Chain recurrence classes with shadowing of three dimensional generic vector fields

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Abstract. For C^1 generic vector field X on a three dimensional manifold which is far from homoclinic tangencies, if the vector field X has the shadowing property on chain recurrence classes then it does not contain singularities and it is hyperbolic.

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1 Introduction

For stability conditions (hyperbolicity, structural stability, Axiom A, etc) of dynamical systems, a main topic is Palis conjecture. He suggested the following problem: Every vector field can be accumulated either by hyperbolic vector fields or by ones with a homoclinic bifurcation or with a singular cycle. For three dimension, the conjecture can be write as follows: Every vector field on three dimensional manifold can be accumulated either by singular hyperbolic vector fields or by one with a homoclinic tangency which associated to a non singular periodic orbit (see [5, 29]). For the relation between the shadowing property and singular hyperbolicity. Komuro [7] proved that geometric Lorenz attractor does not have the fixed-parameter shadowing property, however, it was proved by Kiriki and Soma [6] that is has the parameter-shifted shadowing property. Nevertheless, the shadowing is an important dynamic property which is a useful notion to study of stability condition (see [6, 7, 9, 15, 16, 18, 19, 20, 21, 25, 26, 31, 32]).

Let M be a closed n-dimensional, $n \geq 3$ smooth manifold and let $\mathfrak{X}(M)$ be the set of all vector fields of M endowed with the C^1 topology. Recently, Crovisier and Yang [5] showed that every three dimensional vector field can be accumulated by robustly singular hyperbolic vector fields, or by vector fields with homoclinic tangencies. From the result, they written in the paper [5] as follows.

Theorem 1.1. *For C^1 generic vector field $X \in \mathfrak{X}(M^3)$ which is far from homoclinic tangencies, any chain recurrence class is hyperbolic or is a singular hyperbolic attractor or repeller.*

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The theorem is a motivation of the paper. About a closed X_t-invariant set, many results are published in [1, 10, 11, 12, 13, 14, 17, 22, 23, 24, 27, 28]. Inspired, we consider that if the assumption in Theorem 1.1 holds and a vector field X has the shadowing property on the chain recurrence class then it does not contain singularities and it is hyperbolic.

2 Basic notions and results

Let M be as before and let $X \in \mathfrak{X}(M)$. The flow of X will be denoted by $X^t, t \in \mathbb{R}$. For $X \in \mathfrak{X}(M)$, a point $x \in M$ is *singularity* of X if $X(x) = 0$. Denote by $Sing(X)$ the set of all singular points of X. A point $p \in M$ is *periodic* if there is $\pi(p) > 0$ such that $X^{\pi(p)}(p) = p$, where $\pi(p)$ is the prime period of p. Denote by $P(X)$ the set of all closed orbit of X. Let $Crit(X) = Sing(X) \cup P(X)$. It is clear that $Crit(X) \subset \Omega(X)$, where $\Omega(X)$ is the set of all nonwandering points of X. For any $\delta > 0$, a sequence $\{(x_i, t_i) : x_i \in M, t_i \geq 1, \text{ and } -\infty \leq a < i < b \leq \infty\}$ is a δ-pseudo orbit of X if $d(X^{t_i}(x_i), x_{i+1}) < \delta$ for any $a \leq i \leq b-1$.

An increasing homeomorphism $h : \mathbb{R} \to \mathbb{R}$ with $h(0) = 0$ is called a *reparametrization* of $\mathbb{R}$. Denote by $\mathrm{Rep}(\mathbb{R})$ the set of reparametrizations of $\mathbb{R}$. Fix $\epsilon > 0$ and define $\mathrm{Rep}(\epsilon)$ as follows:

$$\mathrm{Rep}(\epsilon) = \big\{h \in \mathrm{Rep}(\mathbb{R}) : \big|\frac{h(t)}{t} - 1\big| < \epsilon\big\}.$$

For a closed X^t-invariant set $\Lambda \subset M$, we say that X has the *shadowing property* on Λ if for any $\epsilon > 0$, there is $\delta > 0$ satisfies the following property: given any δ-pseudo orbit $\xi = \{(x_i, t_i) : t_i \geq 1, i \in \mathbb{Z}\}$ with $x_i \in \Lambda$, there is a point $y \in M$ and an increasing homeomorphism $h \in \mathrm{Rep}(\epsilon)$ such that $d(X^{h(t)}(y), X^{t-s_i}(x_i)) < \epsilon$ for any $s_i \leq t < s_{i+1}$, where s_i is defined as

$$s_i = \begin{cases} t_0 + t_1 + \cdots + t_{i-1}, & \text{if } i > 0 \\ 0, & \text{if } i = 0 \\ -t_{-1} - t_{-2} - \cdots - t_i, & \text{if } i < 0. \end{cases}$$

The point $y \in M$ is said to be a *shadowing point* of ξ. A point $x \in M$ is called *chain recurrent* if there is a sequence $\{(x_i, t_i) : i = 0, \ldots, n, t_i \geq 1\}$ such that $x_0 = x$ and $d(X^{t_{n-1}}(x_{n-1}), x) < \delta$. Denote by $R(X)$ the set of all chain recurrent points of X, For any $x, y \in M$, we write that $x \rightsquigarrow y$ if for any $\delta > 0$ and $t \geq 1$, there is a $(\delta, 1)$ pseudo orbit from x to y such that $x_0 = x$ and $x_n = y$. Similarly, we write $y \rightsquigarrow x$ as the above. Then $x \leftrightsquigarrow y$ means that $x \rightsquigarrow y$ and $y \rightsquigarrow x$. It is an equivalence relation on $R(X)$. An equivalence class of $\leftrightsquigarrow$ is called a *chain recurrence class* of X, denoted by $C(\cdot, X)$.

Let Λ be a closed X^t-invariant set. The set Λ is called *hyperbolic* for X^t if there are constants $C > 0, \lambda > 0$ and a splitting $T_xM = E^s_x \oplus \langle X(x)\rangle \oplus E^u_x$ such that the tangent flow $DX^t : TM \to TM$ leaves invariant the continuous splitting and

$$\|DX^t|_{E^s_x}\| \leq Ce^{-\lambda t} \text{ and } \|DX^{-t}|_{E^u_x}\| \leq Ce^{-\lambda t}$$

for $t > 0$ and $x \in \Lambda$. We say that $X \in \mathfrak{X}(M)$ is *Anosov* if M is hyperbolic for X. In [10], Lee *et al* proved that if a vector field X has the C^1 robustly shadowing property on $C(\gamma, X)$ then it is hyperbolic, moreover, it does not contains non-hyperbolic singularities. Lee [28] showed that if a vector field X has the C^1 stably shadowing property on $C(\gamma, X)$ then $C(\gamma, X) \cap Sing(X) = \emptyset$ and $C(\gamma, X)$ is hyperbolic.

We say that a subset $\mathcal{G} \subset \mathfrak{X}(M)$ is *residual* if it contains a countable intersection of open and dense subsets of $\mathfrak{X}(M)$. A property is called C^1-*generic* if it holds in a residual subset of $\mathfrak{X}(M)$. Ribeiro [30] proved that if a C^1 generic chain transitive vector field X has the shadowing property in an isolated closed invariant set Λ then it is transitive hyperbolic set.

We say that Λ is *partially hyperbolic* if there are an invariant splitting $T_\Lambda M = E^s \oplus E^c$ and constants $C > 0, \lambda > 0$ such that

(i) $\|DX^t|_{E^s_x}\| \leq Ce^{-\lambda t}$, for all $x \in \Lambda$ and $t > 0$,

(ii) E^s dominates E^c, that is, $E^s_x \neq 0, E^c_x \neq 0$ and $\|DX^t|_{E^s_x}\| \cdot \|DX^{-t}|_{E^c_{X_t(x)}}\| \leq Ce^{-\lambda t}$, for all $x \in \Lambda$ and $t > 0$.

In the above definition, we say that the central bundle E^c is *volume expanding* if the constants $C > 0$ and $\lambda > 0$ satisfy $|J(DX^t|_{E^c_x})| \geq Ce^{\lambda t}$, for all $x \in \Lambda$ and $t > 0$, where $J(\cdot)$ is the Jacobian. A closed X^t-invariant set Λ is *transitive* if there is a point $x \in \Lambda$ such that $\omega(x) = \Lambda$, where $\omega(x)$ is the omega limit set of x. We say that Λ is an *attracting* if there is a compact neighborhood U of Λ such that $\Lambda = \bigcap_{t \geq 0} X^t(U)$. A set Λ is *attractor* if it is a transitive attracting set.

We say that Λ is *singular hyperbolic* if every singularity in Λ is hyperbolic and it is partially hyperbolic with a volume expanding central bundle. A *singular hyperbolic attractor* is an attractor that is also a singular hyperbolic set for X, and a *singular hyperbolic repeller* is an attractor is singular hyperbolic set for $-X$. For a hyperbolic periodic orbit γ, the strong stable manifold $W^{ss}(p)$ of $p \in \gamma$ and stable manifold $W^s(\gamma)$ of γ are defined as follows:

$$W^{ss}(p) = \{y \in M : d(X^t(y), X^t(p)) \to 0 \,\text{as}\, t \to \infty\},$$

and

$$W^s(\gamma) = \bigcup_{t \in \mathbb{R}} W^{ss}(X^t(p)).$$

Analogously we can define the strong unstable manifold, and the unstable manifold.

We say that X has a *homoclinic tangency* if X has a hyperbolic periodic orbit γ such that $W^s(\gamma) \cap W^u(\gamma) \neq \emptyset$ but they are not transverse intersection. In the paper, we assume that the chain recurrence class $C(\gamma, X) \neq \{\gamma\}$. We use the shadowing property in the Theorem 1.1. Then we have the following.

Theorem A For C^1 generic $X \in \mathfrak{X}(M^3)$ which is far from homoclinic tangencies, if X has the shadowing property on the chain recurrence class $C(\gamma, X)$ then $C(\gamma, X) \cap Sing(X) = \emptyset$ and $C(\gamma, X)$ is hyperbolic, for some hyperbolic closed orbit γ.

3 Proof of Theorem A

Let M be as before and let $X \in \mathfrak{X}(M)$. If $\eta > 0$ then the local strong stable manifold $W^{ss}_{\eta(p)}(p)$ of p and the local stable manifolds $W^s_{\eta(\gamma)}(\gamma)$ of γ are defined by

$$W^{ss}_{\eta(p)}(p) = \{y \in M : d(X^t(y), X^t(p)) < \eta(p), \text{if}\; t \geq 0\},$$

and

$$W^s_{\eta(\gamma)}(\gamma) = \{y \in M : d(X^t(y), X^t(\gamma)) < \eta(\gamma),\ \text{if } t \geq 0\}.$$

By the stable manifold theorem, there is $\epsilon = \epsilon(p) > 0$ such that

$$W^{ss}(p) = \bigcup_{t \geq 0} X^{-t}(W^{ss}_\epsilon(X^t(p))).$$

Analogously we can define the strong unstable manifold, unstable manifold, local strong unstable manifold and local unstable manifold. For a hyperbolic $\sigma \in Sing(X)$, there is $\epsilon(\sigma) > 0$ such that

$$W^s_{\epsilon(\sigma)}(\sigma) = \{x \in M : d(X^t(x), \sigma) < \epsilon(\sigma) \text{ if } t \geq 0\}.$$

Moreover,

$$W^s(\sigma) = \bigcup_{t \geq 0} X^{-t}(W^s_{\epsilon(\sigma)}(\sigma)).$$

Analogous definitions hold for the unstable manifolds. Denote by $\text{index}(\sigma) = \dim W^s(\sigma)$.

Lemma 3.1. *Let γ be a hyperbolic closed orbit of X. If X has the shadowing property on the chain recurrence class $C(\gamma, X)$ then for any hyperbolic $\sigma \in C(\gamma, X) \cap Crit(X)$,*

$$W^s(\sigma) \cap W^u(\gamma) \neq \emptyset \ \text{ and } W^u(\sigma) \cap W^s(\gamma) \neq \emptyset\ ,$$

where $Crit(X) = Sing(X) \cup Per(X)$.

Proof. Let $p \in \gamma$. Since σ, γ are hyperbolic, there are $\epsilon(\sigma) > 0$ and $\epsilon(p) > 0$ such that $W^u_{\epsilon(\sigma)}(\sigma)$ and $W^{ss}_{\epsilon(p)}(p)$ are defined as the above. Take $\epsilon = \min\{\epsilon(\sigma), \epsilon(p)\}$. Let $0 < \delta < \epsilon$ be the number of the shadowing property of X^t. Since $\sigma \in C(\gamma, X)$, we can construct a $(\delta, 1)$-pseudo orbit $\{(x_i, t_i) : t_i = 1, i \in \mathbb{Z}\} \subset C(\gamma, X)$ such that (i) $x_0 = \sigma$, $x_{-i} = \sigma, t_{-i} = 1$, for $i \geq 0$ (ii) $x_n = p$, $x_{n+i} = X^{t_i}(p)$, $t_i = 1$ for $i \geq 0$, and (iii) $x_i \in C(\gamma, X)$ for $i = 1, \ldots, n-1$ and $t_i = 1$. Since X^t has the shadowing property on $C(\gamma, X)$, there are $y \in M$ and an increasing homeomorphism $h : \mathbb{R} \to \mathbb{R}$ with $h(0) = 0$ such that

$$d(X^{h(t)}(y), X^{t-s_i}(x_i)) < \epsilon, \quad s_i \leq t < s_{i+1},$$

where $s_i = t_0 + t_1 + \ldots + t_{i-1}$ for $i > 0$, $s_0 = 0$ and $s_i = -t_{-1} - t_{-2} - \ldots - t_i$ for $i < 0$. Since $x_i = \sigma$ for $i < 0$, it is clear that $y \in W^u_\epsilon(\sigma)$. Since X^t has the shadowing property on $C(\gamma, X)$, we know

$$d(X^{h(t)}(y), X^{t-s_n}(x_n)) = d(X^{h(t)}(y), X^{t-s_n}(p)) < \epsilon.$$

We set $h(t) = \tau$. Then we have $d(X^{\tau+i}(y), X^i(p)) < \epsilon$ for $i \geq 0$, and so, $X^\tau(y) \in W^{ss}_\epsilon(p)$. Since $W^{ss}_\epsilon(p) \subset W^s_\epsilon(\gamma) \subset W^s(\gamma)$, we have $y \in X^{-\tau}(W^s_\epsilon(\gamma)) \subset W^s(\gamma)$. Thus $y \in W^u(\sigma) \cap W^s(\gamma)$, that is, $W^u(\sigma) \cap W^s(\gamma) \neq \emptyset$. The another case $W^s(\sigma) \cap W^u(\gamma) \neq \emptyset$ is similar as the previous arguments. □

We say that $X \in \mathfrak{X}(M)$ is *Kupka-Smale* if every critical orbits are hyperbolic and their stable and unstable manifolds are transverse intersection. Denote by $\mathcal{KS}$ the set of all Kupka-Smale vector fields. It is well-known that $\mathcal{KS}$ is a residual set of $\mathfrak{X}(M)$ (see [8]).

Lemma 3.2. *There is a residual set $\mathcal{G} \subset \mathfrak{X}(M)$ such that for any $X \in \mathcal{G}$, if X has the shadowing property on the chain recurrence class $C(\gamma, X)$ which contains a hyperbolic periodic orbit γ then $C(\gamma, X) \cap Sing(X) = \varnothing$.*

Proof. Let $X \in \mathcal{G} = \mathcal{KS}$ and let γ be a hyperbolic periodic orbit of X in $C(\gamma, X)$ with $\dim W^s(\gamma) = j$. To derive a contradiction, for a hyperbolic $\sigma \in C(\gamma, X) \cap Sing(X)$, we assume $\dim W^s(\sigma) = i$.

If $j < i$ then we know $\dim W^s(\sigma) + \dim W^u(\gamma) \leq \dim M$. Since $X \in \mathcal{KS}$, if $\dim W^s(\sigma) + \dim W^u(\gamma) < \dim M$ then $W^s(\sigma) \cap W^u(\gamma) = \varnothing$. Thus we have $\dim W^s(\sigma) + \dim W^u(\gamma) = \dim M$. Then by [3, Lemma 3.4], $W^s(\sigma) \cap W^u(\gamma) = \varnothing$. This is a contradiction by Lemma 3.1.

For other case $j \geq i$, we have $\dim W^u(\sigma) + \dim W^s(\gamma) \leq \dim M$. Then by the previous arguments we have a contradiction. Thus $C(\gamma, X) \cap Sing(X) = \varnothing$. □

Proof of Theorem A Let M be a closed three-dimensional manifold, and let $X \in \mathcal{G} \subset \mathfrak{X}(M)$ which is far from homoclinic tangencies. Suppose that X has the shadowing property on the chain recurrence class $C(\gamma, X)$. Then by Lemma 3.2, we know $C(\gamma, X) \cap Sing(X) = \varnothing$. By Theorem 1.1, the chain recurrence class $C(\gamma, X)$ is hyperbolic. □

We say that Λ is *isolated* if there is a neighborhood U of Λ such that $\Lambda = \bigcap_{t \in \mathbb{R}} X^t(U)$. The closure of the transversal homoclinic points of X^t associated to γ is called the *homoclinic class* of X^t associated to γ, and it is denoted by

$$H(\gamma, X) = \overline{W^s(\gamma) \pitchfork W^u(\gamma)}.$$

It is clear that $H(\gamma, X)$ is a compact, transitive and X^t-invariant set. The following is the version of vector field of a result for diffeomorphism (see [2, 4]).

Lemma 3.3. *There is a residual set $\mathcal{G}_0$ such that for any $X \in \mathcal{G}_0$, if the set Λ is isolated and transitive then $\Lambda = H(\gamma, X) = C(\gamma, X)$, for some hyperbolic periodic orbit γ.*

We say that a X^t-invariant set Δ is *chain transitive* if for any $x, y \in \Delta$, and $\delta > 0$ there is a δ-pseudo orbit $\{(x_i, t_i) : t_i \geq 1, \text{ for } 0 \leq i \leq n\} \subset \Delta$ with $x_0 = x, x_n = y$.

Then we have the following theorem which is a general result since shadowing points will be taken from M.

Corollary 3.4. *For a C^1 generic vector field $X \in \mathfrak{X}(M^3)$ which is far from homoclinic tangencies, if X has the shadowing property on an isolated chain transitive set Δ, then $\Delta \cap Sing(X) = \varnothing$ and Δ is hyperbolic.*

Proof. Take a residual set $\mathcal{TR}$ as in [30, Theorem 4]. Let M be a closed three-dimensional manifold, and let $X \in \mathcal{R} = \mathcal{G} \cap \mathcal{G}_0 \cap \mathcal{TR}$ which is far from homoclinic tangencies. Suppose that X has the shadowing property on an isolated chain transitive set Δ. Since X has the shadowing property on an isolated chain transitive set Δ, as in [30, Lemma 7, Lemma 8], we have $Sing(X) \cap \Delta = \varnothing$. Since $X \in \mathcal{R}$, the chain transitive set Δ is the transitive set Λ. By Lemma 3.3, $\Delta = \Lambda = H(\gamma, X) = C(\gamma, X)$ and by Theorem A, $C(\gamma, X)$ is hyperbolic. □

For the chain recurrence class, Ahn *et al* [1] proved that if a C^1 generic diffeomorphism has the shadowing property on an isolated chain recurrence class which contains a hyperbolic periodic point then it is hyperbolic. Lee and Lee [11] proved that a C^1 generic diffeomorphism has the shadowing property on chain recurrence class which contains a hyperbolic periodic point then it is hyperbolic which is a general result of [1]. Then the following is a version of vector fields from the previous results.

Corollary 3.5. *For C^1 generic vector field $X \in \mathfrak{X}(M^n)(n \geq 3)$, if X has the shadowing property on an isolated chain recurrence class $C(\gamma, X)$ then $C(\gamma, X) \cap Sing(X) = \emptyset$ and it is hyperbolic.*

Proof. Let $X \in \mathcal{R}$ such that $\mathcal{R}$ be as in the Corollary 3.4. Suppose that X has the shadowing property on an isolated chain recurrence class $C(\gamma, X)$. Since $C(\gamma, X)$ is isolated, by Lemma 3.3, $C(\gamma, X)$ is an isolated transitive set which is a chain transitive set Δ. Let U be an isolated neighborhood of Δ. Since Δ is isolated, the shadowing points are taken from Δ. Indeed, For any $\epsilon > 0$, let $0 < \delta < \epsilon$ be the number of the shadowing property of X^t. Then we assume that $\Delta \subset B_\epsilon(\Delta) \subset U$. Then for any $(\delta > 0, T(\geq 1))$ pseudo orbit $\{(x_i, t_i) : t_i \geq T, i \in \mathbb{Z}\} \subset \Delta$ there is $y \in B_\epsilon(x)$ such that y is ϵ shadowing the pseudo orbit. Since Δ is isolated in U, we have that the orbit $Orb_X(y) \in \bigcap_{t \in \mathbb{R}} X^t(U) = \Delta$. Thus as in the proof of [30, Theorem A], $C(\gamma, X)$ is transitive hyperbolic. □

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