{k-10}+....+Q_k d_k+Q_{k+1}d_{k+1}=0.$$

The constraint which determines the transcendental Frobenius solutions is:

$$Q_{k-10}=0 \quad \Rightarrow \quad L_{11}+N_{12}(k-11)=0, \qquad k-11=n>0 \quad (*)$$

and in explicit form it reads

$$-4(M_1-\varepsilon)(M_2+\varepsilon)(M_1-M_2)^2\{(k-10+H)K^3+\alpha\,\varepsilon\,K^2+$$
$$+[\varepsilon^2(H-10+k)+(\frac{9}{2}-\frac{1}{2}k-\frac{1}{2}H)M_1{}^2-\frac{1}{2}(H-11+k)M_2{}^2]K-$$
$$-\frac{1}{2}\alpha\varepsilon(M_1{}^2+M_2{}^2-2\varepsilon^2)\}=0.$$

Let $K=-\sqrt{M_1^2-\varepsilon^2}, \quad H=\nu=1,2,3...$; then the equation (*) takes the form:

$$-2\left(M_1-\varepsilon\right)\left(M_2+\varepsilon\right)\left(M_1-M_2\right)^2\left\{\left(-2k+20-2\nu\right)\left(M_1{}^2-\varepsilon^2\right)^{3/2}+\right.$$

$$+\left[\left(-2k+20-2\nu\right)\varepsilon^2+\left(k-9+\nu\right)M_1{}^2+\left(k-11+\nu\right)M_2{}^2\right]\sqrt{M_1{}^2-\varepsilon^2}+$$

$$\left.+\alpha\,\varepsilon\left(M_1^2-M_2^2\right)\right\}=0,$$

whence we get the roots (write down only the real-valued ones):

$$\varepsilon=+M_1,-M_2,\quad \varepsilon=\pm\frac{M_1}{\sqrt{1+\alpha^2/(k-11+\nu)^2}}.$$

A (quasi-)physical root is:

$$\varepsilon=+\frac{M_1}{\sqrt{1+\alpha^2/(k-11+\nu)^2}}>0,\quad k\geq 12.$$

Let $K=-\sqrt{M_2^2-\varepsilon^2}$, $H=\nu=1,2,3...$; in this case we have the roots

$$\varepsilon = M_1, -M_2, \quad \varepsilon = \pm \frac{M_2}{\sqrt{1+\alpha^2/(k-9+\nu)^2}},$$

and a (quasi-)physical root is:

$$\varepsilon = + \frac{M_2}{\sqrt{1+\alpha^2/(k-9+\nu)^2}}, \quad k \geq 12.$$

It should be noticed that both these spectra cannot be considered as relativistic spectra for a spin 1/2 particle in the Coulomb field, because they do not contain a specific combination of $\sqrt{\nu^2 - \alpha^2}$ depending on the angular momentum.

By this reason, let us study the equation for the variant III; we get the 12-terms recurrent relations of the structure:

$$Q_{k-10} d_{k-10} + Q_{k-9} d_{k-9} + ... + Q_k d_k + Q_{k+1} d_{k+1} = 0.$$

The constraint which determines the transcendental Frobenius takes the form

$$Q_{k-10}=0 \quad \Rightarrow \quad L_{10}+N_{11}(k-10)=0, \qquad k-10=n=1,2,\ldots>0,$$

which explicitly reads:

$$-4(M_1-\varepsilon)(M_2+\varepsilon)(M_1-M_2)^2\{(k-9+H)K^3+\alpha\,\varepsilon\,K^2+$$
$$+[(k-9+H)\varepsilon^2+(4-\frac{1}{2}k-\frac{1}{2}H)M_1{}^2-\frac{1}{2}(k-10+H)M_2{}^2]K-$$
$$-\frac{1}{2}\alpha\varepsilon(M_1{}^2+M_2{}^2-2\varepsilon^2)\}=0.$$

For $K=-\sqrt{M_1^2-\varepsilon^2}, \quad H=\sqrt{\nu^2-\alpha^2}$, we get the equation

$$-4(M_1-\varepsilon)(M_2+\varepsilon)(M_1-M_2)^2\{-(k-9+\sqrt{\nu^2-\alpha^2})(M_1{}^2-\varepsilon^2)^{3/2}+$$
$$+(M_1{}^2-\varepsilon^2)\alpha\,\varepsilon-[(k-9+\sqrt{\nu^2-\alpha^2})\varepsilon^2+(4-\frac{1}{2}k-\frac{1}{2}\sqrt{\nu^2-\alpha^2})M_1{}^2-$$
$$-\frac{1}{2}M_2{}^2(k-10+\sqrt{\nu^2-\alpha^2})]\sqrt{M_1{}^2-\varepsilon^2}-\frac{1}{2}\alpha\,\varepsilon\,(M_1{}^2+M_2{}^2-2\,\varepsilon^2)\}=0,$$

whence we find the roots:

$$\varepsilon = M_1, -M_2, \quad \varepsilon = \pm \frac{M_1}{\sqrt{1+\alpha^2/(k-10+\sqrt{\nu^2-\alpha^2})^2}}.$$

The physical root is

$$\varepsilon = \frac{M_1}{\sqrt{1+\alpha^2/(k-10+\sqrt{\nu^2-\alpha^2})^2}}, \quad k-10 = n = 1,2,3,\dots. \qquad (5)$$

For $K = -\sqrt{M_2^2-\varepsilon^2}, H = \sqrt{\nu^2-\alpha^2}$, the roots are

$$\varepsilon = M_1, -M_2, \quad \varepsilon = \pm \frac{M_2}{\sqrt{1+\alpha^2/(k-8+\sqrt{\nu^2-\alpha^2})^2}},$$

and the physical root is

$$\varepsilon = \frac{M_2}{\sqrt{1+\alpha^2/\left(k-8+\sqrt{\nu^2-\alpha^2}\right)^2}}, \quad k-10 = n = 1,2,3,\dots. \qquad (6)$$

Both spectra (5) and (6) are typical for spin 1/2 particle in the Coulomb field, and they contain the combination $\sqrt{\nu^2-\alpha^2}$.

The energy spectra for states with opposite parity may be found by applying the formal changes:

$$M_1, M_2, \alpha_1, \alpha_2, \beta_1, \beta_2 \Rightarrow -M_1, -M_2, -\alpha_1, -\alpha_2, -\beta_1, -\beta_2.$$

Correspondingly, here we have four possible variants

$$\delta = -1, \qquad I. \qquad L = -2\beta_1 > 0, \quad H = \nu > 0;$$

$$II. \qquad L = +2\beta_1 < 0, \quad H = 1-\nu \le 0;$$

$$III. \qquad L = 0, \quad H = \sqrt{\nu^2-\alpha^2} > 0;$$

$$IV. \qquad L = 0, \quad H = -\sqrt{\nu^2-\alpha^2} < 0.$$

Only variant *III* leads to physical spectra, and they are similar.

Computer simulation. We will use some results from [5].

We consider case III and the following definitions

$$\alpha = \frac{1}{137},\ M = 1, M_1 = \frac{2}{1+\cos\gamma},\ M_2 = \frac{2}{1-\cos\gamma},\ \alpha_1 = -\frac{\beta_1^2}{\beta_2},\ \alpha_2 = -\beta_1. \quad (7)$$

We set the value of the parameter

$$\gamma = \frac{\pi}{6}. \quad (8)$$

Then using (6), (7), (8) we calculate the energy values e/M_1, $M_1 = 1.0718$ for different $n = \overline{1,15}$ (table 1). The visualization of energy values from table 1 is illustrated in fig. 1.

Table 1. Values for energy for different $n = \overline{1,15}$

0.99999834 (*n*=1)	0.99999894 (*n*=2)	0.99999926 (*n*=3)	0.99999946 (*n*=4)	0.99999959 (*n*=5)
0.99999967 (*n*=6)	0.99999974 (*n*=7)	0.99999978 (*n*=8)	0.99999982 (*n*=9)	0.99999984 (*n*=10)
0.99999987 (*n*=11)	0.99999988 (*n*=12)	0.9999999 (*n*=13)	0.99999991 (*n*=14)	0.99999992 (*n*=15)

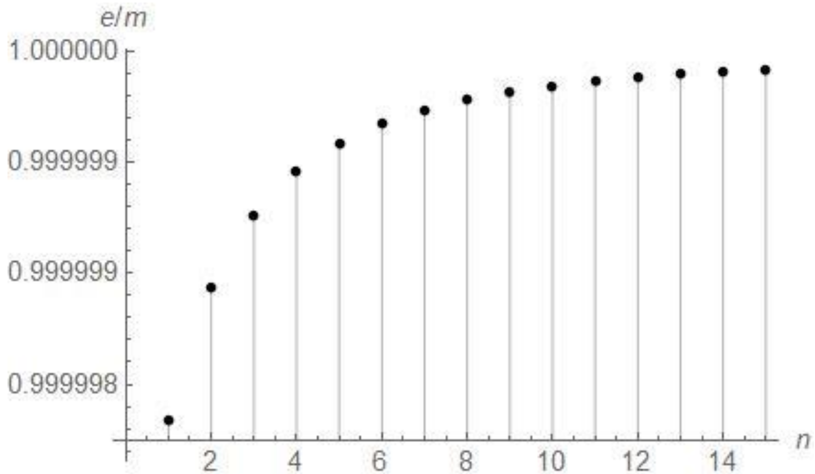

Fig 1. Plot of energy values from table 1

We substitute values (7), (8) and *e*=14.928178 from (6), (7) (*n*=1) in the equation (4).

Using the initial conditions for function $F_1(r)$ in $F(r)=e^{-\sqrt{M_1^2-e^2}r}\times F_1(r)$,

$$r_0=1, F_0=1, F'_0=-6.03\times10^{-1}, F''_0=1, F'''_0=-1$$

we find the corresponding solution $F(r)$. The plot of the function $F(r)=e^{-\sqrt{M_1^2-e^2}r}\times F_1(r)$ is presented in fig. 2.

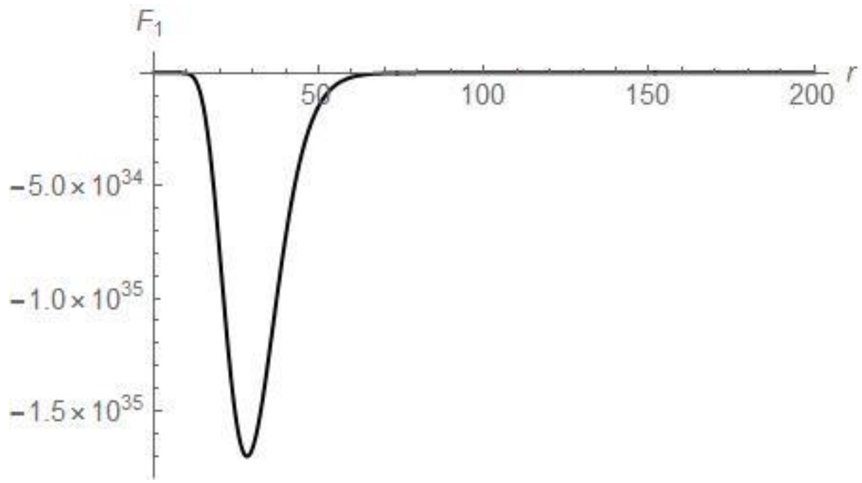

Fig 2. Plot of $F_1(r)=e^{-\sqrt{M_1^2-e^2}r}\times F(r)$ on the interval [0.1, 200]

Conclusions.

The generalized equation for a spin 1/2 particle with two mass parameters is studied in presence of external Coulomb field. After separating the variables, the problem reduces to the system of 8 first order differential equations. Taking into account diagonalization of the space reflection operator, we derive two independent subsystems of 4 equations, referring to states with the opposite parities. In each case, we derive two systems of linked 2-nd order equations. They lead to 4-th order differential equations for separate functions. Their solutions of Frobenius type have been constructed, which involve power series with 13-term (or 12-term) recurrent relations. Two solutions are appropriate to describe physical bound states in the system. As a quantization rule, we apply the known transcendency condition, so deriving two analytical formulas for energy levels. They are similar to relativistic spectra for ordinary spin 1/2 particle, but being governed respectively by different masses, M_1 and M_2.

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