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ECONOMIC GEOMETRIC DYNAMICS

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The book can be used as a text for courses in mathematical economics, as a supplement for courses in economic theory and operations research, and as source for research problems. It should be of interest to economists, engineers, operations research analysts and applied mathematicians.

This book is dedicated to
MARIA TERESA CALAPSO
whose precept and example have
encouraged us to finish this work

FOREWORD

Problems of *Geometric Dynamics* are pervasive in the modern world, appearing in science, engineering and business. Our recent developments in geometric dynamical systems (tools, modelling, linearization, discretization), dynamics of normal economy, queue geometric dynamics, business cycles, thermodynamic and economic systems, nonholonomic optimization theory, and invex functions in mathematical optimization have had many important areas of application and promise to have even wider usage in the future.

This book is intended as a transfer of recent static and dynamic geometry techniques to the economic theory. We hope that the application of tools from modern differential geometry and geometric dynamics in the field of economics will cause an important progress in research, teaching, and in mathematical language of economic theorist. Because each chapter could have been a book in its own right, it was necessary to be selective. While some problems are exercises in manipulating techniques, most are teaching or research problems, suggesting new ideas and offering a challenge to the reader. Each chapter contains a bibliography, and the key references are indicated as the footnotes of the chapters.

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An economy is a collection of certain institutions, each of which faces and solves an economizing problem (e.g., a mathematical optimizing problem). Any real economy contains a myriad of institutions, but the economic theory treats only a few idealized representative institutions:

1) *households* - groups or persons sharing income for consumption purposes (typically family groups);

2) *firms* - entities (proprietorships, partnerships, or corporations) producing goods or services for sale to other firms or final consumers;

3) *trade unions* - groups of employees organized to bargain collectively with

employers for certain ends;

4) *governments* - political entities which often have important economic functions.

In this book *Economics* is considered the application of dynamical systems, geometric dynamics, thermodynamic and economic systems, holonomic or nonholonomic mathematical optimization to the institutions of the economy. Thus, economics is concerned with the allocation of scarce resources among competing ends.

Economic phenomena may often be found to be representable by analogous electromagnetic phenomena. Electromagnetic analogue computing techniques may therefore have broad applications in the study of economic problems. That is why this book presents suitable problems in theoretical economics, using the terminology of geometry and physics.

Chapter 1 contains those concepts and result on geometric dynamics which are used in an essential way in the rest of the book. Chapter 2 introduces and studies the most important flows in electromagnetism and economics described via geometric dynamics, completed by Maple simulations. Chapter 3 focuses on linearization of economic geometric dynamics produced by the Tobin-Benhabib-Miyao flow and by the Euclidean structure of the space. Chapter 4 serves two purposes: one, to introduce the variational integrators for Lagrangian dynamics; and, two, to show that these methods work well for geometric dynamics even if the starting flow is dissipative or chaotic. Chapter 5 describes the nonholonomic evolution of a two-sector economy, supposing that both sectors generate enough profit, and the productivity capacity of the capital goods is sufficient to satisfy their demand. Chapter 6 analyzes three important problems: the application of hybrid 2D Laplace transformation for finding the solution of the state differential system in the theory of queues, the description of a state phenomena by integral manifolds of a Pfaff equation with an infinity of terms, and the prolongation of the state flow to a geometric dynamics associated to a Lagrangian of least squares type, which is a series of functions. Chapter 7 focuses on the Kaldor-Kalecki delay model and the associated geometric dynamics. Chapter 8 underlines some similarities between thermodynamic and economic systems, via two area conditions. The first area condition is used to justify the monopolist role in a market, and the second area condition points out the structure of nonholonomic economic systems. Chapter 9 refers to three ideas: to replace the constraints in an optimum problems by a selector of curves, to reformulate and study extremum problems with point constraints and/or velocity constraints, to extend the saddle point theory and the Kuhn-Tucker theory to extremum with nonholonomic

constraints. Chapter 10 presents new properties of invex functions, insisting on correlations between the invexity, preinvexity and monotonicity notions of real functions.

The work to this book starts with discussions between me, D. Opriş and M. Ferrara regarding some economic topics and their mathematical content. Than the basic economical ideas were introduced in my Lectures that I made at Peloritana Academy and at Faculty of Economics-Faculty of Mathematics, University of Messina, May 12-19, 2002; October 14 - November 14, 2003; April 18-28, 2004, and at University of Salerno, April 18-28, 2004. Developing these Lectures and our common ideas about mathematical economics explained by geometric dynamics, we realized the present book.

I address warmly thanks to Acad. Prof. Dr. Dorel Zugrăvescu, Prof. Dr. Maria Teresa Calapso, Prof. Dr. Vincenzo Ciancio who supported in time my works regarding the geometric dynamics.

Also all authors are indebted to Prof. Dr. Vladimir Balan, Prof. Dr. Constantin Cristescu, Prof. Dr. Oltin Dogaru, Prof. Dr. Florea Hănţilă, Prof. Dr. Alexandru Lupaşcu, Prof. Dr. I. M. Popescu, Prof. Dr. Valeriu Prepeliţă for comments regarding the research papers included in this book. Of course, we are very much obliged to Armando Ciancio, Oltin Dogaru, Gabriela Mircea, Ştefan Mititelu, Mihaela Neamţu, Andreea Niglia, Mihai Postolache, Ionel Ţevy, Sorin Adrian Udrişte, which are authors or coauthors of landmark papers used in each chapter.

My warm thanks go to my wife Aneta Udrişte whose love, support, and encouragement over the years have to a large extent made the writing of this book possible.

We hope that our book will contribute to the comprehensibility and quality of economics education at both the undergraduate and graduate levels. The book includes many new ideas previously available only through specialized journals and also challenging research problems.

Bucharest, November 2004

Prof. Dr. Constantin Udrişte

LETTERA DI APPREZZAMENTO

E' nell'anno 2000 che la Kluwer Academic Publishers pubblica un libro dal titolo "Geometric Dynamics". Autore un brillante ed autorevole matematico rumeno apprezzato e stimato in campo internazionale per i suoi precedenti studi in Geometria differenziale, Teoria dell'ottimizzazione, Sistemi dinamici: Constantin Udrişte. Conversando una sera, dopo una cena in compagnia di alcuni colleghi in vista dello Stretto di Messina, Constantin Udrişte ci confidò che quel volume dal titolo breve ed accattivante gli era costato ben 25 anni di studio. In verità quel libro ha si un titolo breve ma altresì un contenuto matematico importante, bene degno degli anni di studio, tanto importante da coinvolgere non soltanto la matematica in senso tecnico ma anche svariate altre discipline. A Udrişte, infatti, si deve una teoria geometrica originale che permette di cogliere le evoluzioni dinamiche di alcune strutture geometriche: solo alcune già note in letteratura, altre del tutto nuove introdotte per la prima volta dallo stesso Autore. Questa teoria tanto feconda da prestarsi - come gi dicevamo - all'applicazione ai problemi posti da molte discipline: Biologia, Fisica, Chimica, Geologia, Economia. Ecco che, a questo punto, gli studi di Udrişte si "intersecano" con quelli di un giovane matematico italiano Massimiliano Ferrara, già appassionato ai problemi di carattere economico che svolge la sua attività all'Università di Messina. L'occasione si presentata nel novembre del 2001. In quell'anno, infatti, l'Università di Messina decise di celebrare il centenario della nascita di Renato Calapso organizzando un Congresso a carattere internazionale. Nel ricordo degli importanti legami scientifici che per tutto il secolo scorso l'Università di Messina ha mantenuto con i matematici rumeni (Tzitzeica, Gheorghiu, Roşca, Vrănceanu ed oggi Udrişte) il Comitato organizzatore invitò - tra i relatori al Congresso - anche C. Udrişte proprio per ricordare e rinsaldare tali legami. Fu così che, tra il Maestro di Bucarest e il giovane matematico italiano iniziò una collaborazione scientifica che oggi, con questo volume, dona uno dei suoi migliori frutti ma che ai miei occhi ha una valenza maggiore: la continuità di quegli studi di geometria che - iniziati da Pasquale Calapso - portarono l'Università di Messina ad una notorietà internazionale e alla collaborazione con alcuni tra i più significativi geometri dei nostri tempi. E' questa continuità che, sulla scia delle nostre più

nobili tradizioni in Geometria differenziale, viene oggi rafforzata e indirizzata a tematiche più moderne e adeguate al dibattito scientifico più attuale. Così che ancora una volta nei nostri studi passato e presente si accordano profondamente. Il passato è rappresentato dalla sempre feconda tradizione della Geometria differenziale classica, il presente dalla geniale capacità che nuove generazioni di studiosi stanno dimostrando, di scoprire le profonde connessioni con i più rilevanti e attuali problemi posti dalle più diverse branche dell'unica impresa scientifica che dalla matematica trae ancora una volta risolutivi elementi di chiarificazione.

Messina, luglio 2004

Prof. Dr. Maria Teresa Calapso

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