Picture fuzzy soft sets over UP-algebras

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Abstract. This paper aims to extend the concept of picture fuzzy sets in UP-algebras to picture fuzzy soft sets over UP-algebras by merging the concept of picture fuzzy sets and soft sets. Further, we discuss the eight new concepts of picture fuzzy soft UP-subalgebras, picture fuzzy soft near UP-filters, picture fuzzy soft UP-filters, picture fuzzy soft implicative UP-filters, picture fuzzy soft comparative UP-filters, picture fuzzy soft shift UP-filters, picture fuzzy soft UP-ideals, and picture fuzzy soft strong UP-ideals of UP-algebras, and provide some properties.

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Key words: UP-algebra; picture fuzzy set; picture fuzzy UP-subalgebra; picture fuzzy near UP-filter; picture fuzzy UP-filter; picture fuzzy implicative UP-filter; picture fuzzy comparative UP-filter; picture fuzzy shift UP-filter; picture fuzzy UP-ideal; picture fuzzy strong UP-ideal.

1 Introduction

The concept of fuzzy sets was first considered by Zadeh [31] in 1965. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere. After the introduction of the concept of fuzzy sets by Zadeh [31], Atanassov [3, 4] defined a new concept called an intuitionistic fuzzy set which is a generalization of fuzzy set. The concept of picture fuzzy sets was first considered by Cuong and Kreinovich [6] in 2013, which is direct extensions of the fuzzy sets and the intuitionistic fuzzy sets. The picture fuzzy set is characterized by three functions expressing the degree of membership, the degree of neutral membership, and the degree of non-membership. The only constraint is that the sum of the three degrees must not exceed 1. Cuong [5] presented the concept of picture fuzzy sets in the Journal of Computer Science and Cybernetics in 2014. Some operations on picture fuzzy sets with some properties are considered. The Zadeh Extension Principle, picture fuzzy relations, and picture fuzzy soft sets are studied. Several researches were conducted on the generalizations of the concept of picture fuzzy sets in a variety of different fields and its application to a decision-making problem. In 2015, Singh [25] presented a geometrical interpretation of picture fuzzy sets. The author proposed correlation coefficients for picture fuzzy sets which considers the degree of positive membership, degree of neutral membership, degree of negative membership and the

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degree of refusal membership. In 2017, Wei [27] presented another form of eight similarity measures between picture fuzzy sets based on the cosine function between picture fuzzy sets by considering the degree of positive membership, degree of neutral membership, degree of negative membership and degree of refusal membership in picture fuzzy sets. The author applied these weighted cosine function similarity measures between picture fuzzy sets to strategic decision making. In 2018, Wei and Gao [29] presented some novel Dice similarity measures of picture fuzzy sets and the generalized Dice similarity measures of picture fuzzy sets and indicate that the Dice similarity measures and asymmetric measures (projection measures) are the special cases of the generalized Dice similarity measures in some parameter values. Wei [28] presented some novel process to measure the similarity between picture fuzzy sets. The author applied these similarity measures between picture fuzzy sets to building material recognition and minerals field recognition. In 2020, Ganie et al. [8] introduced two correlation coefficients of picture fuzzy sets. These correlation coefficients of picture fuzzy sets are better than existing ones and effective in expressing the nature of correlation (positive or negative correlation).

A soft set over a universe set is a parametrized family of subsets of the universe set. In 1999, to solve complicated problems in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as the probability theory, the theory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets. However, all of these theories have their own difficulties which are pointed out by Molodtsov [19]. In 2001, Maji et al. [18] introduced the concept of fuzzy soft sets as a generalization of the standard soft sets, and presented an application of fuzzy soft sets in a decision making problem. In 2015, Yang et al. [30] introduced the concept of picture fuzzy soft sets and studied some of their relevant properties, especially, a sufficient and necessary condition is presented to ensure that the dual laws are true in picture fuzzy soft theory.

In this paper, we extend the concept of picture fuzzy sets in UP-algebras to picture fuzzy soft sets over UP-algebras by merging the concept of picture fuzzy sets and soft sets. Further, we discuss the eight new concepts of picture fuzzy soft UP-subalgebras, picture fuzzy soft near UP-filters, picture fuzzy soft UP-filters, picture fuzzy soft implicative UP-filters, picture fuzzy soft comparative UP-filters, picture fuzzy soft shift UP-filters, picture fuzzy soft UP-ideals, and picture fuzzy soft strong UP-ideals of UP-algebras, and provide some properties.

2 Basic results on UP-algebras

Before we begin our study, let's review the definition of UP-algebras.

Definition 2.1. [11] An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra*, where X is a nonempty set, $\cdot$ is a binary operation on X, and 0 is a fixed element of

X (i.e., a nullary operation) if it satisfies the following axioms:

$$(\forall x, y, z \in X)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0), \tag{2.1}$$
$$(\forall x \in X)(0 \cdot x = x), \tag{2.2}$$
$$(\forall x \in X)(x \cdot 0 = 0), \text{ and} \tag{2.3}$$
$$(\forall x, y \in X)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y). \tag{2.4}$$

From [11], we know that the concept of UP-algebras is a generalization of KU-algebras (see [20]).

The binary relation $\leq$ on a UP-algebra $X = (X, \cdot, 0)$ is defined as follows:

$$(\forall x, y \in X)(x \leq y \Leftrightarrow x \cdot y = 0) \tag{2.5}$$

and the following assertions are valid (see [11, 12]).

$$(\forall x \in X)(x \leq x), \tag{2.6}$$
$$(\forall x, y, z \in X)(x \leq y, y \leq z \Rightarrow x \leq z), \tag{2.7}$$
$$(\forall x, y, z \in X)(x \leq y \Rightarrow z \cdot x \leq z \cdot y), \tag{2.8}$$
$$(\forall x, y, z \in X)(x \leq y \Rightarrow y \cdot z \leq x \cdot z), \tag{2.9}$$
$$(\forall x, y, z \in X)(x \leq y \cdot x, \text{ in particular, } y \cdot z \leq x \cdot (y \cdot z)), \tag{2.10}$$
$$(\forall x, y \in X)(y \cdot x \leq x \Leftrightarrow x = y \cdot x), \tag{2.11}$$
$$(\forall x, y \in X)(x \leq y \cdot y), \tag{2.12}$$
$$(\forall a, x, y, z \in X)(x \cdot (y \cdot z) \leq x \cdot ((a \cdot y) \cdot (a \cdot z))), \tag{2.13}$$
$$(\forall a, x, y, z \in X)(((a \cdot x) \cdot (a \cdot y)) \cdot z \leq (x \cdot y) \cdot z), \tag{2.14}$$
$$(\forall x, y, z \in X)((x \cdot y) \cdot z \leq y \cdot z), \tag{2.15}$$
$$(\forall x, y, z \in X)(x \leq y \Rightarrow x \leq z \cdot y), \tag{2.16}$$
$$(\forall x, y, z \in X)((x \cdot y) \cdot z \leq x \cdot (y \cdot z)), \text{ and} \tag{2.17}$$
$$(\forall a, x, y, z \in X)((x \cdot y) \cdot z \leq y \cdot (a \cdot z)). \tag{2.18}$$

Example 2.2. [22] Let U be a nonempty set and let $X \in \mathcal{P}(U)$, where $\mathcal{P}(U)$ means the power set of U. Let $\mathcal{P}_X(U) = \{A \in \mathcal{P}(U) \mid X \subseteq A\}$. Define a binary operation $\triangle$ on $\mathcal{P}_X(U)$ by putting $A \triangle B = B \cap (A^C \cup X)$ for all $A, B \in \mathcal{P}_X(U)$, where A^C means the complement of a subset A. Then $(\mathcal{P}_X(U), \triangle, X)$ is a UP-algebra. Let $\mathcal{P}^X(U) = \{A \in \mathcal{P}(U) \mid A \subseteq X\}$. Define a binary operation $\blacktriangle$ on $\mathcal{P}^X(U)$ by putting $A \blacktriangle B = B \cup (A^C \cap X)$ for all $A, B \in \mathcal{P}^X(U)$. Then $(\mathcal{P}^X(U), \blacktriangle, X)$ is a UP-algebra.

Example 2.3. [7] Let $\mathbb{Z}^*$ be the set of all nonnegative integers. Define two binary operations $\circ$ and $\star$ on $\mathbb{Z}^*$ by:

$$(\forall m, n \in \mathbb{Z}^*)\left(m \circ n = \begin{cases} n & \text{if } m < n, \\ 0 & \text{otherwise} \end{cases}\right)$$

and

$$(\forall m, n \in \mathbb{Z}^*)\left(m \star n = \begin{cases} n & \text{if } m > n \text{ or } m = 0, \\ 0 & \text{otherwise} \end{cases}\right).$$

Then $(\mathbb{Z}^*, \circ, 0)$ and $(\mathbb{Z}^*, \star, 0)$ are UP-algebras.

For more examples of UP-algebras, see [1, 2, 12, 13, 21, 22, 23, 24].

For a nonempty subset S of a UP-algebra $X = (X, \cdot, 0)$ which satisfies the following condition:

$$(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S). \tag{2.19}$$

Then the constant 0 of X is in S. Indeed, let $x \in S$. By (2.6) and (2.19), we have $0 = x \cdot x \in S$.

Definition 2.4. [9, 10, 11, 14, 15, 16, 26] A nonempty subset S of a UP-algebra $X = (X, \cdot, 0)$ is called

(1) a *UP-subalgebra* of X if it satisfies the following condition:

$$(\forall x, y \in S)(x \cdot y \in S), \tag{2.20}$$

(2) a *near UP-filter* of X if it satisfies the condition (2.19),

(3) a *UP-filter* of X if it satisfies the following conditions:

$$\text{the constant } 0 \text{ of } X \text{ is in } S, \tag{2.21}$$

$$(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S), \tag{2.22}$$

(4) an *implicative UP-filter* of X if it satisfies the condition (2.21) and the following condition:

$$(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, x \cdot y \in S \Rightarrow x \cdot z \in S), \tag{2.23}$$

(5) a *comparative UP-filter* of X if it satisfies the condition (2.21) and the following condition:

$$(\forall x, y, z \in X)(x \cdot ((y \cdot z) \cdot y) \in S, x \in S \Rightarrow y \in S), \tag{2.24}$$

(6) a *shift UP-filter* of X if it satisfies the condition (2.21) and the following condition:

$$(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, x \in S \Rightarrow ((z \cdot y) \cdot y) \cdot z \in S), \tag{2.25}$$

(7) a *UP-ideal* of X if it satisfies the condition (2.21) and the following condition:

$$(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S), \tag{2.26}$$

(8) a *strong UP-ideal* of X if it satisfies the condition (2.21) and the following condition:

$$(\forall x, y, z \in X)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S). \tag{2.27}$$

Guntasow et al. [9] proved that the only strong UP-ideal of a UP-algebra X is X.

3 PFSs in UP-algebras

In 2013, Cuong and Kreinovich [6] introduced the concept of picture fuzzy sets as the following definition.

A *picture fuzzy set* (briefly, PFS) in a nonempty set X is a structure of the form:

$$P = \{(x, r_P(x), g_P(x), b_P(x)) \mid x \in X\},$$

where $r_P : X \to [0,1]$ is a *positive membership*, $g_P : X \to [0,1]$ is a *neutral membership*, and $b_P : X \to [0,1]$ is a *negative membership* satisfy the following condition:

$$(\forall x \in X)(r_P(x) + g_P(x) + b_P(x) \leq 1).$$

For our convenience, we will denote a PFS as $P = (X, r_P, g_P, b_P)$.

A PFS P in X is said to be *constant* if P is a constant function from X to $[0,1]^3$. That is, r_P, g_P, and b_P are constant functions from X to $[0,1]$.

In what follows, let X denote a UP-algebra $(X, \cdot, 0)$ unless otherwise specified.

Kankaew et al. [17] introduced the eight new concepts of picture fuzzy sets in UP-algebras: picture fuzzy UP-subalgebras, picture fuzzy near UP-filters, picture fuzzy UP-filters, picture fuzzy implicative UP-filters, picture fuzzy comparative UP-filters, picture fuzzy shift UP-filters, picture fuzzy UP-ideals, and picture fuzzy strong UP-ideals.

Definition 3.1. A PFS P in X is called

(1) a *picture fuzzy UP-subalgebra* of X if it satisfies the following conditions:

$$(\forall x, y \in X)(r_P(x \cdot y) \geq \min\{r_P(x), r_P(y)\}), \tag{3.1}$$
$$(\forall x, y \in X)(g_P(x \cdot y) \geq \min\{g_P(x), g_P(y)\}), \tag{3.2}$$
$$(\forall x, y \in X)(b_P(x \cdot y) \leq \max\{b_P(x), b_P(y)\}), \tag{3.3}$$

(2) a *picture fuzzy near UP-filter* of X if it satisfies the following conditions:

$$(\forall x, y \in X)(r_P(x \cdot y) \geq r_P(y)), \tag{3.4}$$
$$(\forall x, y \in X)(g_P(x \cdot y) \geq g_P(y)), \tag{3.5}$$
$$(\forall x, y \in X)(b_P(x \cdot y) \leq b_P(y)), \tag{3.6}$$

(3) a *picture fuzzy UP-filter* of X if it satisfies the following conditions:

$$(\forall x \in X)(r_P(0) \geq r_P(x)), \tag{3.7}$$
$$(\forall x \in X)(g_P(0) \geq g_P(x)), \tag{3.8}$$
$$(\forall x \in X)(b_P(0) \leq b_P(x)), \tag{3.9}$$
$$(\forall x, y \in X)(r_P(y) \geq \min\{r_P(x \cdot y), r_P(x)\}), \tag{3.10}$$
$$(\forall x, y \in X)(g_P(y) \geq \min\{g_P(x \cdot y), g_P(x)\}), \tag{3.11}$$
$$(\forall x, y \in X)(b_P(y) \leq \max\{b_P(x \cdot y), b_P(x)\}), \tag{3.12}$$

(4) a *picture fuzzy implicative UP-filter* of X if it satisfies the following conditions: (3.7), (3.8), (3.9), and

$$(3.13) \qquad (\forall x, y, z \in X)(r_P(x \cdot z) \geq \min\{r_P(x \cdot (y \cdot z)), r_P(x \cdot y)\}),$$
$$(3.14) \qquad (\forall x, y, z \in X)(g_P(x \cdot z) \geq \min\{g_P(x \cdot (y \cdot z)), g_P(x \cdot y)\}),$$
$$(3.15) \qquad (\forall x, y, z \in X)(b_P(x \cdot z) \leq \max\{b_P(x \cdot (y \cdot z)), b_P(x \cdot y)\}),$$

(5) a *picture fuzzy comparative UP-filter* of X if it satisfies the following conditions: (3.7), (3.8), (3.9), and

$$(3.16) \qquad (\forall x, y, z \in X)(r_P(y) \geq \min\{r_P(x \cdot ((y \cdot z) \cdot y)), r_P(x)\}),$$
$$(3.17) \qquad (\forall x, y, z \in X)(g_P(y) \geq \min\{g_P(x \cdot ((y \cdot z) \cdot y)), g_P(x)\}),$$
$$(3.18) \qquad (\forall x, y, z \in X)(b_P(y) \leq \max\{b_P(x \cdot ((y \cdot z) \cdot y)), b_P(x)\}),$$

(6) a *picture fuzzy shift UP-filter* of X if it satisfies the following conditions: (3.7), (3.8), (3.9), and

$$(3.19) \qquad (\forall x, y, z \in X)(r_P(((z \cdot y) \cdot y) \cdot z) \geq \min\{r_P(x \cdot (y \cdot z)), r_P(x)\}),$$
$$(3.20) \qquad (\forall x, y, z \in X(g_P(((z \cdot y) \cdot y) \cdot z) \geq \min\{g_P(x \cdot (y \cdot z)), g_P(x)\}),$$
$$(3.21) \qquad (\forall x, y, z \in X)(b_P(((z \cdot y) \cdot y) \cdot z) \leq \max\{b_P(x \cdot (y \cdot z)), b_P(x)\}),$$

(7) a *picture fuzzy UP-ideal* of X if it satisfies the following conditions: (3.7), (3.8), (3.9), and

$$(3.22) \qquad (\forall x, y, z \in X)(r_P(x \cdot z) \geq \min\{r_P(x \cdot (y \cdot z)), r_P(y)\}),$$
$$(3.23) \qquad (\forall x, y, z \in X)(g_P(x \cdot z) \geq \min\{g_P(x \cdot (y \cdot z)), g_P(y)\}),$$
$$(3.24) \qquad (\forall x, y, z \in X)(b_P(x \cdot z) \leq \max\{b_P(x \cdot (y \cdot z)), b_P(y)\}),$$

(8) a *picture fuzzy strong UP-ideal* of X if it satisfies the following conditions: (3.7), (3.8), (3.9), and

$$(3.25) \qquad (\forall x, y, z \in X)(r_P(x) \geq \min\{r_P((z \cdot y) \cdot (z \cdot x)), r_P(y)\}),$$
$$(3.26) \qquad (\forall x, y, z \in X)(g_P(x) \geq \min\{g_P((z \cdot y) \cdot (z \cdot x)), g_P(y)\}),$$
$$(3.27) \qquad (\forall x, y, z \in X)(b_P(x) \leq \max\{b_P((z \cdot y) \cdot (z \cdot x)), b_P(y)\}).$$

Kankaew et al. [17] proved the generalization that the concept of picture fuzzy UP-subalgebras is a generalization of picture fuzzy near UP-filters, picture fuzzy near UP-filters is a generalization of picture fuzzy UP-filters, picture fuzzy UP-filters is a generalization of picture fuzzy comparative UP-filters, picture fuzzy UP-filters is a generalization of picture fuzzy shift UP-filters, picture fuzzy UP-filters is a generalization of picture fuzzy UP-ideals, picture fuzzy UP-ideals is a generalization of picture fuzzy implicative UP-filters, and picture fuzzy implicative UP-filters, picture fuzzy comparative UP-filters, and picture fuzzy shift UP-filters is a generalization of picture fuzzy strong UP-ideals. Moreover, they proved that picture fuzzy strong UP-ideals and constant picture fuzzy sets coincide.

4 PFSSs over UP-algebras

In 2015, Yang et al. [30] introduced the concept of picture fuzzy soft sets as the following definition.

Let X be a reference set (or an initial universe set) and P be a set of parameters. Let $\mathrm{PFS}(X)$ be the set of all PFSs in X and Y be a nonempty subset of P. A pair $(\widetilde{\mathrm{P}}, Y)$ is called a *picture fuzzy soft set* (briefly, PFSS) over X, where $\widetilde{\mathrm{P}}$ is a mapping given by

$$\widetilde{\mathrm{P}}\colon Y \to \mathrm{PFS}(X), p \mapsto \widetilde{\mathrm{P}}[p].$$

Now, we introduce the eight new concepts of picture fuzzy soft sets over UP-algebras: picture fuzzy soft UP-subalgebras, picture fuzzy soft near UP-filters, picture fuzzy soft UP-filters, picture fuzzy soft implicative UP-filters, picture fuzzy soft comparative UP-filters, picture fuzzy soft shift UP-filters, picture fuzzy soft UP-ideals, and picture fuzzy soft strong UP-ideals, provide the necessary examples, investigate their properties, and prove their generalizations.

Definition 4.1. Let Y be a nonempty subset of P. A picture fuzzy soft set $(\widetilde{\mathrm{P}}, Y)$ over X is called a *picture fuzzy soft UP-subalgebra* (resp., picture fuzzy soft near UP-filter, picture fuzzy soft UP-filter, picture fuzzy soft implicative UP-filter, picture fuzzy soft comparative UP-filter, picture fuzzy soft shift UP-filter, picture fuzzy soft UP-ideal, picture fuzzy soft strong UP-ideal) based on $p \in Y$ (we shortly call a *p-picture fuzzy soft UP-subalgebra* (resp., p-picture fuzzy soft near UP-filter, p-picture fuzzy soft UP-filter, p-picture fuzzy soft implicative UP-filter, p-picture fuzzy soft comparative UP-filter, p-picture fuzzy soft shift UP-filter, p-picture fuzzy soft UP-ideal, p-picture fuzzy soft strong UP-ideal) of X if the picture fuzzy set

$$\widetilde{\mathrm{P}}[p] := (X, r_{\widetilde{\mathrm{P}}[p]}, g_{\widetilde{\mathrm{P}}[p]}, b_{\widetilde{\mathrm{P}}[p]})$$

in X is a picture fuzzy UP-subalgebra (resp., picture fuzzy near UP-filter, picture fuzzy UP-filter, picture fuzzy implicative UP-filter, picture fuzzy comparative UP-filter, picture fuzzy shift UP-filter, picture fuzzy UP-ideal, picture fuzzy strong UP-ideal) of X. If $(\widetilde{\mathrm{P}}, Y)$ is a p-picture fuzzy soft UP-subalgebra (resp., p-picture fuzzy soft near UP-filter, p-picture fuzzy soft UP-filter, p-picture fuzzy soft implicative UP-filter, p-picture fuzzy soft comparative UP-filter, p-picture fuzzy soft shift UP-filter, p-picture fuzzy soft UP-ideal, p-picture fuzzy soft strong UP-ideal) of X for all $p \in Y$, we state that $(\widetilde{\mathrm{P}}, Y)$ is a *picture fuzzy soft UP-subalgebra* (resp., picture fuzzy soft near UP-filter, picture fuzzy soft UP-filter, picture fuzzy soft implicative UP-filter, picture fuzzy soft comparative UP-filter, picture fuzzy soft shift UP-filter, picture fuzzy soft UP-ideal, picture fuzzy soft strong UP-ideal) of X.

Theorem 4.1. *If $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-subalgebra of X, then it satisfies the following condition:*

$$(\forall p \in Y, \forall x \in X) \begin{pmatrix} r_{\widetilde{\mathrm{P}}[p]}(0) \geq r_{\widetilde{\mathrm{P}}[p]}(x) \\ g_{\widetilde{\mathrm{P}}[p]}(0) \geq g_{\widetilde{\mathrm{P}}[p]}(x) \\ b_{\widetilde{\mathrm{P}}[p]}(0) \leq b_{\widetilde{\mathrm{P}}[p]}(x) \end{pmatrix}. \tag{4.1}$$

That is, for any $p \in Y, \widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7)*,* (3.8)*, and* (3.9)*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-subalgebra of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-subalgebra of X. Let $x \in X$. Then

by (2.6) and (3.1) $\quad r_{\widetilde{\mathrm{P}}[p]}(0) = r_{\widetilde{\mathrm{P}}[p]}(x \cdot x) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(x)\} = r_{\widetilde{\mathrm{P}}[p]}(x),$

by (2.6) and (3.2) $\quad g_{\widetilde{\mathrm{P}}[p]}(0) = g_{\widetilde{\mathrm{P}}[p]}(x \cdot x) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(x)\} = g_{\widetilde{\mathrm{P}}[p]}(x),$

by (2.6) and (3.3) $\quad b_{\widetilde{\mathrm{P}}[p]}(0) = b_{\widetilde{\mathrm{P}}[p]}(x \cdot x) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(x)\} = b_{\widetilde{\mathrm{P}}[p]}(x).$

Hence, $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9), that is, $(\widetilde{\mathrm{P}}, Y)$ satisfies the condition (4.1). ☐

Theorem 4.2. *A PFSS $(\widetilde{\mathrm{P}}, Y)$ over X is a picture fuzzy soft strong UP-ideal of X if and only if for all $p \in Y, \widetilde{\mathrm{P}}[p]$ is constant.*

Proof. Assume that $\widetilde{\mathrm{P}}[p]$ is constant for all $p \in Y$. Let $p \in Y$. Then for all $x \in X, r_{\widetilde{\mathrm{P}}[p]}(x) = r_{\widetilde{\mathrm{P}}[p]}(0), g_{\widetilde{\mathrm{P}}[p]}(x) = g_{\widetilde{\mathrm{P}}[p]}(0)$, and $b_{\widetilde{\mathrm{P}}[p]}(x) = b_{\widetilde{\mathrm{P}}[p]}(0)$ and so $r_{\widetilde{\mathrm{P}}[p]}(0) \geq r_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(0) \geq g_{\widetilde{\mathrm{P}}[p]}(x)$, and $b_{\widetilde{\mathrm{P}}[p]}(0) \leq b_{\widetilde{\mathrm{P}}[p]}(x)$. Next, let $x, y, z \in X$. Then

$$r_{\widetilde{\mathrm{P}}[p]}(x) \geq r_{\widetilde{\mathrm{P}}[p]}(0) = \min\{r_{\widetilde{\mathrm{P}}[p]}(0), r_{\widetilde{\mathrm{P}}[p]}(0)\} = \min\{r_{\widetilde{\mathrm{P}}[p]}((z \cdot y) \cdot (z \cdot x)), r_{\widetilde{\mathrm{P}}[p]}(y)\},$$

$$g_{\widetilde{\mathrm{P}}[p]}(x) \geq g_{\widetilde{\mathrm{P}}[p]}(0) = \min\{g_{\widetilde{\mathrm{P}}[p]}(0), g_{\widetilde{\mathrm{P}}[p]}(0)\} = \min\{g_{\widetilde{\mathrm{P}}[p]}((z \cdot y) \cdot (z \cdot x)), g_{\widetilde{\mathrm{P}}[p]}(y)\},$$

$$b_{\widetilde{\mathrm{P}}[p]}(x) \leq b_{\widetilde{\mathrm{P}}[p]}(0) = \max\{b_{\widetilde{\mathrm{P}}[p]}(0), b_{\widetilde{\mathrm{P}}[p]}(0)\} = \max\{b_{\widetilde{\mathrm{P}}[p]}((z \cdot y) \cdot (z \cdot x)), b_{\widetilde{\mathrm{P}}[p]}(y)\}.$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy strong UP-ideal of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft strong UP-ideal of X.

Conversely, assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft strong UP-ideal of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy strong UP-ideal of X. Then for any $x \in X$,

$$\begin{aligned}
&\text{by (3.25)} & r_{\widetilde{\mathrm{P}}[p]}(x) &\geq \min\{r_{\widetilde{\mathrm{P}}[p]}((x \cdot 0) \cdot (x \cdot x)), r_{\widetilde{\mathrm{P}}[p]}(0)\} \\
&\text{by (2.3)} & &= \min\{r_{\widetilde{\mathrm{P}}[p]}(0 \cdot (x \cdot x)), r_{\widetilde{\mathrm{P}}[p]}(0)\} \\
&\text{by (2.2)} & &= \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot x), r_{\widetilde{\mathrm{P}}[p]}(0)\} \\
&\text{by (2.6)} & &= \min\{r_{\widetilde{\mathrm{P}}[p]}(0), r_{\widetilde{\mathrm{P}}[p]}(0)\} \\
& & &= r_{\widetilde{\mathrm{P}}[p]}(0), \\
&\text{by (3.26)} & g_{\widetilde{\mathrm{P}}[p]}(x) &\geq \min\{g_{\widetilde{\mathrm{P}}[p]}((x \cdot 0) \cdot (x \cdot x)), g_{\widetilde{\mathrm{P}}[p]}(0)\} \\
&\text{by (2.3)} & &= \min\{g_{\widetilde{\mathrm{P}}[p]}(0 \cdot (x \cdot x)), g_{\widetilde{\mathrm{P}}[p]}(0)\} \\
&\text{by (2.2)} & &= \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot x), g_{\widetilde{\mathrm{P}}[p]}(0)\} \\
&\text{by (2.6)} & &= \min\{g_{\widetilde{\mathrm{P}}[p]}(0), g_{\widetilde{\mathrm{P}}[p]}(0)\} \\
& & &= g_{\widetilde{\mathrm{P}}[p]}(0), \\
&\text{by (3.27)} & b_{\widetilde{\mathrm{P}}[p]}(x) &\leq \max\{b_{\widetilde{\mathrm{P}}[p]}((x \cdot 0) \cdot (x \cdot x)), b_{\widetilde{\mathrm{P}}[p]}(0)\} \\
&\text{by (2.3)} & &= \max\{b_{\widetilde{\mathrm{P}}[p]}(0 \cdot (x \cdot x)), b_{\widetilde{\mathrm{P}}[p]}(0)\} \\
&\text{by (2.2)} & &= \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot x), b_{\widetilde{\mathrm{P}}[p]}(0)\} \\
&\text{by (2.6)} & &= \max\{b_{\widetilde{\mathrm{P}}[p]}(0), b_{\widetilde{\mathrm{P}}[p]}(0)\} \\
& & &= b_{\widetilde{\mathrm{P}}[p]}(0).
\end{aligned}$$

By (3.7), (3.8), and (3.9), we have $r_{\widetilde{\mathrm{P}}[p]}(x) = r_{\widetilde{\mathrm{P}}[p]}(0)$, $g_{\widetilde{\mathrm{P}}[p]}(x) = g_{\widetilde{\mathrm{P}}[p]}(0)$, and $b_{\widetilde{\mathrm{P}}[p]}(x) = b_{\widetilde{\mathrm{P}}[p]}(0)$ for all $x \in X$. Hence, $\widetilde{\mathrm{P}}[p]$ is constant. □

Theorem 4.3. *Every picture fuzzy soft near UP-filter of X is a picture fuzzy soft UP-subalgebra.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft near UP-filter of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy near UP-filter of X. Let $x, y \in X$. Then

by (3.4) $\quad r_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \geq r_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(y)\}$,

by (3.5) $\quad g_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \geq g_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(y)\}$,

by (3.6) $\quad b_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \leq b_{\widetilde{\mathrm{P}}[p]}(y) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(y)\}$.

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-subalgebra of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-subalgebra of X. □

The following example shows that the converse of Theorem 4.3 is not true.

Example 4.2. Let X be the set of five species of the cat, that is,

$$X = \{\text{Persian}, \text{Sphynx}, \text{Munchkin}, \text{Bengal}, \text{Thai}\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	Persian	Sphynx	Munchkin	Bengal	Thai
Persian	Persian	Sphynx	Munchkin	Bengal	Thai
Sphynx	Persian	Persian	Munchkin	Bengal	Thai
Munchkin	Persian	Persian	Persian	Bengal	Bengal
Bengal	Persian	Sphynx	Munchkin	Persian	Bengal
Thai	Persian	Sphynx	Munchkin	Persian	Persian

Then $(X, \cdot, \text{Persian})$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{price}, \text{playful}, \text{clean}\}$$

with $\widetilde{\mathrm{P}}[\text{price}]$, $\widetilde{\mathrm{P}}[\text{playful}]$, and $\widetilde{\mathrm{P}}[\text{clean}]$ are picture fuzzy sets in X defined as follows:

$\widetilde{\mathrm{P}}$	Persian	Sphynx	Munchkin	Bengal	Thai
$r_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.4	0.5	0.3	0.3
$g_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.4	0.3	0.2	0.1
$b_{\widetilde{\mathrm{P}}[\text{price}]}$	0	0.1	0.2	0.1	0
$r_{\widetilde{\mathrm{P}}[\text{playful}]}$	0.5	0.4	0.4	0.3	0.2
$g_{\widetilde{\mathrm{P}}[\text{playful}]}$	0.5	0.4	0.3	0.1	0
$b_{\widetilde{\mathrm{P}}[\text{playful}]}$	0	0.2	0.2	0.1	0.1
$r_{\widetilde{\mathrm{P}}[\text{clean}]}$	0.4	0.4	0.3	0.2	0.1
$g_{\widetilde{\mathrm{P}}[\text{clean}]}$	0.4	0.3	0.3	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{clean}]}$	0.1	0.2	0.2	0.1	0.2

Then $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-subalgebra of X. But $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft near UP-filter of X. Indeed,

$$\begin{aligned} b_{\widetilde{\mathrm{P}}[\text{price}]}(\text{Bengal} \cdot \text{Thai}) &= b_{\widetilde{\mathrm{P}}[\text{price}]}(\text{Bengal}) \\ &= 0.1 \\ &> 0 \\ &= b_{\widetilde{\mathrm{P}}[\text{price}]}(\text{Thai}). \end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[\text{price}]$ is not a picture fuzzy near UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft near UP-filter of X.

Theorem 4.4. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft near UP-filter of* X*, then it satisfies the condition* (4.1).

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft near UP-filter of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy near UP-filter of X. Let $x \in X$. Then

by (2.6) and (3.1) $\quad r_{\widetilde{\mathrm{P}}[p]}(0) = r_{\widetilde{\mathrm{P}}[p]}(x \cdot x) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(x)\} = r_{\widetilde{\mathrm{P}}[p]}(x),$

by (2.6) and (3.2) $\quad g_{\widetilde{\mathrm{P}}[p]}(0) = g_{\widetilde{\mathrm{P}}[p]}(x \cdot x) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(x)\} = g_{\widetilde{\mathrm{P}}[p]}(x),$

by (2.6) and (3.3) $\quad b_{\widetilde{\mathrm{P}}[p]}(0) = b_{\widetilde{\mathrm{P}}[p]}(x \cdot x) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(x)\} = b_{\widetilde{\mathrm{P}}[p]}(x).$

Hence, $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9), that is, $(\widetilde{\mathrm{P}}, Y)$ satisfies the condition (4.1). □

Theorem 4.5. *Every picture fuzzy soft UP-filter of* X *is a picture fuzzy soft near UP-filter.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-filter of X. Let $x, y \in X$. Then

$$\begin{aligned}
&\text{by (3.10)} && r_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot y)), r_{\widetilde{\mathrm{P}}[p]}(y)\} \\
&\text{by (2.10)} && \qquad = \min\{r_{\widetilde{\mathrm{P}}[p]}(0), r_{\widetilde{\mathrm{P}}[p]}(y)\} \\
&\text{by (3.7)} && \qquad = r_{\widetilde{\mathrm{P}}[p]}(y), \\
&\text{by (3.11)} && g_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot y)), g_{\widetilde{\mathrm{P}}[p]}(y)\} \\
&\text{by (2.10)} && \qquad = \min\{g_{\widetilde{\mathrm{P}}[p]}(0), g_{\widetilde{\mathrm{P}}[p]}(y)\} \\
&\text{by (3.8)} && \qquad = g_{\widetilde{\mathrm{P}}[p]}(y), \\
&\text{by (3.12)} && b_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot y)), b_{\widetilde{\mathrm{P}}[p]}(y)\} \\
&\text{by (2.10)} && \qquad = \max\{b_{\widetilde{\mathrm{P}}[p]}(0), b_{\widetilde{\mathrm{P}}[p]}(y)\} \\
&\text{by (3.9)} && \qquad = b_{\widetilde{\mathrm{P}}[p]}(y).
\end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy near UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft near UP-filter of X. □

The following example shows that the converse of Theorem 4.5 is not true.

Example 4.3. Let X be the set of four coffees, that is,

$$X = \{\text{Espresso}, \text{Mocca}, \text{Latte}, \text{Americano}\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	Espresso	Mocca	Latte	Americano
Espresso	Espresso	Mocca	Latte	Americano
Mocca	Espresso	Espresso	Latte	Latte
Latte	Espresso	Mocca	Espresso	Mocca
Americano	Espresso	Espresso	Espresso	Espresso

Then $(X, \cdot, \text{Espresso})$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{aroma}, \text{strong}, \text{sweetness}, \text{bitter}\}$$

with $\widetilde{\mathrm{P}}[\text{aroma}], \widetilde{\mathrm{P}}[\text{strong}], \widetilde{\mathrm{P}}[\text{sweetness}]$, and $\widetilde{\mathrm{P}}[\text{bitter}]$ are picture fuzzy sets in X defined as follows:

$\widetilde{\mathrm{P}}$	Espresso	Mocca	Latte	Americano
$r_{\widetilde{\mathrm{P}}[\text{aroma}]}$	0.5	0.3	0.4	0.1
$g_{\widetilde{\mathrm{P}}[\text{aroma}]}$	0.5	0.4	0.3	0.3
$b_{\widetilde{\mathrm{P}}[\text{aroma}]}$	0	0.2	0.3	0.4
$r_{\widetilde{\mathrm{P}}[\text{strong}]}$	0.4	0.4	0.1	0
$g_{\widetilde{\mathrm{P}}[\text{strong}]}$	0.5	0.3	0.2	0.1
$b_{\widetilde{\mathrm{P}}[\text{strong}]}$	0	0	0.5	0.7
$r_{\widetilde{\mathrm{P}}[\text{sweetness}]}$	0.5	0.3	0.4	0.3
$g_{\widetilde{\mathrm{P}}[\text{sweetness}]}$	0.3	0.3	0.3	0.2
$b_{\widetilde{\mathrm{P}}[\text{sweetness}]}$	0.1	0.2	0.2	0.3
$r_{\widetilde{\mathrm{P}}[\text{bitter}]}$	0.5	0.4	0.4	0.2
$g_{\widetilde{\mathrm{P}}[\text{bitter}]}$	0.5	0.4	0.3	0.3
$b_{\widetilde{\mathrm{P}}[\text{bitter}]}$	0	0.1	0.3	0.3

Then $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft near UP-filter of X. But $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft UP-filter of X. Indeed,

$$\begin{aligned}
& g_{\widetilde{\mathrm{P}}[\text{sweetness}]}(\text{Americano}) \\
& = 0.2 \\
& < 0.3 \\
& = \min\{0.3, 0.3\} \\
& = \min\{g_{\widetilde{\mathrm{P}}[\text{sweetness}]}(\text{Mocca}), g_{\widetilde{\mathrm{P}}[\text{sweetness}]}(\text{Latte})\} \\
& = \min\{g_{\widetilde{\mathrm{P}}[\text{sweetness}]}(\text{Latte} \cdot \text{Americano}), g_{\widetilde{\mathrm{P}}[\text{sweetness}]}(\text{Latte})\}.
\end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[\text{sweetness}]$ is not a picture fuzzy UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft UP-filter of X.

Theorem 4.6. *Every picture fuzzy soft UP-ideal of X is a picture fuzzy soft UP-filter.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-ideal of X, so $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y \in X$. Then

$$\begin{array}{lrl}
\text{by (2.2)} & r_{\widetilde{\mathrm{P}}[p]}(y) & = r_{\widetilde{\mathrm{P}}[p]}(0 \cdot y) \\
\text{by (3.22)} & & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(0 \cdot (x \cdot y)), r_{\widetilde{\mathrm{P}}[p]}(x)\} \\
\text{by (2.2)} & & = \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot y), r_{\widetilde{\mathrm{P}}[p]}(x)\}, \\
\text{by (2.2)} & g_{\widetilde{\mathrm{P}}[p]}(y) & = g_{\widetilde{\mathrm{P}}[p]}(0 \cdot y) \\
\text{by (3.23)} & & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(0 \cdot (x \cdot y)), g_{\widetilde{\mathrm{P}}[p]}(x)\} \\
\text{by (2.2)} & & = \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot y), g_{\widetilde{\mathrm{P}}[p]}(x)\}, \\
\text{by (2.2)} & b_{\widetilde{\mathrm{P}}[p]}(y) & = b_{\widetilde{\mathrm{P}}[p]}(0 \cdot y) \\
\text{by (3.24)} & & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(0 \cdot (x \cdot y)), b_{\widetilde{\mathrm{P}}[p]}(x)\} \\
\text{by (2.2)} & & = \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot y), b_{\widetilde{\mathrm{P}}[p]}(x)\}.
\end{array}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X. □

The following example shows that the converse of Theorem 4.6 is not true.

Example 4.4. Let X be the set of five brands of the car, that is,

$$X = \{\boxminus, \boxtimes, \boxdot, \circledast, \circledcirc\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxminus$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxtimes$	$\boxminus$	$\boxminus$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxdot$	$\boxminus$	$\boxminus$	$\boxminus$	$\circledast$	$\circledast$
$\circledast$	$\boxminus$	$\boxtimes$	$\boxdot$	$\boxminus$	$\circledast$
$\circledcirc$	$\boxminus$	$\boxtimes$	$\boxdot$	$\boxminus$	$\boxminus$

Then $(X, \cdot, \boxminus)$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{price}, \text{speed}, \text{durable}\}$$

with $\widetilde{\mathrm{P}}[\text{price}], \widetilde{\mathrm{P}}[\text{speed}]$, and $\widetilde{\mathrm{P}}[\text{durable}]$ are picture fuzzy sets in X defined as follows:

$\widetilde{\mathrm{P}}$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$r_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.4	0.3	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{price}]}$	0	0.1	0.3	0.6	0.6
$r_{\widetilde{\mathrm{P}}[\text{speed}]}$	0.5	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{speed}]}$	0.4	0.4	0.3	0	0
$b_{\widetilde{\mathrm{P}}[\text{speed}]}$	0	0.3	0.5	0.6	0.6
$r_{\widetilde{\mathrm{P}}[\text{durable}]}$	0.4	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{durable}]}$	0.5	0.4	0.1	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{durable}]}$	0	0.1	0.3	0.5	0.5

Then $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X. But $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft UP-ideal of X. Indeed,

$$\begin{aligned}
b_{\widetilde{\mathrm{P}}[\text{speed}]}(\circledast \cdot \circledcirc) &= b_{\widetilde{\mathrm{P}}[\text{speed}]}(\circledast) \\
&= 0.6 \\
&> 0.5 \\
&= \max\{0, 0.5\} \\
&= \max\{b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxminus), b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxtimes)\} \\
&= \max\{b_{\widetilde{\mathrm{P}}[\text{speed}]}(\circledast \cdot (\boxtimes \cdot \circledcirc)), b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxtimes)\}.
\end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[\text{speed}]$ is not a picture fuzzy UP-ideal of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft UP-ideal of X.

Theorem 4.7. *Every picture fuzzy soft implicative UP-filter of X is a picture fuzzy soft UP-filter.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft implicative UP-filter of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy implicative UP-filter of X, so $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y \in X$. Then

$$\begin{aligned}
&\text{by (2.2)} && r_{\widetilde{\mathrm{P}}[p]}(y) = r_{\widetilde{\mathrm{P}}[p]}(0 \cdot y) \\
&\text{by (3.13)} && \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(0 \cdot (x \cdot y)), r_{\widetilde{\mathrm{P}}[p]}(0 \cdot x)\} \\
&\text{by (2.2)} && = \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot y), r_{\widetilde{\mathrm{P}}[p]}(x)\}, \\
&\text{by (2.2)} && g_{\widetilde{\mathrm{P}}[p]}(y) = g_{\widetilde{\mathrm{P}}[p]}(0 \cdot y) \\
&\text{by (3.14)} && \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(0 \cdot (x \cdot y)), g_{\widetilde{\mathrm{P}}[p]}(0 \cdot x)\} \\
&\text{by (2.2)} && = \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot y), g_{\widetilde{\mathrm{P}}[p]}(x)\}, \\
&\text{by (2.2)} && b_{\widetilde{\mathrm{P}}[p]}(y) = b_{\widetilde{\mathrm{P}}[p]}(0 \cdot y) \\
&\text{by (3.15)} && \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(0 \cdot (x \cdot y)), b_{\widetilde{\mathrm{P}}[p]}(0 \cdot x)\} \\
&\text{by (2.2)} && = \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot y), b_{\widetilde{\mathrm{P}}[p]}(x)\}.
\end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X. □

The following example shows that the converse of Theorem 4.7 is not true.

Example 4.5. Let X be the set of five brands of the car, that is,

$$X = \{\boxminus, \boxtimes, \boxdot, \circledast, \circledcirc\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxminus$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxtimes$	$\boxminus$	$\boxminus$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxdot$	$\boxminus$	$\boxminus$	$\boxminus$	$\circledast$	$\circledast$
$\circledast$	$\boxminus$	$\boxtimes$	$\boxdot$	$\boxminus$	$\circledast$
$\circledcirc$	$\boxminus$	$\boxtimes$	$\boxdot$	$\boxminus$	$\boxminus$

Then $(X, \cdot, \boxminus)$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{price}, \text{speed}, \text{durable}\}$$

with $\widetilde{\mathrm{P}}[\text{price}], \widetilde{\mathrm{P}}[\text{speed}]$, and $\widetilde{\mathrm{P}}[\text{durable}]$ are picture fuzzy sets in X defined as follows:

$\widetilde{\mathrm{P}}$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$r_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.4	0.3	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{price}]}$	0	0.1	0.3	0.6	0.6
$r_{\widetilde{\mathrm{P}}[\text{speed}]}$	0.5	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{speed}]}$	0.4	0.4	0.3	0	0
$b_{\widetilde{\mathrm{P}}[\text{speed}]}$	0	0.3	0.5	0.6	0.6
$r_{\widetilde{\mathrm{P}}[\text{durable}]}$	0.4	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{durable}]}$	0.5	0.4	0.1	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{durable}]}$	0	0.1	0.3	0.5	0.5

Then $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X. But $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft implicative UP-filter of X. Indeed,

$$\begin{aligned} g_{\widetilde{\mathrm{P}}[\text{speed}]}(\circledast \cdot \circledcirc) &= g_{\widetilde{\mathrm{P}}[\text{speed}]}(\circledast) \\ &= 0 \\ &< 0.4 \\ &= \min\{0.4, 0.4\} \\ &= \min\{g_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxminus), g_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxminus)\} \\ &= \min\{g_{\widetilde{\mathrm{P}}[\text{speed}]}(\circledast \cdot (\circledast \cdot \circledcirc)), g_{\widetilde{\mathrm{P}}[\text{speed}]}(\circledast \cdot \circledast)\}. \end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[\text{speed}]$ is not a picture fuzzy implicative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft implicative UP-filter of X.

Theorem 4.8. *Every picture fuzzy soft implicative UP-filter of X is a picture fuzzy soft UP-ideal.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft implicative UP-filter of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy implicative UP-filter of X, so $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y, z \in X$. Then

$$
\begin{array}{lrl}
\text{by (3.13)} & r_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\} \\
\text{by Theorems 4.7 and 4.5 and (3.4)} & & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(y)\}, \\
\text{by (3.14)} & g_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\} \\
\text{by Theorems 4.7 and 4.5 and (3.5)} & & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(y)\}, \\
\text{by (3.15)} & b_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\} \\
\text{by Theorems 4.7 and 4.5 and (3.6)} & & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(y)\}.
\end{array}
$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-ideal of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X. □

The following example shows that the converse of Theorem 4.8 is not true.

Example 4.6. Let X be the set of five brands of the bag, that is,

$$X = \{\varnothing, \triangle, \triangledown, \blacktriangle, \blacktriangledown\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	$\varnothing$	$\triangle$	$\triangledown$	$\blacktriangle$	$\blacktriangledown$
$\varnothing$	$\varnothing$	$\triangle$	$\triangledown$	$\blacktriangle$	$\blacktriangledown$
$\triangle$	$\varnothing$	$\varnothing$	$\triangledown$	$\blacktriangle$	$\blacktriangledown$
$\triangledown$	$\varnothing$	$\varnothing$	$\varnothing$	$\blacktriangle$	$\blacktriangledown$
$\blacktriangle$	$\varnothing$	$\varnothing$	$\triangle$	$\varnothing$	$\blacktriangledown$
$\blacktriangledown$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

Then $(X, \cdot, \varnothing)$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{beauty}, \text{price}, \text{luxurious}, \text{lightweight}\}$$

with $\widetilde{\mathrm{P}}[\text{beauty}], \widetilde{\mathrm{P}}[\text{price}], \widetilde{\mathrm{P}}[\text{luxurious}]$, and $\widetilde{\mathrm{P}}[\text{lightweight}]$ are picture fuzzy sets in X

defined as follows:

$\widetilde{\mathrm{P}}$	$\varnothing$	$\triangle$	$\triangledown$	$\blacktriangle$	$\blacktriangledown$
$r_{\widetilde{\mathrm{P}}[\text{beauty}]}$	0.5	0.4	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{beauty}]}$	0.4	0.4	0.3	0.1	0
$b_{\widetilde{\mathrm{P}}[\text{beauty}]}$	0.1	0.2	0.4	0.6	0.7
$r_{\widetilde{\mathrm{P}}[\text{price}]}$	0.4	0.4	0.2	0.1	0
$g_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.5	0.3	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{price}]}$	0.1	0.1	0.4	0.6	0.7
$r_{\widetilde{\mathrm{P}}[\text{luxurious}]}$	0.5	0.4	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{luxurious}]}$	0.5	0.5	0.4	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{luxurious}]}$	0	0.1	0.2	0.5	0.8
$r_{\widetilde{\mathrm{P}}[\text{lightweight}]}$	0.4	0.4	0.3	0.1	0
$g_{\widetilde{\mathrm{P}}[\text{lightweight}]}$	0.5	0.4	0.4	0.2	0.1
$b_{\widetilde{\mathrm{P}}[\text{lightweight}]}$	0	0	0.3	0.4	0.6

Then $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X. But $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft implicative UP-filter of X. Indeed,

$$\begin{aligned}
r_{\widetilde{\mathrm{P}}[\text{beauty}]}(\blacktriangle \cdot \triangledown) &= r_{\widetilde{\mathrm{P}}[\text{beauty}]}(\triangle) \\
&= 0.4 \\
&< 0.5 \\
&= \min\{0.5, 0.5\} \\
&= \min\{r_{\widetilde{\mathrm{P}}[\text{beauty}]}(\varnothing), r_{\widetilde{\mathrm{P}}[\text{beauty}]}(\varnothing)\} \\
&= \min\{r_{\widetilde{\mathrm{P}}[\text{beauty}]}(\blacktriangle \cdot (\blacktriangle \cdot \triangledown)), r_{\widetilde{\mathrm{P}}[\text{beauty}]}(\blacktriangle \cdot \blacktriangle)\}.
\end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[\text{beauty}]$ is not a picture fuzzy implicative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft implicative UP-filter of X.

Theorem 4.9. *Every picture fuzzy soft comparative UP-filter of X is a picture fuzzy soft UP-filter.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft comparative UP-filter of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy comparative UP-filter of X, so $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y \in X$. Then

$$\begin{array}{lrl}
\text{by (3.16)} & r_{\widetilde{\mathrm{P}}[p]}(y) & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot y) \cdot y)), r_{\widetilde{\mathrm{P}}[p]}(x)\} \\
\text{by (2.6)} & & = \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (0 \cdot y)), r_{\widetilde{\mathrm{P}}[p]}(x)\} \\
\text{by (2.2)} & & = \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot y), r_{\widetilde{\mathrm{P}}[p]}(x)\}, \\
\text{by (3.17)} & g_{\widetilde{\mathrm{P}}[p]}(y) & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot y) \cdot y)), g_{\widetilde{\mathrm{P}}[p]}(x)\} \\
\text{by (2.6)} & & = \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (0 \cdot y)), g_{\widetilde{\mathrm{P}}[p]}(x)\} \\
\text{by (2.2)} & & = \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot y), g_{\widetilde{\mathrm{P}}[p]}(x)\}, \\
\text{by (3.18)} & b_{\widetilde{\mathrm{P}}[p]}(y) & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot y) \cdot y)), b_{\widetilde{\mathrm{P}}[p]}(x)\} \\
\text{by (2.6)} & & = \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (0 \cdot y)), b_{\widetilde{\mathrm{P}}[p]}(x)\} \\
\text{by (2.2)} & & = \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot y), b_{\widetilde{\mathrm{P}}[p]}(x)\}.
\end{array}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X. □

The following example shows that the converse of Theorem 4.9 is not true.

Example 4.7. Let X be the set of five brands of the car, that is,

$$X = \{\boxminus, \boxtimes, \boxdot, \circledast, \circledcirc\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxminus$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxtimes$	$\boxminus$	$\boxminus$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxdot$	$\boxminus$	$\boxminus$	$\boxminus$	$\circledast$	$\circledast$
$\circledast$	$\boxminus$	$\boxtimes$	$\boxdot$	$\boxminus$	$\circledast$
$\circledcirc$	$\boxminus$	$\boxtimes$	$\boxdot$	$\boxminus$	$\boxminus$

Then $(X, \cdot, \boxminus)$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{price}, \text{speed}, \text{durable}\}$$

with $\widetilde{\mathrm{P}}[\text{price}], \widetilde{\mathrm{P}}[\text{speed}]$, and $\widetilde{\mathrm{P}}[\text{durable}]$ are picture fuzzy sets in X defined as follows:

$\widetilde{\mathrm{P}}$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$r_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.4	0.3	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{price}]}$	0	0.1	0.3	0.6	0.6
$r_{\widetilde{\mathrm{P}}[\text{speed}]}$	0.5	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{speed}]}$	0.4	0.4	0.3	0	0
$b_{\widetilde{\mathrm{P}}[\text{speed}]}$	0	0.3	0.5	0.6	0.6
$r_{\widetilde{\mathrm{P}}[\text{durable}]}$	0.4	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{durable}]}$	0.5	0.4	0.1	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{durable}]}$	0	0.1	0.3	0.5	0.5

Then $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X. But $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft comparative UP-filter of X. Indeed,

$$\begin{aligned} b_{\widetilde{\mathrm{P}}[\text{speed}]}(\circledast) &= 0.6 \\ &> 0.5 \\ &= \max\{0, 0.5\} \\ &= \max\{b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxminus), b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxdot)\} \\ &= \max\{b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxdot \cdot ((\circledast \cdot \circledcirc) \cdot \circledast)), b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxdot)\}. \end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[\text{speed}]$ is not a picture fuzzy comparative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft comparative UP-filter of X.

Theorem 4.10. *Every picture fuzzy soft shift UP-filter of X is a picture fuzzy soft UP-filter.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft shift UP-filter of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy shift UP-filter of X, so $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y \in X$. Then

$$\begin{aligned} &\text{by (2.2) and (2.3)} & r_{\widetilde{\mathrm{P}}[p]}(y) &= r_{\widetilde{\mathrm{P}}[p]}(((y \cdot 0) \cdot 0) \cdot y) \\ &\text{by (3.19)} & &\geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (0 \cdot y)), r_{\widetilde{\mathrm{P}}[p]}(x)\} \\ &\text{by (2.2)} & &= \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot y), r_{\widetilde{\mathrm{P}}[p]}(x)\}, \\ &\text{by (2.2) and (2.3)} & g_{\widetilde{\mathrm{P}}[p]}(y) &= g_{\widetilde{\mathrm{P}}[p]}(((y \cdot 0) \cdot 0) \cdot y) \\ &\text{by (3.20)} & &\geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (0 \cdot y)), g_{\widetilde{\mathrm{P}}[p]}(x)\} \\ &\text{by (2.2)} & &= \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot y), g_{\widetilde{\mathrm{P}}[p]}(x)\}, \\ &\text{by (2.2) and (2.3)} & b_{\widetilde{\mathrm{P}}[p]}(y) &= b_{\widetilde{\mathrm{P}}[p]}(((y \cdot 0) \cdot 0) \cdot y) \\ &\text{by (3.21)} & &\leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (0 \cdot y)), b_{\widetilde{\mathrm{P}}[p]}(x)\} \\ &\text{by (2.2)} & &= \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot y), b_{\widetilde{\mathrm{P}}[p]}(x)\}. \end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X. $\square$

The following example shows that the converse of Theorem 4.10 is not true.

Example 4.8. Let X be the set of five brands of the car, that is,

$$X = \{\boxminus, \boxtimes, \boxdot, \circledast, \circledcirc\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxminus$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxtimes$	$\boxminus$	$\boxminus$	$\boxdot$	$\circledast$	$\circledcirc$
$\boxdot$	$\boxminus$	$\boxminus$	$\boxminus$	$\circledast$	$\circledast$
$\circledast$	$\boxminus$	$\boxtimes$	$\boxdot$	$\boxminus$	$\circledast$
$\circledcirc$	$\boxminus$	$\boxtimes$	$\boxdot$	$\boxminus$	$\boxminus$

Then $(X, \cdot, \boxminus)$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{price}, \text{speed}, \text{durable}\}$$

with $\widetilde{\mathrm{P}}[\text{price}], \widetilde{\mathrm{P}}[\text{speed}]$, and $\widetilde{\mathrm{P}}[\text{durable}]$ are picture fuzzy sets in X defined as follows:

$\widetilde{\mathrm{P}}$	$\boxminus$	$\boxtimes$	$\boxdot$	$\circledast$	$\circledcirc$
$r_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.4	0.3	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{price}]}$	0	0.1	0.3	0.6	0.6
$r_{\widetilde{\mathrm{P}}[\text{speed}]}$	0.5	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{speed}]}$	0.4	0.4	0.3	0	0
$b_{\widetilde{\mathrm{P}}[\text{speed}]}$	0	0.3	0.5	0.6	0.6
$r_{\widetilde{\mathrm{P}}[\text{durable}]}$	0.4	0.3	0.2	0.1	0.1
$g_{\widetilde{\mathrm{P}}[\text{durable}]}$	0.5	0.4	0.1	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{durable}]}$	0	0.1	0.3	0.5	0.5

Then $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X. But $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft shift UP-filter of X. Indeed,

$$\begin{aligned}
b_{\widetilde{\mathrm{P}}[\text{speed}]}(((\boxtimes \cdot \boxdot) \cdot \boxdot) \cdot \boxtimes) &= 0.3 \\
&> 0 \\
&= \max\{0, 0\} \\
&= \max\{b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxminus), b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxminus)\} \\
&= \max\{b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxminus \cdot (\boxdot \cdot \boxtimes)), b_{\widetilde{\mathrm{P}}[\text{speed}]}(\boxminus)\}.
\end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[\text{speed}]$ is not a picture fuzzy shift UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is not a picture fuzzy soft shift UP-filter of X.

Theorem 4.11. *Every picture fuzzy soft strong UP-ideal of X is a picture fuzzy soft implicative UP-filter.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft strong UP-ideal of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy strong UP-ideal of X, so $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). By Theorem 4.2, we have $\widetilde{\mathrm{P}}[p]$ is constant. Next, let $x, y, z \in X$. Then

$$\begin{aligned}
&\text{by } r_{\widetilde{\mathrm{P}}[p]} \text{ is constant} && r_{\widetilde{\mathrm{P}}[p]}(x \cdot z) = r_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \\
& && \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}, \\
&\text{by } g_{\widetilde{\mathrm{P}}[p]} \text{ is constant} && g_{\widetilde{\mathrm{P}}[p]}(x \cdot z) = g_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \\
& && \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}, \\
&\text{by } b_{\widetilde{\mathrm{P}}[p]} \text{ is constant} && b_{\widetilde{\mathrm{P}}[p]}(x \cdot z) = b_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \\
& && \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}.
\end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy implicative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft implicative UP-filter of X. □

Theorem 4.12. *Every picture fuzzy soft strong UP-ideal of X is a picture fuzzy soft comparative UP-filter.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft strong UP-ideal of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy strong UP-ideal of X, so $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). By Theorem 4.2, we have $\widetilde{\mathrm{P}}[p]$ is constant. Next, let $x, y, z \in X$. Then

$$\begin{aligned}
&\text{by } r_{\widetilde{\mathrm{P}}[p]} \text{ is constant} && r_{\widetilde{\mathrm{P}}[p]}(y) = r_{\widetilde{\mathrm{P}}[p]}(x)\\
&&& \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), r_{\widetilde{\mathrm{P}}[p]}(x)\},\\
&\text{by } g_{\widetilde{\mathrm{P}}[p]} \text{ is constant} && g_{\widetilde{\mathrm{P}}[p]}(y) = g_{\widetilde{\mathrm{P}}[p]}(x)\\
&&& \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), g_{\widetilde{\mathrm{P}}[p]}(x)\},\\
&\text{by } b_{\widetilde{\mathrm{P}}[p]} \text{ is constant} && b_{\widetilde{\mathrm{P}}[p]}(y) = b_{\widetilde{\mathrm{P}}[p]}(x)\\
&&& \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), b_{\widetilde{\mathrm{P}}[p]}(x)\}.
\end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy comparative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft comparative UP-filter of X. □

Theorem 4.13. *Every picture fuzzy soft strong UP-ideal of X is a picture fuzzy soft shift UP-filter.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft strong UP-ideal of X and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy strong UP-ideal of X, so $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). By Theorem 4.2, we have $\widetilde{\mathrm{P}}[p]$ is constant. Next, let $x, y, z \in X$. Then

$$\begin{aligned}
&\text{by } r_{\widetilde{\mathrm{P}}[p]} \text{ is constant} && r_{\widetilde{\mathrm{P}}[p]}(((z \cdot y) \cdot y) \cdot z) = r_{\widetilde{\mathrm{P}}[p]}(x)\\
&&& \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x)\},\\
&\text{by } g_{\widetilde{\mathrm{P}}[p]} \text{ is constant} && g_{\widetilde{\mathrm{P}}[p]}(((z \cdot y) \cdot y) \cdot z) = g_{\widetilde{\mathrm{P}}[p]}(x)\\
&&& \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x)\},\\
&\text{by } b_{\widetilde{\mathrm{P}}[p]} \text{ is constant} && b_{\widetilde{\mathrm{P}}[p]}(((z \cdot y) \cdot y) \cdot z) = b_{\widetilde{\mathrm{P}}[p]}(x)\\
&&& \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x)\}.
\end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy shift UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft shift UP-filter of X. □

By Theorems 4.3, 4.5, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12, and 4.13 and Examples 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, and 4.8, we have that the concept of picture fuzzy soft UP-subalgebras is a generalization of picture fuzzy soft near UP-filters, picture fuzzy near soft UP-filters is a generalization of picture fuzzy soft UP-filters, picture fuzzy soft UP-filters is a generalization of picture fuzzy soft comparative UP-filters, picture fuzzy soft UP-filters is a generalization of picture fuzzy soft shift UP-filters, picture fuzzy soft UP-filters is a generalization of picture fuzzy soft UP-ideals, picture fuzzy soft UP-ideals is a generalization of picture fuzzy soft implicative UP-filters, and picture fuzzy soft implicative UP-filters, picture fuzzy soft comparative UP-filters, and picture fuzzy soft shift UP-filters is a generalization of picture fuzzy soft strong UP-ideals.

Theorem 4.14. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft UP-subalgebra of* X *satisfying the following condition:*

$$(\forall p \in Y, \forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} r_{\widetilde{\mathrm{P}}[p]}(x) \geq r_{\widetilde{\mathrm{P}}[p]}(y) \\ g_{\widetilde{\mathrm{P}}[p]}(x) \geq g_{\widetilde{\mathrm{P}}[p]}(y) \\ b_{\widetilde{\mathrm{P}}[p]}(x) \leq b_{\widetilde{\mathrm{P}}[p]}(y) \end{cases}\right), \tag{4.2}$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft near UP-filter of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-subalgebra of X satisfying the condition (4.2) and let $p \in Y$. Let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

by (3.7) $\qquad r_{\widetilde{\mathrm{P}}[p]}(x \cdot y) = r_{\widetilde{\mathrm{P}}[p]}(0) \geq r_{\widetilde{\mathrm{P}}[p]}(y),$

by (3.8) $\qquad g_{\widetilde{\mathrm{P}}[p]}(x \cdot y) = g_{\widetilde{\mathrm{P}}[p]}(0) \geq g_{\widetilde{\mathrm{P}}[p]}(y),$

by (3.9) $\qquad b_{\widetilde{\mathrm{P}}[p]}(x \cdot y) = b_{\widetilde{\mathrm{P}}[p]}(0) \leq b_{\widetilde{\mathrm{P}}[p]}(y).$

Case 2: $x \cdot y \neq 0$. Then

by (3.1) and (4.2) for $r_{\widetilde{\mathrm{P}}[p]}$ $\qquad r_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(y)\} = r_{\widetilde{\mathrm{P}}[p]}(y),$

by (3.2) and (4.2) for $g_{\widetilde{\mathrm{P}}[p]}$ $\qquad g_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(y)\} = g_{\widetilde{\mathrm{P}}[p]}(y),$

by (3.3) and (4.2) for $b_{\widetilde{\mathrm{P}}[p]}$ $\qquad b_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(y)\} = b_{\widetilde{\mathrm{P}}[p]}(y).$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy near UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft near UP-filter of X. □

Theorem 4.15. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft near UP-filter of* X *satisfying the following condition:*

$$(\forall p \in Y, \forall x \in X)\begin{pmatrix} r_{\widetilde{\mathrm{P}}[p]}(0) = g_{\widetilde{\mathrm{P}}[p]}(0) = b_{\widetilde{\mathrm{P}}[p]}(0) \\ r_{\widetilde{\mathrm{P}}[p]}(x) \geq b_{\widetilde{\mathrm{P}}[p]}(x) \\ g_{\widetilde{\mathrm{P}}[p]}(x) \geq b_{\widetilde{\mathrm{P}}[p]}(x) \end{pmatrix}, \tag{4.3}$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft strong UP-ideal of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft near UP-filter of X satisfying the condition (4.3) and let $p \in Y$. By Theorem 4.4, we have $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x \in X$. Then

$$r_{\widetilde{\mathrm{P}}[p]}(0) \geq r_{\widetilde{\mathrm{P}}[p]}(x) \geq b_{\widetilde{\mathrm{P}}[p]}(x) \geq b_{\widetilde{\mathrm{P}}[p]}(0) = r_{\widetilde{\mathrm{P}}[p]}(0),$$
$$g_{\widetilde{\mathrm{P}}[p]}(0) \geq g_{\widetilde{\mathrm{P}}[p]}(x) \geq b_{\widetilde{\mathrm{P}}[p]}(x) \geq b_{\widetilde{\mathrm{P}}[p]}(0) = g_{\widetilde{\mathrm{P}}[p]}(0),$$
$$b_{\widetilde{\mathrm{P}}[p]}(0) \leq b_{\widetilde{\mathrm{P}}[p]}(x) \leq r_{\widetilde{\mathrm{P}}[p]}(x) \leq r_{\widetilde{\mathrm{P}}[p]}(0) = b_{\widetilde{\mathrm{P}}[p]}(0).$$

Thus $r_{\widetilde{\mathrm{P}}[p]}(x) = r_{\widetilde{\mathrm{P}}[p]}(0), g_{\widetilde{\mathrm{P}}[p]}(x) = g_{\widetilde{\mathrm{P}}[p]}(0)$, and $b_{\widetilde{\mathrm{P}}[p]}(x) = b_{\widetilde{\mathrm{P}}[p]}(0)$ for all $x \in X$, that is, $\widetilde{\mathrm{P}}[p]$ is constant. By Theorem 4.2, we have $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft strong UP-ideal of X. □

Theorem 4.16. *If $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X satisfying the following condition:*

$$(4.4) \qquad (\forall p \in Y, \forall x, y, z \in X) \left(\begin{array}{c} r_{\widetilde{\mathrm{P}}[p]}(y) \geq r_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot z)) \\ \Rightarrow r_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot z)) \geq r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \\ g_{\widetilde{\mathrm{P}}[p]}(y) \geq g_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot z)) \\ \Rightarrow g_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot z)) \geq g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \\ b_{\widetilde{\mathrm{P}}[p]}(y) \leq b_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot z)) \\ \Rightarrow b_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot z)) \leq b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \end{array} \right),$$

then $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X.

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X satisfying the condition (4.4) and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y, z \in X$. Then

$$\begin{array}{lrl} \text{by (3.10)} & r_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(y)\} \\ \text{by (4.4) for } r_{\widetilde{\mathrm{P}}[p]} & & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(y)\}, \\ \text{by (3.11)} & g_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(y)\} \\ \text{by (4.4) for } g_{\widetilde{\mathrm{P}}[p]} & & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(y)\}, \\ \text{by (3.12)} & b_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(y \cdot (x \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(y)\} \\ \text{by (4.4) for } b_{\widetilde{\mathrm{P}}[p]} & & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(y)\}. \end{array}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-ideal of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X. □

Theorem 4.17. *If $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the following condition:*

$$(4.5) \qquad (\forall p \in Y, \forall x, y, z \in X) \left(\begin{array}{c} r_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot y), r_{\widetilde{\mathrm{P}}[p]}(x)\} \\ g_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot y), g_{\widetilde{\mathrm{P}}[p]}(x)\} \\ b_{\widetilde{\mathrm{P}}[p]}(y) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot y), b_{\widetilde{\mathrm{P}}[p]}(x)\} \\ r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \geq r_{\widetilde{\mathrm{P}}[p]}((x \cdot (y \cdot z)) \cdot (x \cdot z)) \\ \Rightarrow r_{\widetilde{\mathrm{P}}[p]}((x \cdot (y \cdot z)) \cdot (x \cdot z)) \geq r_{\widetilde{\mathrm{P}}[p]}(y) \\ g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \geq g_{\widetilde{\mathrm{P}}[p]}((x \cdot (y \cdot z)) \cdot (x \cdot z)) \\ \Rightarrow g_{\widetilde{\mathrm{P}}[p]}((x \cdot (y \cdot z)) \cdot (x \cdot z)) \geq g_{\widetilde{\mathrm{P}}[p]}(y) \\ b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \leq b_{\widetilde{\mathrm{P}}[p]}((x \cdot (y \cdot z)) \cdot (x \cdot z)) \\ \Rightarrow b_{\widetilde{\mathrm{P}}[p]}((x \cdot (y \cdot z)) \cdot (x \cdot z)) \leq b_{\widetilde{\mathrm{P}}[p]}(y) \end{array} \right),$$

then $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X.

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the condition (4.5) and let $p \in Y$. If $z = 0$ and by (4.5), we have $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y, z \in X$. Then

$$\begin{array}{lrl}
\text{by (3.10)} & r_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}((x \cdot (y \cdot z)) \cdot (x \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z))\} \\
\text{by (4.5) for } r_{\widetilde{\mathrm{P}}[p]} & & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(y)\}, \\
\text{by (3.11)} & g_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}((x \cdot (y \cdot z)) \cdot (x \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z))\} \\
\text{by (4.5) for } g_{\widetilde{\mathrm{P}}[p]} & & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(y)\}, \\
\text{by (3.12)} & b_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}((x \cdot (y \cdot z)) \cdot (x \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z))\} \\
\text{by (4.5) for } b_{\widetilde{\mathrm{P}}[p]} & & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(y)\}.
\end{array}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-ideal of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X. □

Theorem 4.18. *If $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X satisfying the following condition:*

$$(4.6)\quad (\forall p \in Y, \forall x, y, z \in X) \begin{pmatrix} r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \geq r_{\widetilde{\mathrm{P}}[p]}(y) \Rightarrow r_{\widetilde{\mathrm{P}}[p]}(y) \geq r_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \\ g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \geq g_{\widetilde{\mathrm{P}}[p]}(y) \Rightarrow g_{\widetilde{\mathrm{P}}[p]}(y) \geq g_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \\ b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \leq b_{\widetilde{\mathrm{P}}[p]}(y) \Rightarrow b_{\widetilde{\mathrm{P}}[p]}(y) \leq b_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \end{pmatrix},$$

then $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft implicative UP-filter of X.

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X satisfying the condition (4.6) and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y, z \in X$. Then

$$\begin{array}{lrl}
\text{by (3.22)} & r_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(y)\} \\
\text{by (4.6) for } r_{\widetilde{\mathrm{P}}[p]} & & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}, \\
\text{by (3.23)} & g_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(y)\} \\
\text{by (4.6) for } g_{\widetilde{\mathrm{P}}[p]} & & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}, \\
\text{by (3.24)} & b_{\widetilde{\mathrm{P}}[p]}(x \cdot z) & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(y)\} \\
\text{by (4.6) for } b_{\widetilde{\mathrm{P}}[p]} & & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}.
\end{array}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy implicative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft implicative UP-filter of X. □

Theorem 4.19. *If $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X satisfying the following condition:*

$$(4.7)\qquad (\forall p \in Y, \forall x, y, z \in X) \begin{pmatrix} r_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \geq r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((x \cdot y) \cdot z)) \\ \Rightarrow r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((x \cdot y) \cdot z)) \geq r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \\ g_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \geq g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((x \cdot y) \cdot z)) \\ \Rightarrow g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((x \cdot y) \cdot z)) \geq g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \\ b_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \leq b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((x \cdot y) \cdot z)) \\ \Rightarrow b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((x \cdot y) \cdot z)) \leq b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \end{pmatrix},$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft implicative UP-filter of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X satisfying the condition (4.7) and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y, z \in X$. Then

$$
\begin{aligned}
&\text{by (3.22)} && r_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((x \cdot y) \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\} \\
&\text{by (4.7) for } r_{\widetilde{\mathrm{P}}[p]} && \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}, \\
&\text{by (3.23)} && g_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((x \cdot y) \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\} \\
&\text{by (4.7) for } g_{\widetilde{\mathrm{P}}[p]} && \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}, \\
&\text{by (3.24)} && b_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((x \cdot y) \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\} \\
&\text{by (4.7) for } b_{\widetilde{\mathrm{P}}[p]} && \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}.
\end{aligned}
$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy implicative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft implicative UP-filter of X. □

Theorem 4.20. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft UP-filter of* X *satisfying the following condition:*
(4.8)

$$
(\forall p \in Y, \forall x, y, z \in X) \begin{pmatrix} r_{\widetilde{\mathrm{P}}[p]}(x) \geq r_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \Rightarrow r_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \geq r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)) \\ g_{\widetilde{\mathrm{P}}[p]}(x) \geq g_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \Rightarrow g_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \geq g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)) \\ b_{\widetilde{\mathrm{P}}[p]}(x) \leq b_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \Rightarrow b_{\widetilde{\mathrm{P}}[p]}(x \cdot y) \leq b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)) \end{pmatrix},
$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft comparative UP-filter of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X satisfying the condition (4.8) and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y, z \in X$. Then

$$
\begin{aligned}
&\text{by (3.10)} && r_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot y), r_{\widetilde{\mathrm{P}}[p]}(x)\} \\
&\text{by (4.8) for } r_{\widetilde{\mathrm{P}}[p]} && \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), r_{\widetilde{\mathrm{P}}[p]}(x)\}, \\
&\text{by (3.11)} && g_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot y), g_{\widetilde{\mathrm{P}}[p]}(x)\} \\
&\text{by (4.8) for } g_{\widetilde{\mathrm{P}}[p]} && \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), g_{\widetilde{\mathrm{P}}[p]}(x)\}, \\
&\text{by (3.12)} && b_{\widetilde{\mathrm{P}}[p]}(y) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot y), b_{\widetilde{\mathrm{P}}[p]}(x)\} \\
&\text{by (4.8) for } b_{\widetilde{\mathrm{P}}[p]} && \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), b_{\widetilde{\mathrm{P}}[p]}(x)\}.
\end{aligned}
$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy comparative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft comparative UP-filter of X. □

Theorem 4.21. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a PFSS over* X *satisfying the following condition:*

$$(4.9) \qquad (\forall p \in Y, \forall x, y, z \in X) \left(\begin{array}{c} r_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot y), r_{\widetilde{\mathrm{P}}[p]}(x)\} \\ g_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot y), g_{\widetilde{\mathrm{P}}[p]}(x)\} \\ b_{\widetilde{\mathrm{P}}[p]}(y) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot y), b_{\widetilde{\mathrm{P}}[p]}(x)\} \\ r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)) \geq r_{\widetilde{\mathrm{P}}[p]}((x \cdot ((y \cdot z) \cdot y)) \cdot y) \\ \Rightarrow r_{\widetilde{\mathrm{P}}[p]}((x \cdot ((y \cdot z) \cdot y)) \cdot y) \geq r_{\widetilde{\mathrm{P}}[p]}(x) \\ g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)) \geq g_{\widetilde{\mathrm{P}}[p]}((x \cdot ((y \cdot z) \cdot y)) \cdot y) \\ \Rightarrow g_{\widetilde{\mathrm{P}}[p]}((x \cdot ((y \cdot z) \cdot y)) \cdot y) \geq g_{\widetilde{\mathrm{P}}[p]}(x) \\ b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)) \leq b_{\widetilde{\mathrm{P}}[p]}((x \cdot ((y \cdot z) \cdot y)) \cdot y) \\ \Rightarrow b_{\widetilde{\mathrm{P}}[p]}((x \cdot ((y \cdot z) \cdot y)) \cdot y) \leq b_{\widetilde{\mathrm{P}}[p]}(x) \end{array} \right),$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft comparative UP-filter of* X.

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the condition (4.9) and let $p \in Y$. If $y = 0$ and by (4.9), we have $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y, z \in X$. Then

$$\begin{array}{lrl} \text{by (3.10)} & r_{\widetilde{\mathrm{P}}[p]}(y) & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}((x \cdot ((y \cdot z) \cdot y)) \cdot y), r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y))\} \\ \text{by (4.9) for } r_{\widetilde{\mathrm{P}}[p]} & & \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), r_{\widetilde{\mathrm{P}}[p]}(x)\}, \\ \text{by (3.11)} & g_{\widetilde{\mathrm{P}}[p]}(y) & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}((x \cdot ((y \cdot z) \cdot y)) \cdot y), g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y))\} \\ \text{by (4.9) for } g_{\widetilde{\mathrm{P}}[p]} & & \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), g_{\widetilde{\mathrm{P}}[p]}(x)\}, \\ \text{by (3.12)} & b_{\widetilde{\mathrm{P}}[p]}(y) & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}((x \cdot ((y \cdot z) \cdot y)) \cdot y), b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y))\} \\ \text{by (4.9) for } b_{\widetilde{\mathrm{P}}[p]} & & \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), b_{\widetilde{\mathrm{P}}[p]}(x)\}. \end{array}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy comparative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft comparative UP-filter of X. □

Theorem 4.22. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft UP-filter of* X *satisfying the following condition:*

$$(4.10) \qquad (\forall p \in Y, \forall x, y, z \in X) \left(\begin{array}{c} r_{\widetilde{\mathrm{P}}[p]}(x) \geq r_{\widetilde{\mathrm{P}}[p]}(x \cdot (((z \cdot y) \cdot y) \cdot z)) \\ \Rightarrow r_{\widetilde{\mathrm{P}}[p]}(x \cdot (((z \cdot y) \cdot y) \cdot z)) \geq r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \\ g_{\widetilde{\mathrm{P}}[p]}(x) \geq g_{\widetilde{\mathrm{P}}[p]}(x \cdot (((z \cdot y) \cdot y) \cdot z)) \\ \Rightarrow g_{\widetilde{\mathrm{P}}[p]}(x \cdot (((z \cdot y) \cdot y) \cdot z)) \geq g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \\ b_{\widetilde{\mathrm{P}}[p]}(x) \leq b_{\widetilde{\mathrm{P}}[p]}(x \cdot (((z \cdot y) \cdot y) \cdot z)) \\ \Rightarrow b_{\widetilde{\mathrm{P}}[p]}(x \cdot (((z \cdot y) \cdot y) \cdot z)) \leq b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)) \end{array} \right),$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft shift UP-filter of* X.

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X satisfying the condition (4.10) and let $p \in Y$. Then $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9).

Next, let $x, y, z \in X$. Then

$$
\begin{aligned}
&\text{by (3.10)} && r_{\widetilde{\mathrm{P}}[p]}(((z\cdot y)\cdot y)\cdot z) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x\cdot(((z\cdot y)\cdot y)\cdot z), r_{\widetilde{\mathrm{P}}[p]}(x)\}\\
&\text{by (4.10) for } r_{\widetilde{\mathrm{P}}[p]} && \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x)\},\\
&\text{by (3.11)} && g_{\widetilde{\mathrm{P}}[p]}(((z\cdot y)\cdot y)\cdot z) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x\cdot(((z\cdot y)\cdot y)\cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x)\}\\
&\text{by (4.10) for } g_{\widetilde{\mathrm{P}}[p]} && \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x)\},\\
&\text{by (3.12)} && b_{\widetilde{\mathrm{P}}[p]}(((z\cdot y)\cdot y)\cdot z) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x\cdot(((z\cdot y)\cdot y)\cdot z), b_{\widetilde{\mathrm{P}}[p]}(x)\}\\
&\text{by (4.10) for } b_{\widetilde{\mathrm{P}}[p]} && \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x)\}.
\end{aligned}
$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy shift UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft shift UP-filter of X. □

Theorem 4.23. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a PFSS over* X *satisfying the following condition:*
(4.11)

$$
(\forall p \in Y, \forall x, y, z \in X)\left(\begin{array}{c}
r_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x\cdot y), r_{\widetilde{\mathrm{P}}[p]}(x)\}\\
g_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x\cdot y), g_{\widetilde{\mathrm{P}}[p]}(x)\}\\
b_{\widetilde{\mathrm{P}}[p]}(y) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x\cdot y), b_{\widetilde{\mathrm{P}}[p]}(x)\}\\
r_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z)) \geq r_{\widetilde{\mathrm{P}}[p]}((x\cdot(y\cdot z))\cdot(((z\cdot y)\cdot y)\cdot z)\\
\Rightarrow r_{\widetilde{\mathrm{P}}[p]}((x\cdot(y\cdot z))\cdot(((z\cdot y)\cdot y)\cdot z) \geq r_{\widetilde{\mathrm{P}}[p]}(x)\\
g_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z)) \geq g_{\widetilde{\mathrm{P}}[p]}((x\cdot(y\cdot z))\cdot(((z\cdot y)\cdot y)\cdot z)\\
\Rightarrow g_{\widetilde{\mathrm{P}}[p]}((x\cdot(y\cdot z))\cdot(((z\cdot y)\cdot y)\cdot z) \geq g_{\widetilde{\mathrm{P}}[p]}(x)\\
b_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z)) \leq b_{\widetilde{\mathrm{P}}[p]}((x\cdot(y\cdot z))\cdot(((z\cdot y)\cdot y)\cdot z)\\
\Rightarrow b_{\widetilde{\mathrm{P}}[p]}((x\cdot(y\cdot z))\cdot(((z\cdot y)\cdot y)\cdot z) \leq b_{\widetilde{\mathrm{P}}[p]}(x)
\end{array}\right),
$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft shift UP-filter of* X.

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the condition (4.11) and let $p \in Y$. If $z = 0$ and by (4.11), we have $\widetilde{\mathrm{P}}[p]$ satisfies the conditions (3.7), (3.8), and (3.9). Next, let $x, y, z \in X$. Then

by (3.10)

$$
\begin{aligned}
&r_{\widetilde{\mathrm{P}}[p]}(((z\cdot y)\cdot y)\cdot z) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}((x\cdot(y\cdot z))\cdot(((z\cdot y)\cdot y)\cdot z), r_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z))\}\\
&\text{by (4.11) for } r_{\widetilde{\mathrm{P}}[p]} \quad \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x)\},
\end{aligned}
$$

by (3.11)

$$
\begin{aligned}
&g_{\widetilde{\mathrm{P}}[p]}(((z\cdot y)\cdot y)\cdot z) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}((x\cdot(y\cdot z))\cdot(((z\cdot y)\cdot y)\cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z))\}\\
&\text{by (4.11) for } g_{\widetilde{\mathrm{P}}[p]} \quad \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x)\},
\end{aligned}
$$

by (3.12)

$$
\begin{aligned}
&b_{\widetilde{\mathrm{P}}[p]}(((z\cdot y)\cdot y)\cdot z) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}((x\cdot(y\cdot z))\cdot(((z\cdot y)\cdot y)\cdot z), b_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z))\}\\
&\text{by (4.11) for } b_{\widetilde{\mathrm{P}}[p]} \quad \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x\cdot(y\cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x)\}.
\end{aligned}
$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy shift UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft shift UP-filter of X. □

Theorem 4.24. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a PFSS over* X *satisfying the following condition:*

$$(\forall p \in Y, \forall x, y, z \in X)\left(\begin{array}{c} z \leq x \cdot y \\ \Rightarrow \begin{cases} r_{\widetilde{\mathrm{P}}[p]}(z) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(y)\} \\ g_{\widetilde{\mathrm{P}}[p]}(z) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(y)\} \\ b_{\widetilde{\mathrm{P}}[p]}(z) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(y)\} \end{cases} \end{array}\right), \tag{4.12}$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft UP-subalgebra of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the condition (4.12) and let $p \in Y$. Next, let $x, y \in X$. By (2.6), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (4.12) that

$$\begin{aligned} r_{\widetilde{\mathrm{P}}[p]}(x \cdot y) &\geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(y)\}, \\ g_{\widetilde{\mathrm{P}}[p]}(x \cdot y) &\geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(y)\}, \\ b_{\widetilde{\mathrm{P}}[p]}(x \cdot y) &\leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(y)\}. \end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-subalgebra of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-subalgebra of X. □

Theorem 4.25. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a PFSS over* X *satisfying the following condition:*

$$(\forall p \in Y, \forall x, y, z \in X)\left(\begin{array}{c} z \leq x \cdot y \\ \Rightarrow \begin{cases} r_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(z), r_{\widetilde{\mathrm{P}}[p]}(x)\} \\ g_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(z), g_{\widetilde{\mathrm{P}}[p]}(x)\} \\ b_{\widetilde{\mathrm{P}}[p]}(y) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(z), b_{\widetilde{\mathrm{P}}[p]}(x)\} \end{cases} \end{array}\right), \tag{4.13}$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft UP-filter of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the condition (4.13) and let $p \in Y$. Next, let $x \in X$. By (2.3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leq x \cdot 0$. It follows from (4.13) that

$$\begin{aligned} r_{\widetilde{\mathrm{P}}[p]}(0) &\geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(x)\} = r_{\widetilde{\mathrm{P}}[p]}(x), \\ g_{\widetilde{\mathrm{P}}[p]}(0) &\geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(x)\} = g_{\widetilde{\mathrm{P}}[p]}(x), \\ b_{\widetilde{\mathrm{P}}[p]}(0) &\leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(x)\} = b_{\widetilde{\mathrm{P}}[p]}(x). \end{aligned}$$

Next, let $x, y \in X$. By (2.6), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (4.13) that

$$\begin{aligned} r_{\widetilde{\mathrm{P}}[p]}(y) &\geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot y), r_{\widetilde{\mathrm{P}}[p]}(x)\}, \\ g_{\widetilde{\mathrm{P}}[p]}(y) &\geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot y), g_{\widetilde{\mathrm{P}}[p]}(x)\}, \\ b_{\widetilde{\mathrm{P}}[p]}(y) &\leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot y), b_{\widetilde{\mathrm{P}}[p]}(x)\}. \end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X. □

Theorem 4.26. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a PFSS over* X *satisfying the following condition:*

$$(4.14) \qquad (\forall p \in Y, \forall a, x, y, z \in X) \left(\begin{array}{c} a \leq x \cdot (y \cdot z) \\ \Rightarrow \begin{cases} r_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(a), r_{\widetilde{\mathrm{P}}[p]}(y)\} \\ g_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(a), g_{\widetilde{\mathrm{P}}[p]}(y)\} \\ b_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(a), b_{\widetilde{\mathrm{P}}[p]}(y)\} \end{cases} \end{array} \right),$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft UP-ideal of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the condition (4.14) and let $p \in Y$. Next, let $x \in X$. By (2.3), we have $x \cdot (0 \cdot (x \cdot 0)) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (4.14) that

by (2.2) $\qquad r_{\widetilde{\mathrm{P}}[p]}(0) = r_{\widetilde{\mathrm{P}}[p]}(0 \cdot 0) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(x)\} = r_{\widetilde{\mathrm{P}}[p]}(x),$

by (2.2) $\qquad g_{\widetilde{\mathrm{P}}[p]}(0) = g_{\widetilde{\mathrm{P}}[p]}(0 \cdot 0) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(x)\} = g_{\widetilde{\mathrm{P}}[p]}(x),$

by (2.2) $\qquad b_{\widetilde{\mathrm{P}}[p]}(0) = b_{\widetilde{\mathrm{P}}[p]}(0 \cdot 0) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(x)\} = b_{\widetilde{\mathrm{P}}[p]}(x).$

Next, let $x, y, z \in X$. By (2.6), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \leq x \cdot (y \cdot z)$. It follows from (4.14) that

$$r_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(y)\},$$
$$g_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(y)\},$$
$$b_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(y)\}.$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy UP-ideal of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X. □

Theorem 4.27. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a PFSS over* X *satisfying the following condition:*

$$(4.15) \quad (\forall p \in Y, \forall a, x, y, z \in X) \left(\begin{array}{c} a \leq x \cdot (y \cdot z) \\ \Rightarrow \begin{cases} r_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(a), r_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\} \\ g_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(a), g_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\} \\ b_{\widetilde{\mathrm{P}}[p]}(x \cdot z) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(a), b_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\} \end{cases} \end{array} \right),$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft implicative UP-filter of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the condition (4.15) and let $p \in Y$. Next, let $x \in X$. By (2.3), we have $x \cdot (0 \cdot (x \cdot 0)) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (4.15) that

by (2.2)
$r_{\widetilde{\mathrm{P}}[p]}(0) = r_{\widetilde{\mathrm{P}}[p]}(0 \cdot 0) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(0 \cdot x)\} = \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(x)\} = r_{\widetilde{\mathrm{P}}[p]}(x),$

by (2.2)
$g_{\widetilde{\mathrm{P}}[p]}(0) = g_{\widetilde{\mathrm{P}}[p]}(0 \cdot 0) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(0 \cdot x)\} = \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(x)\} = g_{\widetilde{\mathrm{P}}[p]}(x),$

by (2.2)
$b_{\widetilde{\mathrm{P}}[p]}(0) = b_{\widetilde{\mathrm{P}}[p]}(0 \cdot 0) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(0 \cdot x)\} = \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(x)\} = b_{\widetilde{\mathrm{P}}[p]}(x).$

Next, let $x, y, z \in X$. By (2.6), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \leq x \cdot (y \cdot z)$. It follows from (4.15) that

$$\begin{aligned} r_{\widetilde{\mathrm{P}}[p]}(x \cdot z) &\geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}, \\ g_{\widetilde{\mathrm{P}}[p]}(x \cdot z) &\geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}, \\ b_{\widetilde{\mathrm{P}}[p]}(x \cdot z) &\leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x \cdot y)\}. \end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy implicative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft implicative UP-filter of X. □

Theorem 4.28. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a PFSS over* X *satisfying the following condition:*

$$(4.16) \qquad (\forall p \in Y, \forall a, x, y, z \in X) \left(\begin{array}{c} a \leq x \cdot ((y \cdot z) \cdot y) \\ \Rightarrow \begin{cases} r_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(a), r_{\widetilde{\mathrm{P}}[p]}(x)\} \\ g_{\widetilde{\mathrm{P}}[p]}(y) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(a), g_{\widetilde{\mathrm{P}}[p]}(x)\} \\ b_{\widetilde{\mathrm{P}}[p]}(y) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(a), b_{\widetilde{\mathrm{P}}[p]}(x)\} \end{cases} \end{array} \right),$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft comparative UP-filter of* X.

Proof. . Assume that $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the condition (4.16) and let $p \in Y$. Next, let $x \in X$. By (2.3), we have $x \cdot (x \cdot ((0 \cdot x) \cdot 0)) = 0$, that is, $x \leq x \cdot ((0 \cdot x) \cdot 0)$. It follows from (4.16) that

$$\begin{aligned} r_{\widetilde{\mathrm{P}}[p]}(0) &\geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(x)\} = r_{\widetilde{\mathrm{P}}[p]}(x), \\ g_{\widetilde{\mathrm{P}}[p]}(0) &\geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(x)\} = g_{\widetilde{\mathrm{P}}[p]}(x), \\ b_{\widetilde{\mathrm{P}}[p]}(0) &\leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(x)\} = b_{\widetilde{\mathrm{P}}[p]}(x). \end{aligned}$$

Next, let $x, y, z \in X$. By (2.6), we have $(x \cdot ((y \cdot z) \cdot y)) \cdot (x \cdot ((y \cdot z) \cdot y)) = 0$, that is, $x \cdot ((y \cdot z) \cdot y) \leq x \cdot ((y \cdot z) \cdot y)$. It follows from (4.16) that

$$\begin{aligned} r_{\widetilde{\mathrm{P}}[p]}(y) &\geq \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), r_{\widetilde{\mathrm{P}}[p]}(x)\}, \\ g_{\widetilde{\mathrm{P}}[p]}(y) &\geq \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), g_{\widetilde{\mathrm{P}}[p]}(x)\}, \\ b_{\widetilde{\mathrm{P}}[p]}(y) &\leq \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot ((y \cdot z) \cdot y)), b_{\widetilde{\mathrm{P}}[p]}(x)\}. \end{aligned}$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy comparative UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft comparative UP-filter of X. □

Theorem 4.29. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a PFSS over* X *satisfying the following condition:*
(4.17)

$$(\forall p \in Y, \forall a, x, y, z \in X) \left(\begin{array}{c} a \leq x \cdot (y \cdot z) \\ \Rightarrow \begin{cases} r_{\widetilde{\mathrm{P}}[p]}(((z \cdot y) \cdot y) \cdot z) \geq \min\{r_{\widetilde{\mathrm{P}}[p]}(a), r_{\widetilde{\mathrm{P}}[p]}(x)\} \\ g_{\widetilde{\mathrm{P}}[p]}(((z \cdot y) \cdot y) \cdot z) \geq \min\{g_{\widetilde{\mathrm{P}}[p]}(a), g_{\widetilde{\mathrm{P}}[p]}(x)\} \\ b_{\widetilde{\mathrm{P}}[p]}(((z \cdot y) \cdot y) \cdot z) \leq \max\{b_{\widetilde{\mathrm{P}}[p]}(a), b_{\widetilde{\mathrm{P}}[p]}(x)\} \end{cases} \end{array} \right),$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft shift UP-filter of* X.

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the condition (4.17) and let $p \in Y$. Next, let $x \in X$. By (2.3), we have $x \cdot (x \cdot (x \cdot 0)) = 0$, that is, $x \le x \cdot (x \cdot 0)$. It follows from (4.17) that

$$\text{by (2.3)} \qquad r_{\widetilde{\mathrm{P}}[p]}(((0 \cdot x) \cdot x) \cdot 0) = r_{\widetilde{\mathrm{P}}[p]}(0) \ge \min\{r_{\widetilde{\mathrm{P}}[p]}(x), r_{\widetilde{\mathrm{P}}[p]}(x)\} = r_{\widetilde{\mathrm{P}}[p]}(x),$$
$$\text{by (2.3)} \qquad g_{\widetilde{\mathrm{P}}[p]}(((0 \cdot x) \cdot x) \cdot 0) = g_{\widetilde{\mathrm{P}}[p]}(0) \ge \min\{g_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(x)\} = g_{\widetilde{\mathrm{P}}[p]}(x),$$
$$\text{by (2.3)} \qquad b_{\widetilde{\mathrm{P}}[p]}(((0 \cdot x) \cdot x) \cdot 0) = b_{\widetilde{\mathrm{P}}[p]}(0) \le \max\{b_{\widetilde{\mathrm{P}}[p]}(x), b_{\widetilde{\mathrm{P}}[p]}(x)\} = b_{\widetilde{\mathrm{P}}[p]}(x).$$

Next, let $x, y, z \in X$. By (2.6), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \le x \cdot (y \cdot z)$. It follows from (4.17) that

$$r_{\widetilde{\mathrm{P}}[p]}(((z \cdot y) \cdot y) \cdot z) \ge \min\{r_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), r_{\widetilde{\mathrm{P}}[p]}(x)\},$$
$$g_{\widetilde{\mathrm{P}}[p]}(((z \cdot y) \cdot y) \cdot z) \ge \min\{g_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), g_{\widetilde{\mathrm{P}}[p]}(x)\},$$
$$b_{\widetilde{\mathrm{P}}[p]}(((z \cdot y) \cdot y) \cdot z) \le \max\{b_{\widetilde{\mathrm{P}}[p]}(x \cdot (y \cdot z)), b_{\widetilde{\mathrm{P}}[p]}(x)\}.$$

Hence, $\widetilde{\mathrm{P}}[p]$ is a picture fuzzy shift UP-filter of X, that is, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft shift UP-filter of X. □

Theorem 4.30. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a PFSS over* X *satisfying the following condition:*

$$(4.18) \qquad (\forall p \in Y, \forall x, y, z \in X)\left(z \le x \cdot y \Rightarrow \begin{cases} r_{\widetilde{\mathrm{P}}[p]}(z) \ge r_{\widetilde{\mathrm{P}}[p]}(y) \\ g_{\widetilde{\mathrm{P}}[p]}(z) \ge g_{\widetilde{\mathrm{P}}[p]}(y) \\ b_{\widetilde{\mathrm{P}}[p]}(z) \le b_{\widetilde{\mathrm{P}}[p]}(y) \end{cases}\right),$$

then $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft strong UP-ideal of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a PFSS over X satisfying the condition (4.18) and let $p \in Y$. Next, let $x, y \in X$. By (2.3) and (2.6), we have $x \cdot (y \cdot y) = 0$, that is, $x \le y \cdot y$. It follows from (4.18) that $r_{\widetilde{\mathrm{P}}[p]}(x) \ge r_{\widetilde{\mathrm{P}}[p]}(y), g_{\widetilde{\mathrm{P}}[p]}(x) \ge g_{\widetilde{\mathrm{P}}[p]}(y)$, and $b_{\widetilde{\mathrm{P}}[p]}(x) \le b_{\widetilde{\mathrm{P}}[p]}(y)$. Similarly, $r_{\widetilde{\mathrm{P}}[p]}(y) \ge r_{\widetilde{\mathrm{P}}[p]}(x), g_{\widetilde{\mathrm{P}}[p]}(y) \ge g_{\widetilde{\mathrm{P}}[p]}(x)$, and $b_{\widetilde{\mathrm{P}}[p]}(y) \le b_{\widetilde{\mathrm{P}}[p]}(x)$. Then $r_{\widetilde{\mathrm{P}}[p]}(x) = r_{\widetilde{\mathrm{P}}[p]}(y), g_{\widetilde{\mathrm{P}}[p]}(x) = g_{\widetilde{\mathrm{P}}[p]}(y)$, and $b_{\widetilde{\mathrm{P}}[p]}(x) = b_{\widetilde{\mathrm{P}}[p]}(y)$. Thus $\widetilde{\mathrm{P}}[p]$ is constant. By Theorem 4.2, we have $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft strong UP-ideal of X. □

Corollary 4.31. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a PFSS over* X *which satisfies the condition* (4.12) *(resp.,* (4.13)*,* (4.14)*,* (4.15)*,* (4.16)*,* (4.17)*, and* (4.18)*), then it satisfies the condition* (4.1).

Proof. It is straightforward form Theorems 4.1 and 4.24 (resp., Theorem 4.25, Theorem 4.26, Theorem 4.27, Theorem 4.28, Theorem 4.29, and Theorem 4.30). □

Theorem 4.32. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft UP-subalgebra of* X *and* N *is a nonempty subset of* Y*, then* $(\widetilde{\mathrm{P}}|_N, N)$ *is a picture fuzzy soft UP-subalgebra of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-subalgebra of X and $\varnothing \neq N \subseteq Y$. Then $(\widetilde{\mathrm{P}}, Y)$ is a p-picture fuzzy soft UP-subalgebra of X for all $p \in Y$. Since $N \subseteq Y$, we have $(\widetilde{\mathrm{P}}|_N, N)$ is a p-picture fuzzy soft UP-subalgebra of X for all $p \in N$. Therefore, $(\widetilde{\mathrm{P}}|_N, N)$ is a picture fuzzy soft UP-subalgebra of X. □

Theorem 4.33. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft near UP-filter of* X *and* N *is a nonempty subset of* Y*, then* $(\widetilde{\mathrm{P}}|_N, N)$ *is a picture fuzzy soft near UP-filter of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft near UP-filter of X and $\varnothing \neq N \subseteq Y$. Then $(\widetilde{\mathrm{P}}, Y)$ is a p-picture fuzzy soft near UP-filter of X for all $p \in Y$. Since $N \subseteq Y$, we have $(\widetilde{\mathrm{P}}|_N, N)$ is a p-picture fuzzy soft near UP-filter of X for all $p \in N$. Therefore, $(\widetilde{\mathrm{P}}|_N, N)$ is a picture fuzzy soft near UP-filter of X. □

Theorem 4.34. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft UP-filter of* X *and* N *is a nonempty subset of* Y*, then* $(\widetilde{\mathrm{P}}|_N, N)$ *is a picture fuzzy soft UP-filter of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-filter of X and $\varnothing \neq N \subseteq Y$. Then $(\widetilde{\mathrm{P}}, Y)$ is a p-picture fuzzy soft UP-filter of X for all $p \in Y$. Since $N \subseteq Y$, we have $(\widetilde{\mathrm{P}}|_N, N)$ is a p-picture fuzzy soft UP-filter of X for all $p \in N$. Therefore, $(\widetilde{\mathrm{P}}|_N, N)$ is a picture fuzzy soft UP-filter of X. □

Theorem 4.35. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft implicative UP-filter of* X *and* N *is a nonempty subset of* Y*, then* $(\widetilde{\mathrm{P}}|_N, N)$ *is a picture fuzzy soft implicative UP-filter of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft implicative UP-filter of X and $\varnothing \neq N \subseteq Y$. Then $(\widetilde{\mathrm{P}}, Y)$ is a p-picture fuzzy soft implicative UP-filter of X for all $p \in Y$. Since $N \subseteq Y$, we have $(\widetilde{\mathrm{P}}|_N, N)$ is a p-picture fuzzy soft implicative UP-filter of X for all $p \in N$. Therefore, $(\widetilde{\mathrm{P}}|_N, N)$ is a picture fuzzy soft implicative UP-filter of X. □

Theorem 4.36. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft comparative UP-filter of* X *and* N *is a nonempty subset of* Y*, then* $(\widetilde{\mathrm{P}}|_N, N)$ *is a picture fuzzy soft comparative UP-filter of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft comparative UP-filter of X and $\varnothing \neq N \subseteq Y$. Then $(\widetilde{\mathrm{P}}, Y)$ is a p-picture fuzzy soft comparative UP-filter of X for all $p \in Y$. Since $N \subseteq Y$, we have $(\widetilde{\mathrm{P}}|_N, N)$ is a p-picture fuzzy soft comparative UP-filter of X for all $p \in N$. Therefore, $(\widetilde{\mathrm{P}}|_N, N)$ is a picture fuzzy soft comparative UP-filter of X. □

Theorem 4.37. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft shift UP-filter of* X *and* N *is a nonempty subset of* Y*, then* $(\widetilde{\mathrm{P}}|_N, N)$ *is a picture fuzzy soft shift UP-filter of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft shift UP-filter of X and $\varnothing \neq N \subseteq Y$. Then $(\widetilde{\mathrm{P}}, Y)$ is a p-picture fuzzy soft shift UP-filter of X for all $p \in Y$. Since $N \subseteq Y$, we have $(\widetilde{\mathrm{P}}|_N, N)$ is a p-picture fuzzy soft shift UP-filter of X for all $p \in N$. Therefore, $(\widetilde{\mathrm{P}}|_N, N)$ is a picture fuzzy soft shift UP-filter of X. □

Theorem 4.38. *If* $(\widetilde{\mathrm{P}}, Y)$ *is a picture fuzzy soft UP-ideal of* X *and* N *is a nonempty subset of* Y*, then* $(\widetilde{\mathrm{P}}|_N, N)$ *is a picture fuzzy soft UP-ideal of* X*.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft UP-ideal of X and $\emptyset \neq N \subseteq Y$. Then $(\widetilde{\mathrm{P}}, Y)$ is a p-picture fuzzy soft UP-ideal of X for all $p \in Y$. Since $N \subseteq Y$, we have $(\widetilde{\mathrm{P}}|_N, N)$ is a p-picture fuzzy soft UP-ideal of X for all $p \in N$. Therefore, $(\widetilde{\mathrm{P}}|_N, N)$ is a picture fuzzy soft UP-ideal of X. □

Theorem 4.39. *If $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft strong UP-ideal of X and N is a nonempty subset of Y, then $(\widetilde{\mathrm{P}}|_N, N)$ is a picture fuzzy soft strong UP-ideal of X.*

Proof. Assume that $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft strong UP-ideal of X and $\emptyset \neq N \subseteq Y$. Then $(\widetilde{\mathrm{P}}, Y)$ is a p-picture fuzzy soft strong UP-ideal of X for all $p \in Y$. Since $N \subseteq Y$, we have $(\widetilde{\mathrm{P}}|_N, N)$ is a p-picture fuzzy soft strong UP-ideal of X for all $p \in N$. Therefore, $(\widetilde{\mathrm{P}}|_N, N)$ is a picture fuzzy soft strong UP-ideal of X. □

Example 4.9. Let X be the set of four brands of a phone, that is,

$$X = \{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	$\clubsuit$	$\diamondsuit$	$\heartsuit$	$\spadesuit$
$\clubsuit$	$\clubsuit$	$\diamondsuit$	$\heartsuit$	$\spadesuit$
$\diamondsuit$	$\clubsuit$	$\clubsuit$	$\clubsuit$	$\clubsuit$
$\heartsuit$	$\clubsuit$	$\diamondsuit$	$\clubsuit$	$\spadesuit$
$\spadesuit$	$\clubsuit$	$\diamondsuit$	$\heartsuit$	$\clubsuit$

Then $(X, \cdot, \clubsuit)$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{price}, \text{beauty}, \text{stability}\}$$

with $\widetilde{\mathrm{P}}[\text{price}], \widetilde{\mathrm{P}}[\text{beauty}]$, and $\widetilde{\mathrm{P}}[\text{stability}]$ are picture fuzzy sets in X defined as follows:

$\widetilde{\mathrm{P}}$	$\clubsuit$	$\diamondsuit$	$\heartsuit$	$\spadesuit$
$r_{\widetilde{\mathrm{P}}[\text{price}]}$	0.5	0.4	0.4	0.4
$g_{\widetilde{\mathrm{P}}[\text{price}]}$	0.4	0.3	0.3	0.3
$b_{\widetilde{\mathrm{P}}[\text{price}]}$	0	0.2	0.2	0.2
$r_{\widetilde{\mathrm{P}}[\text{beauty}]}$	0.4	0.3	0.3	0.3
$g_{\widetilde{\mathrm{P}}[\text{beauty}]}$	0.5	0.2	0.3	0.3
$b_{\widetilde{\mathrm{P}}[\text{beauty}]}$	0	0.3	0.1	0.1
$r_{\widetilde{\mathrm{P}}[\text{stability}]}$	0.4	0.3	0.3	0.3
$g_{\widetilde{\mathrm{P}}[\text{stability}]}$	0.6	0.3	0.3	0.3
$b_{\widetilde{\mathrm{P}}[\text{stability}]}$	0	0.3	0.2	0.2

Hence, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft implicative UP-filter of X.

Example 4.10. Let X be the set of four foods, that is,

$$X = \{\text{carrot}, \text{banana}, \text{rice}, \text{meat}\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	carrot	banana	rice	meat
carrot	carrot	banana	rice	meat
banana	carrot	carrot	banana	meat
rice	carrot	carrot	carrot	meat
meat	carrot	carrot	banana	carrot

Then $(X, \cdot, \text{carrot})$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{giraffe}, \text{pig}, \text{elephant}\}$$

with $\widetilde{\mathrm{P}}[\text{giraffe}], \widetilde{\mathrm{P}}[\text{pig}]$, and $\widetilde{\mathrm{P}}[\text{elephant}]$ are picture fuzzy sets in X defined as follows:

$\widetilde{\mathrm{P}}$	carrot	banana	rice	meat
$r_{\widetilde{\mathrm{P}}[\text{giraffe}]}$	0.5	0.5	0.5	0.4
$g_{\widetilde{\mathrm{P}}[\text{giraffe}]}$	0.3	0.3	0.3	0.1
$b_{\widetilde{\mathrm{P}}[\text{giraffe}]}$	0.1	0.1	0.1	0.2
$r_{\widetilde{\mathrm{P}}[\text{pig}]}$	0.4	0.4	0.4	0.2
$g_{\widetilde{\mathrm{P}}[\text{pig}]}$	0.3	0.3	0.3	0.2
$b_{\widetilde{\mathrm{P}}[\text{pig}]}$	0	0	0	0.1
$r_{\widetilde{\mathrm{P}}[\text{elephant}]}$	0.4	0.4	0.4	0.3
$g_{\widetilde{\mathrm{P}}[\text{elephant}]}$	0.3	0.3	0.3	0.1
$b_{\widetilde{\mathrm{P}}[\text{elephant}]}$	0.2	0.2	0.2	0.4

Hence, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft comparative UP-filter of X.

Example 4.11. Let X be the set of five types of a movie, that is,

$$X = \{\text{war}, \text{thriller}, \text{crime}, \text{action}\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	war	thriller	crime	action
war	war	thriller	crime	action
thriller	war	war	crime	crime
crime	war	war	war	crime
action	war	war	war	war

Then $(X, \cdot, \text{war})$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{horror}, \text{battle}, \text{adventure}, \text{mystery}\}$$

with $\widetilde{\mathrm{P}}[\text{horror}], \widetilde{\mathrm{P}}[\text{battle}], \widetilde{\mathrm{P}}[\text{adventure}]$, and $\widetilde{\mathrm{P}}[\text{mystery}]$ are picture fuzzy sets in X

defined as follows:

$\widetilde{\mathrm{P}}$	war	thriller	crime	action
$r_{\widetilde{\mathrm{P}}[\text{horror}]}$	0.5	0.5	0.4	0.4
$g_{\widetilde{\mathrm{P}}[\text{horror}]}$	0.3	0.3	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{horror}]}$	0.2	0.2	0.3	0.3
$r_{\widetilde{\mathrm{P}}[\text{battle}]}$	0.6	0.6	0.4	0.4
$g_{\widetilde{\mathrm{P}}[\text{battle}]}$	0.3	0.3	0.2	0.2
$b_{\widetilde{\mathrm{P}}[\text{battle}]}$	0.1	0.1	0.3	0.3
$r_{\widetilde{\mathrm{P}}[\text{adventure}]}$	0.7	0.7	0.5	0.5
$g_{\widetilde{\mathrm{P}}[\text{adventure}]}$	0.3	0.3	0.2	0.2
$b_{\widetilde{\mathrm{P}}[\text{adventure}]}$	0	0	0.1	0.1
$r_{\widetilde{\mathrm{P}}[\text{mystery}]}$	0.4	0.4	0.3	0.3
$g_{\widetilde{\mathrm{P}}[\text{mystery}]}$	0.5	0.5	0.1	0.1
$b_{\widetilde{\mathrm{P}}[\text{mystery}]}$	0.1	0.1	0.4	0.4

Hence, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft shift UP-filter of X.

Example 4.12. Let X be the set of four types of a music, that is,

$$X = \{\text{pop}, \text{rock}, \text{classic}, \text{jazz}\}.$$

Define a binary operation $\cdot$ on X as the following Cayley table:

$\cdot$	pop	rock	classic	jazz
pop	pop	rock	classic	jazz
rock	pop	pop	rock	rock
classic	pop	pop	pop	rock
jazz	pop	pop	rock	pop

Then $(X, \cdot, \text{pop})$ is a UP-algebra. Let $(\widetilde{\mathrm{P}}, Y)$ be a picture fuzzy soft set over X, where

$$Y := \{\text{modernity}, \text{enjoyment}, \text{sorrow}\}$$

with $\widetilde{\mathrm{P}}[\text{modernity}], \widetilde{\mathrm{P}}[\text{enjoyment}]$, and $\widetilde{\mathrm{P}}[\text{sorrow}]$ are picture fuzzy sets in X defined as follows:

$\widetilde{\mathrm{P}}$	pop	rock	classic	jazz
$r_{\widetilde{\mathrm{P}}[\text{modernity}]}$	0.3	0.3	0.3	0.3
$g_{\widetilde{\mathrm{P}}[\text{modernity}]}$	0.4	0.4	0.4	0.4
$b_{\widetilde{\mathrm{P}}[\text{modernity}]}$	0.2	0.2	0.2	0.2
$r_{\widetilde{\mathrm{P}}[\text{enjoyment}]}$	0.4	0.4	0.4	0.4
$g_{\widetilde{\mathrm{P}}[\text{enjoyment}]}$	0.5	0.5	0.5	0.5
$b_{\widetilde{\mathrm{P}}[\text{enjoyment}]}$	0	0	0	0
$r_{\widetilde{\mathrm{P}}[\text{sorrow}]}$	0.2	0.2	0.2	0.2
$g_{\widetilde{\mathrm{P}}[\text{sorrow}]}$	0.3	0.3	0.3	0.3
$b_{\widetilde{\mathrm{P}}[\text{sorrow}]}$	0.1	0.1	0.1	0.1

Hence, $(\widetilde{\mathrm{P}}, Y)$ is a picture fuzzy soft strong UP-ideal of X.

5 Conclusions

In this paper, we have introduced the eight new concepts of picture fuzzy soft sets over UP-algebras: picture fuzzy soft UP-subalgebras, picture fuzzy soft near UP-filters, picture fuzzy soft UP-filters, picture fuzzy soft implicative UP-filters, picture fuzzy soft comparative UP-filters, picture fuzzy soft shift UP-filters, picture fuzzy soft UP-ideals, and picture fuzzy soft strong UP-ideals and investigated some of their important properties. Then, we get the diagram of generalization of PFSSs over UP-algebras as shown in Figure 1 and sufficient conditions of PFSSs as shown in Figure 2.

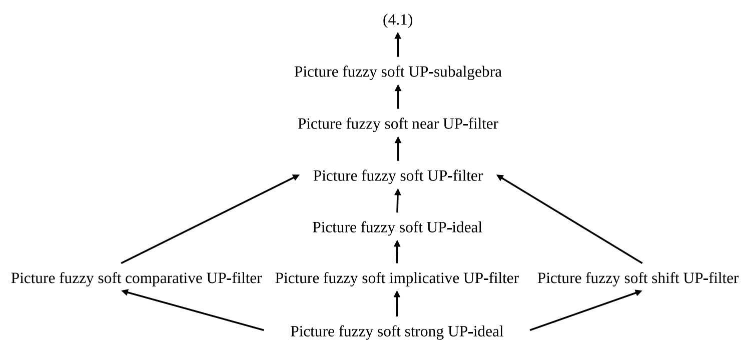

Figure 1: PFSSs over UP-algebras

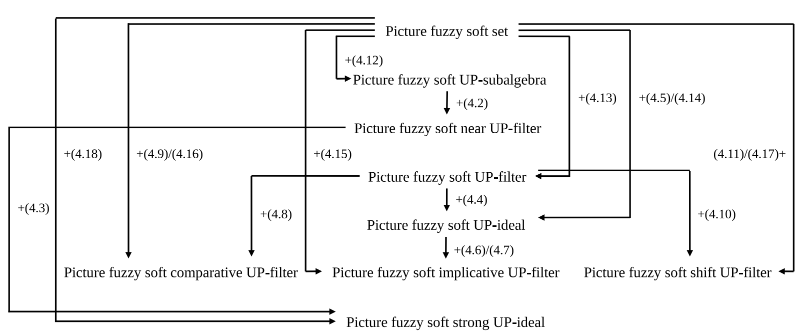

Figure 2: Sufficient conditions of PFSSs over UP-algebras

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