Abstracts [6 September 2014]

**Genome of Descartes folium via normalization**

Adrian Constantinescu, Constantin Udriște, Steluța Pricopie

**Abstract.** The Folium of Descartes carries group laws, defined entirely in terms of algebraic operations over the field. The problems discussed in this paper include: normalization of Descartes Folium, group laws and morphisms, exotic structures, second exotic structure, some topologies on Descartes Folium, differential structure on Descartes Folium, first isomorphism of algebraic Lie groups, second isomorphism of algebraic Lie groups, derived structures of algebraic Lie groups, a differential/complex analytic structure on Descartes Folium, Descartes Folium as a topological field, etc. For predicting these terms, we focus on methods that exploit diagram manipulation techniques (as alternatives to algebraic method of proofs). All our results confirm that the Descartes Folium stores natural group structures, unsuspected till now.

**Some algebraic and C-Lie group structures on Descartes Folium**

Adrian Constantinescu, Constantin Udriște, Steluța Pricopie

**Abstract.** Let $K$ be a field with characteristic $K \neq 3$. The classic Descartes folium over $K$ is the affine plane cubic $DF: x^3 + y^3 - 3axy = 0 \subset \mathbb{A}^2_K$, with $a \in K, a \neq 0$ (having only one singular point, namely $O=(0,0)$) as well as its projective closure $\overline{DF}: x^3 + y^3 - 3axy = 0 \subset P^2_K$, with $a \in K, a \neq 0$.

The aim of this talk is to find all the $K$-algebraic (Lie) group structures on $\overline{DF} \setminus \{O\}$ in the case when $K=C$.

The main result is the following: "Let $K$ be an algebraically closed field with characteristic $K \neq 3$ (respectively $K=C$) and $\overline{DF} \setminus \{O\}$, then there exists a unique $K$-algebraic group structure (respectively a $C$-Lie group structure) on $\overline{DF} \setminus \{O\}$ whose neutral element is $A$. Moreover, in the case $K=C$, for each $A \in \overline{DF} \setminus \{O\}$, the corresponding $C$-Lie group structure is canonically associated to the corresponding $C$-algebraic group structure."

For all such structures we have isomorphisms with the $K$-algebraic group $G_K(K) = K \setminus \{O\}$ (respectively $G_C(C) = C \setminus \{O\}$) and explicit formulas for the underlying group composition laws. For $A = I = (1, -1, 0)$, as for $A = V = (3a, 3a, 2)$, we recover the $K$-algebraic group or the $C$-Lie group structures from [4], respectively [2] or [3].

**References**


**Magnetic geometric dynamics around an infinite rectilinear wire**

Adriana Caloată, Nona Crăciun

**Abstract.** In this paper we compare the trajectories of particles described by the Lorentz Law and the trajectories of geometric dynamics generated by the magnetic flow around an infinite rectilinear circuit.

**Spherical multiergodic averages from Markkov action groups**

Alejandro Meson and Fernando Vericat

**Short abstract.** We study the norm convergence of multiple ergodic spherical averages from actions of word hyperbolic groups or more generally strongly Markov groups. The objective is generalize, the results by Bufetov, Khristoforov and Klimenko and by Pollicott and Sharp. We use the techniques that Walsh introduced for proving the norm convergence of multiple ergodic averages from measure-preserving transformations and for integer valued polynomials.

**Shape optimization based on Riemannian optimal control techniques**

Andreea Bejenaru, Constantin Udriște

**Short abstract.** The aim of this paper is to adapt the general mutitime maximum principle to a Riemannian setting. More precisely, we intend to study geometric optimal control problems constrained by the metric compatibility evolution PDE system. We apply the obtained results in order to solve two flow-type optimal control problems on Riemannian setting: first, we maximize the total divergence of a fixed vector field; secondly, we optimize the total Laplacian (the gradient flux) of a fixed differentiable function. Each time, the result is a bang-bang-type optimal linear connection. Moreover, we emphasize the possibility of choosing at least two soliton-type optimal (semi-)Riemannian structures. Finally, these theoretical examples help us to conclude about the geometric optimal shape of pipes, induced by the direction of the flow passing through them.
Solitons of Regularized Long Waves equation obtained by the method of expanding $G'/G$

Andreea Novăacescu

Abstract. The objective of this article was to use the extension method $G'/G$ with general method $G'/G$ for finding exact solutions of nonlinear equations of evolution of size $(2+1)$, necessary for finding solitons of RWL equation. RWL plays an important role in mathematical physics, applied mathematics, and engineering. Is very important equation in physics, because it describes phenomena with a weak nonlinearity and dispersion waves, including nonlinear cross waves of water surface, magnetic hydrodynamic waves and acoustic plasma or photon packets in nonlinear crystals.

Comparing variants of single-time stochastic maximum principle

Constantin Udrişte

Abstract. This paper is concerned with comparison between the well-known forward-backward stochastic maximum principle and the simplified stochastic maximum principle realized in our papers. Some examples illuminate our ideas and certifies that what we said is correct, since sophisticated and reach it may be, the forward-backward stochastic control theory could only handle pathetically limited problems. In short, we point out that, despite a stochastic optimal control problem has a unique solution, finding these solution techniques can be different due to the diffusion parts of stochastic processes.

The time waves are born on a bed of standing waves

Constantin Udrişte, Florin Munteanu, Dorel Zugrăvescu

Abstract. The purpose of this paper is a new description of standing waves and their role in actual theories. Waves are achieved by imposing a special form of solutions of the system of PDEs describing electromagnetic waves. One obtains a frequency dependent EDP system. Results: the time waves are born on a bed of standing waves; the set of all frequencies is discrete; genesis of waves etc.

Informational origin of gravity

Constantin Udrişte, Florin Munteanu, Dorel Zugrăvescu

Abstract. Information theory is a branch of applied mathematics, electrical engineering, bio-informatics, and computer science involving the quantification of information. A key measure of information is entropy, which is usually expressed by the average number of bits needed to store or communicate one symbol in a message. Entropy quantifies the uncertainty involved in predicting the value of a random variable. Entropy is a measure of unpredictability or information content. From entropy, energy and temperature we obtain the Newton Law, the Newton Law of Masses and informational origin of gravity since changes in the amount of information, measured by entropy, can lead to a force.

A mathematical model adapted to a $P$-swarm to detect malevolent action

Corina Cipu, Simona Bibic

Short abstract. Robotic swarms could solve, without the need for a central controller, a wide range of difficult applications in real-world environments. In first part of the paper we extend the notion of a $P$ colony to a colony of $P$ colonies used for robotic swarms, called $P$ swarms. The second part of our research is the development of a biologically inspired BIS approach for enabling the distributed security of a robotic swarm. Thus we propose in AIS (a computational system constructed to take certain characteristics found in natural immune system) an algorithm based on negative selection for Pcol automaton. In this paper we define a mathematical model for security issues of robotic swarms combining bio-inspired computational models with evolutive systems.

Harmonicity of vector fields and one-forms on a (pseudo-)Riemannian manifold

Cornelia Livia Bejan

Short abstract. Some maps from a Riemannian manifold $M$ to an Euclidean space are lifted to $TM$ and to $T^*M$. Then we study the harmonicity of these new maps with respect to certain metrics. After that we investigate if these maps are harmonic morphisms.

Discrete multitime recurrences and their application in economics

Cristian Ghiu, Raluca Tuliga, Constantin Udrişte, Ionel Tevy

Abstract. Our original results refer to three multivariate linear recurrences: discrete multitime diagonal recurrence, discrete multitime multiple recurrence, discrete multitime recurrence of way required. We find the solutions, we clarify the structural background and provides short, conceptual proofs. The results are applied to the discrete multitime Samuelson-Hicks models, with constant coefficients respectively with multi-periodic coefficients. For constant coefficients case, it was found also the generating function.

A note on the existence of $F$-holomorphic connections

Cristian Ida, Paul Popescu

Short abstract. Given a $F$-holomorphic vector bundle $E→(M,F)$ over a manifold $M$ endowed with a complex foliation $F$, in this note we give a characterization for the existence of $F$-holomorphic connections on $E$ by vanishing of a tangential Atiyah cohomology class. Our results generalizes the same characterization for usual holomorphic vector bundles over complex manifolds, that is the case when the complex foliation $F$ have codimension 0.
Properties of \( \Sigma \)-metallic Riemannian structures

**Cristina Elena Hrețcanu, Mircea Crăsmăreanu**

**Abstract.** The purpose of this paper is to give some properties of \( \Sigma \)-metallic Riemannian structures induced on submanifolds by metallic structures \( J \) (i.e. \( J^2 = pJ + qI \), where \( I \) is the identity operator on the Lie algebra of the vector fields on a manifold \( M \) and \( p, q \) are fixed positive integer numbers) given on a Riemannian manifold \((M, g)\) (\( J \) is \( J \)-compatible, i.e. \( g(IX, Y) = g(X, JY) \)).

A \( \Sigma \)-metallic Riemannian structure is a structure \( \Sigma = (P, g, u_\alpha, (a_{\alpha\beta})) \) induced on a submanifold of a metallic Riemannian manifold \((M, g, J)\), where \( P \) is an \((l, 1)\)-tensor field on the submanifold, \( \xi_\alpha \in \mathcal{X}(M) \), \( u_\alpha \) are 1-forms on \( M \) and \((a_{\alpha\beta})\) is an \( r \times r \) matrix of smooth real functions on \( M \) (\( r \) is the codimension of submanifold in the manifold). We define the normal \( \Sigma \)-metallic Riemannian structure on a submanifold in a metallic Riemannian manifolds and we find conditions for this structure to be normal. Also, we study some properties of structures induced on a product manifold of two metallic Riemannian manifolds.

Dynamical systems. Applications

**Dumitru Bălă**

**Abstract.** In this paper there is studied the stability of dynamical systems with applications in economics and technique. Methods are applied to study the stability of systems of concrete differential equations.

Compactness in FK Spaces

**Eberhard Malkowsky**

**Abstract.** Compactness plays an important role in metric fixed point theory and the theory of FK spaces which constitute a fairly general class of sequence spaces. The important concept is that of measures of noncompactness. We give an introduction to FK spaces and to measures of noncompactness on bounded sets in complete metric spaces, and a survey of the most important basic properties of certain measures of noncompactness, in particular, we consider the Kuratowski, Hausdorff and separation measures of noncompactness. We also define and study the Hausdorff measure of noncompactness of operators between Banach spaces. Furthermore, we demonstrate how measures of noncompactness can be applied in fixed point theory, in FK spaces, and in the characterizations of classes of compact operators between certain sequence spaces.

Multitime differential games

**Elena-Laura Otobîcu, Constantin Udriște, Ionel Țevy**

**Short abstract.** The multitime differential game theory is the aim of our paper. We benefit from a recent literature dedicated to multitime maximum principle, accomplished by the research group of Prof. Dr. Constantin Udriște. This literature is enough to design multitime cooperative and non-cooperative games. Theories are distinguished by functional choice: either multiple integral or curvilinear integral.

Geometrical methods for the study of mathematical models of the multi-species interactions

**Florian Munteanu, Romulus Militaru**

**Short abstract.** The purpose of this paper is to find symmetries and conservation laws for the mathematical models of the multi-species interactions, given by Volterra-Lotka equations and other systems. We will use the Hamilton-Poisson realizations of 2D and 3D Volterra-Lotka systems for obtain new conservation laws starting from symmetries.

Adapted metrics for Newtonian gravity models

**Gabriel-Teodor Pripoae**

**Abstract.** We use geodesivable vector fields (i.e. those vector fields for which there exists a Riemannian metric whose geodesics are their trajectories) in order to detail the geometrization of the Newtonian force. Several variations on the two-bodies dynamics are included: the Pioneer 11 abnormal acceleration, perturbations of the planets orbits, etc.

Geometry of pre-contrast functions

**Hiroshi Matsuzoe**

**Short abstract.** A contrast function is an asymmetric squared distance like function, which is an important object in geometric theory of statistical inferences. It is known that a contrast function induces a statistical manifold structure. A pre-contrast function is a differential or a quantum version of contrast functions. It is known that a pre-contrast function induces a statistical manifold admitting torsion, which naturally arises in con-conservative estimation theory and in quantum information theory. In this talk, geometry of pre-contrast functions is discussed.

On some Kahler-Riemann type flows

**Iulia Elena Hiriță**

**Abstract.** The Kahler-Ricci flow is the analogue of the Ricci flow in the complex case. The Kahler-Riemann flow generalizes the notion of Riemann flow. These extensions are natural, since they resemble the results of Ricci and Riemann flows. Using the properties of the \( h \)-projective, \( h \)-concircular and \( h \)-conharmonic curvature tensor fields on Kahler manifolds, it is possible to find some families of Kahler metrics produced by certain Kahler-Riemann type flows. Holomorphically projective, concircular and conharmonic mappings between Kahler manifolds are characterized.
Dynamics of nonholonomic systems

Jana Musilová, Michal Čech

Short abstract. Geometric approach to mechanical systems under nonholonomic constraints. From this perspective the equations of motion arise as extremals on constrained submanifold in the jet bundle, where admissible variations are defined by the Chetaev bundle arising from the canonical distribution corresponding to every constraint. As an example this model is used to study the solutions and conserved quantities of the Chaplygin's sledge motion.

Multitime Schrodinger spatial solitons

Lavinia Petrescu

Abstract. We introduce and analyze a multitime nonlinear Schrodinger PDE. The original results refer to multitime Schrodinger spatial solitons.

Homothetical hypersurfaces in Euclidean spaces from the point of view of production theory in economics

Muhittin Evren Aydin and Mahmut Ergut

Abstract. The authors [1] obtained a classification for the composite functions of the form \( f = F(h_1(x_1), \ldots, h_n(x_n)) \) via Allen determinants for production functions in economics. In this talk, we adopt this classification to the homothetical hypersurfaces in Euclidean spaces. Several applications for the homothetical hypersurfaces are also given.

References


Local characterizations of surfaces in various spaces by induced metrics and principal distributions

Naoya Ando

Short abstract. We obtain local characterizations of minimal surfaces in 4-dimensional Riemannian space forms, in terms of the induced metrics and principal distributions. We also obtain a local characterization of space-like surfaces with zero mean curvature vector in 4-dimensional Lorentz space forms. Each of these characterizations gives a unique surface, up to an isometry of the space. On the other hand, we obtain characterizations of surfaces in 3-dimensional Riemannian space forms which are not uniquely determined by the induced metrics and principal distributions.

Hamilton-De Donder equations for harmonic maps

Olga Rossi and Jakub Dvorsky

Abstract. We study harmonic maps with help of Hamilton-De Donder theory in dual jet bundles. The aim is to obtain the corresponding Hamilton equations, and to investigate their geometric properties.

Metric dimensional reduction at singularities with implications to Quantum Gravity

Ovidiu Cristinel Stoica

Abstract. A series of old and recent theoretical observations suggests that the quantization of gravity would be feasible, and some problems of Quantum Field Theory would go away if, somehow, the spacetime would undergo a dimensional reduction at high energy scales. But an identification of the deep mechanism causing this dimensional reduction would still be desirable. The main contribution of this article is to show that dimensional reduction effects are due to General Relativity at singularities, and do not need to be postulated ad-hoc. Recent advances in understanding the geometry of singularities do not require modification of General Relativity, being just non-singular extensions of its mathematics to the limit cases. They turn out to work fine for some known types of cosmological singularities (black holes and FLRW Big-Bang), allowing a choice of the fundamental geometric invariants and physical quantities which remain regular. The resulting equations are equivalent to the standard ones outside the singularities. One consequence of this mathematical approach to the singularities in General Relativity is a special, (geo)metric type of dimensional reduction: at singularities, the metric tensor becomes degenerate in certain spacetime directions, and some properties of the fields become independent of those directions. Effectively, it is like one or more dimensions of spacetime just vanish at singularities. This suggests that it is worth exploring the possibility that the geometry of singularities leads naturally to the spontaneous dimensional reduction needed by Quantum Gravity.

On discrete geometry of non-holonomic spaces

Paul Popescu and Marcela Popescu

Short abstract. New algorithms for discrete integration of non-holonomic dynamics equations with linear and non-linear constraints are proposed.
On non-holonomic dynamics with linear and non-linear constraints

Paul Popescu and Cristian Ida

Short abstract. A new form of non-holonomic dynamics equations, using linear and non-linear constraints, is given. The setting is that of foliations and the Lagrange multipliers are not involved explicitly.

A characterization of homogeneous real hypersurfaces of types (C), (D) and (E) in a complex projective space

Sadahiro Maeda

Short abstract. We give a necessary and sufficient condition that a Riemannian manifold $\mathbb{M}^{2n-1}$ isometrically immersed into a complex projective space $\mathbb{CP}^n(c)$ is locally congruent to one of homogeneous real hypersurfaces of types (C), (D) and (E) in this ambient space in terms of its restricted principal curvature distributions.

Isoperimetric constrained optimal control problems

Savin Treanţă

Short abstract. Using corresponding variational and adjoint differential systems, necessary optimality conditions for a feasible point in isoperimetric constrained optimal control problems are derived.

Optimal control problems for stress tensor in perfect plastic plane medium

Simona Dinu, Andreea Bejenaru

Short abstract. This study joins some concepts from Mechanics, Partial Differential Equations and Control Theory in order to solve bi-time optimization problems related to stress tensor in plastic deformations. The main goal is to analyze some optimal control problems constrained by the equilibrium equations of the stress tensor in perfect plastic plane medium. As consequence of this approach, we emphasize a natural split of the constraints which leads to integrability conditions and changes a classical variational problem into an optimal control one. The final outcomes confirm all the expectations related to the physical features of plastic deformations phenomenon.

Efficiency for multitime vector variational problems on Riemannian manifolds involving geodesic invex functionals

Ştefan Mititelu, Constantin Udrişte

Abstract. We study the connection between a multitime scalar variational problem (SVP), a multitime vector variational problem (VVP) and a multitime vector fractional variational problem (VFP). For (SVP) we establish necessary optimality conditions. For both vector variational problems we define the notions of efficient (that is, Pareto) solution and of normal efficient solution and we establish necessary efficiency conditions for (VVP) and (VFP) using these two notions. The main purpose of the paper is establishment of sufficient efficiency conditions for the vector problems (VVP) and (VFP). Moreover, we obtain sufficient optimality conditions for (SVP). The sufficient conditions are based on our original notion of $(\rho, b)$-geodesic quasiinvexity.

Observable, controllable and minimal realizations for 2D hybrid systems

Tiberiu Vasilache, Valeriu Prepelită, Mona Doroftei

Short abstract. The state-space realization problem is considered in a class of 2D continuous-discrete LTI systems. An algorithm is provided to determine an observable realization. Similarly, a controllable realization is presented. Finally, an extension of the Ho's algorithm is used to obtain a realization which is both observable and controllable, i.e., minimal.

Bivariate diagonal Fibonacci numbers

Vasile Arsinte, Constantin Udrişte

Abstract. Having in mind the usually Fibonacci numbers, we introduce a bivariate recurrence which define the bivariate diagonal Fibonacci numbers. We find the solutions and the generating functions.

Line Graphics versus Polygon Mesh

Vesna Velickovic

Abstract. Nowadays, Polygon Mesh is a standard way for the graphical representation of surfaces in Computer Graphics. We discuss the advantages and disadvantages of this approach and compare it with the Line Graphics methods, which we use. Furthermore, we give the basis properties of Line Graphics.

Duality for multitime vector fractional variational problems on manifolds via $(\rho, b)$-geodesic quasi-invexity

Vladimir Balan, Ştefan Mititelu

Abstract. We consider a multitime scalar variational problem (SVP), a multitime vector (or multiobjective) variational problem(VVP) and a multitime vector fractional variational problem (VFP). For (SVP) we present necessary optimality conditions and we define the notion of normal optimal solution. For the two vector variational problems we define the notions of efficient (Pareto) solution and of normal efficient solution and using these two notions we also present necessary efficiency conditions for (VVP) and (VFP). Moreover, in the following, we developed a duality of Mond-Weir-Zalmai type for the fractional problem (VFP) through weak, direct and converse duality theorems. Also we give a generalization to this duality. The presentation of this duality involves the notion of $(\rho, b)$-geodesic quasi-invexity defined for the variational problems on Riemannian manifolds.
On MKdV equations related to simple Lie algebras

Vladimir Gerdjikov

Abstract. We have derived a one-parameter family of MKdV equations related to the simple Lie algebra $g$ using a Coxeter $Z_h$ reduction, where $h$ is the Coxeter number of $g$. They admit the Lax pair

$$L \psi = i \frac{\partial \psi}{\partial x} + U(x,t,\lambda) \psi = 0, \quad M \psi = i \frac{\partial \psi}{\partial t} + v(x,t,\lambda) \psi = 0,$$

$$U(x,t,\lambda) = Q(x,t) - \lambda J, \quad V(x,t,\lambda) = \sum_{s=0}^{2} \lambda^s V_{2s}(x,t) - \lambda^3 K,$$  

(1)

satisfying the reduction condition

$$CU(x,t,\lambda)C^{-1} = U(x,t,\omega \lambda), \quad CV(x,t,\lambda)C^{-1} = V(x,t,\omega \lambda),$$  

(2)

where $C$ is the Coxeter automorphism of $g$, $C^h = 1$ and $\omega^h = 1$. Thus we are able to derive the corresponding system of MKdV equations and discuss their properties. We discuss two of the possible choices for $g$: i) $sl(n)$ and ii) $g = so(8)$. In the second case we derive a one-parameter family of MKdV-type equations.

Twisted surfaces with null rotation axis

Wendy Goemans

Abstract. In [1] and [2], the authors revived the study of twisted surfaces. These surfaces are generated by synchronized rotations of a planar curve in its supporting plane and of this supporting plane about some axis, possibly at different speeds. Since this construction is a natural generalization of the one that is used to construct a surface of revolution and similar to the construction of helicoid surfaces, it is remarkable that these twisted surfaces have not been studied thoroughly before. In [3], twisted surfaces are mentioned to generalize two well-known examples: the Möbius strip and the twisted Klein bottle. But only in [4] twisted surfaces are studied for their own right, as the authors examine the orientability of twisted surfaces.

This gap in the theory of classical differential geometry has been filled partially in [1] and [2] in which the authors classify twisted surfaces that possess certain curvature properties in Euclidean and Minkowski 3-space. When translating the construction of twisted surfaces to Minkowski 3-space in [1] and [2], only non-null rotation axes are used.

In this talk twisted surfaces constructed using a null rotation axis are defined. For these twisted surfaces we present several classifications concerning interesting curvature properties. These classifications contain some new examples of special twisted surfaces, among which a $B$-scroll.

References


Gromov hyperbolicity and Barbilian spaces

Wladimir-Georges Boskoff, Bogdan Suceavă

Abstract. We explore the connection between the geometries generated by logarithmic oscillations and the class of metric spaces satisfying the condition of Gromov hyperbolicity. We start our discussion with the most fundamental examples, inspired from classical geometries, e.g. the Euclidean distance on the infinite strip or Hilbert’s distance on the unit disk, and we continue our study with Barbilian’s distance, which historically appeared as a natural extension of a model of hyperbolic geometry. We introduce a new metric, called the stabilizing metric, and study its properties. Continuing this study, we explore a class of extensions of this distance and show that, under some analytic conditions, infinitely many new examples of Gromov hyperbolic metric spaces can be constructed. Using similar procedures, we construct Vuorinen’s stabilizing metric and its extensions and we discuss their Gromov hyperbolicity.

Geometry of reliability models

Zahir Abdul Haddi Hassan, Vladimir Balan, Constantin Udrişte

Short abstract. Geometric modeling of reliability models is based on algebraic hypersurfaces. Intrinsic properties of such surfaces select the models which are relevant for applications.

How many are torsionless affine connections in general dimension

Zdenek Dusek, Oldrich Kowalski

Abstract. On an n-dimensional manifold M, we consider locally: a) The family of all real analytic torsion-free affine connections, b) the subfamily of those with skew-symmetric Ricci tensor and c) the subfamily of those with symmetric Ricci tensor. In all cases, the corresponding family is characterized in terms of some number of arbitrary functions of n variables and arbitrary functions of n-1 variables. A paradox is shown that the families a), b), c) depend asymptotically on the same number of functions of n variables (for n going to infinity). The Cauchy-Kowalevski Theorem is used for this purpose.