Bipolar \((\lambda, \delta)\)-fuzzy ideals in LA-semigroups

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**Abstract.** In this paper, the notation of bipolar \((\lambda, \delta)\)-fuzzy LA-subsemigroups is introduced and also some properties of bipolar-\((\lambda, \delta)\)-fuzzy LA-subsemigroup, left (right generalized bi, bi-) ideals of LA-semigroups have been discussed.

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1 Introduction

The concept of a fuzzy set was introduced by Zadeh in 1965. Since its inception, the theory has developed in many directions and found applications in a wide variety of fields. There has been a rapid growth in the interest of fuzzy set theory and its applications from the past several years. Many researchers published high-quality research articles on fuzzy sets in a variety of international journals. Lee [2] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval \([0, 1]\) to \([-1, 1]\). In a bipolar-valued fuzzy set, the membership degree indicate that elements are irrelevant to the corresponding property, the membership degrees on \((0, 1]\) assign that elements somewhat satisfy the property, and the membership degrees on \([-1, 0)\) assign that elements somewhat satisfy the implicit counter-property. They introduced the concept of bipolar fuzzy subalgebras and bipolar fuzzy ideals of a BCH-algebra. Lee applied the notion of bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras. Also some results on bipolar-valued fuzzy BCK/BCI-algebras are introduced by Saeid.

B. Yao [5], introduced \((\lambda, \delta)\)-fuzzy normal subfields. Khan et al. [5] characterized ordered semigroups in terms of \((\lambda, \delta)\)-fuzzy bi-ideals.

In 1972, M. A. Kazim and M. Naseerudin [7], have introduced a pseudo associate law or left invertive law in a groupoid \(\mathcal{G}\) by \((ab)c = (cb)a\) for all \(a, b, c \in \mathcal{G}\), and called the groupoid a left almost semigroup (abbreviated as a LA-semigroup). An LA-semigroup satisfies the medial law \((ab)(cd) = (ac)(bd)\) for all \(a, b, c, d \in \mathcal{G}\). It is a nonassociative algebraic structure midway between a groupoid and a commutative semigroup. An LA-semigroup \(\mathcal{G}\) with left identity the satisfies paramedial law, that is, \((ab)(cd) = (db)(ca)\) and \(\mathcal{G}\) satisfies the following law \(a(bc) = b(ac)\) for all \(a, b, c, d \in \mathcal{G}\). A non-empty subset \(\mathcal{A}\) of an LA-semigroup \(S\) is called a left (right) ideal of \(\mathcal{G}\) if \(\mathcal{G}\mathcal{A} \subseteq \mathcal{A}(\mathcal{A}\mathcal{G} \subseteq \mathcal{A})\) and it is called a two-sided ideal if it is both left and a right ideal of \(\mathcal{G}\).
In 2011, N. Yaqoob [4] have introduced a bipolar valued fuzzy LA-subsemigroups and bipolar-valued fuzzy left (right, bi-, interior) ideals in LA-semigroups. In this paper we defined the notation bipolar \((\lambda, \delta)\)-fuzzy ideal in LA-semigroups and some properties of these ideals are studied.

2 Preliminaries

Definition 2.1. [7] A groupoid \(S\) is called a left almost semigroup (abbreviated as an LA-semigroup) if it satisfy the left invertive law:

\[(ab)c = (cb)a\] for all \(a, b, c \in S\).

Lemma 2.1. [7] If \(S\) is a unitary LA-semigroup, then the medial law holds, that is,

\[(ab)(cd) = (ac)(bd),\] for all \(a, b, c, d \in S\).

Definition 2.2. If there is an element 0 of an LA-semigroup \(S\) such that \(x0 = 0x = x\) for all \(x \in S\), we call 0 a zero element of \(S\).

Proposition 2.2. [8] If \(S\) is an LA-semigroup with left identity, then

\[a(bc) = b(ac),\] for all \(a, b, c \in S\).

Definition 2.3. [8] An LA-semigroup \(S\) is called a paramedial if its satisfies

\[(ab)(cd) = (dc)(ba),\] for all \(a, b, c, d \in S\).

Definition 2.4. [4] An LA-semigroup \(S\) is called a locally associative LA-semigroup if it satisfies

\[(aa)a = a(aa),\] for all \(a \in S\).

Definition 2.5. Let \(S\) be an LA-semigroup. A non-empty subset \(A\) of \(S\) is called an LA-subsemigroup of \(S\) if \(A^2 \subseteq A\).

Definition 2.6. [9] A non-empty subset \(A\) of an LA-semigroup \(S\) is called a left (right) ideal of \(S\) if \(SA \subseteq A (AS \subseteq A)\), and is called an ideal of \(S\) if it is both left and right ideal of \(S\).

Now we will recall the concept of bipolar-valued fuzzy sets.

Definition 2.7. [4] Let \(X\) be a nonempty set. A bipolar-valued fuzzy subset (BVF-subset, in short) \(B\) of \(X\) is an object having the form

\[(2.1) B = \{\langle x, \mu^+_B(x), \mu^-_B(x) \rangle : x \in X\},\]

where \(\mu^+_B(x) : X \to [0, 1]\) and \(\mu^-_B(x) : X \to [-1, 0]\)

The positive membership degree \(\mu^+_B(x)\) denotes the satisfaction degree of an element \(x\) to the property corresponding to a bipolar-valued fuzzy set \(B\) od the form (2.1) and the negative membership degree \(\mu^-_B(x)\) denotes the satisfaction degree of \(x\) to some implicit counter property of \(B = \{\langle x, \mu^+_B(x), \mu^-_B(x) \rangle : x \in X\}\). For the sake of simplicity, we shall use the symbol \(B = \{\langle x, \mu^+_B(x), \mu^-_B(x) \rangle \}\) for the bipolar-valued fuzzy set \(B = \{\langle x, \mu^+_B(x), \mu^-_B(x) \rangle : x \in X\}\).
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Definition 2.8. [4] Let \(\mathcal{B}_1 = \langle \mu_{\mathcal{B}_1}^+, \mu_{\mathcal{B}_1}^- \rangle\) and \(\mathcal{B}_2 = \langle \mu_{\mathcal{B}_2}^+, \mu_{\mathcal{B}_2}^- \rangle\) be two BVF-subsets of a nonempty set \(\mathcal{X}\). Then the product of two BVF-subsets is denoted by \(\mathcal{B}_1 \circ \mathcal{B}_2\) and defined as:

\[
\mu_{\mathcal{B}_1}^+ \circ \mu_{\mathcal{B}_2}^+(x) = \begin{cases} \bigvee_{y=z} \{ \mu_{\mathcal{B}_1}^+(y) \lor \mu_{\mathcal{B}_2}^+(z) \}, & \text{if } x = yz \text{ for some } y, z \in \mathcal{S} \\ 0, & \text{otherwise.} \end{cases}
\]

\[
\mu_{\mathcal{B}_1}^- \circ \mu_{\mathcal{B}_2}^-(x) = \begin{cases} \bigwedge_{y=z} \{ \mu_{\mathcal{B}_1}^-(y) \land \mu_{\mathcal{B}_2}^-(z) \}, & \text{if } x = yz \text{ for some } y, z \in \mathcal{S} \\ 0, & \text{otherwise.} \end{cases}
\]

Note that an LA-semigroup \(\mathcal{S}\) can be considered as a BVF-subset of itself and let

\[
\Gamma = \langle \mathcal{S}_{\Gamma}^+(x), \mathcal{S}_{\Gamma}^-(x) \rangle
\]

be a BVF-subset and \(\Gamma = \langle \mathcal{S}_{\Gamma}^+(x), \mathcal{S}_{\Gamma}^-(x) \rangle\) will be carried out in operations with a BVF-subset \(\mathcal{B} = \langle \mu_{\mathcal{B}}^+(x), \mu_{\mathcal{B}}^-(x) \rangle\) such that \(\mathcal{S}^+\) and \(\mathcal{S}^-\) will be used in collaboration with \(\mu_{\mathcal{B}}^+\) and \(\mu_{\mathcal{B}}^-\), respectively. Let \(\text{BVF}(\mathcal{S})\) denote the set of all BVF-subsets of an LA-semigroup \(\mathcal{S}\).

Proposition 2.3. [4] Let \(\mathcal{S}\) be an LA-semigroup, then the set \((\text{BVF}(\mathcal{S}), \circ)\) is an LA-semigroup.

Corollary 2.4. [4] If \(\mathcal{S}\) is an LA-semigroup, then the medial law holds in \(\text{BVF}(\mathcal{S})\).

Definition 2.9. [5] Let \(\mathcal{B} = \langle x, \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle\) be a bipolar fuzzy set and \((s, t) \in [-1, 0] \times [0, 1]\). Define:

1. the set \(\mathcal{B}^+_x = \{ x \in \mathcal{X} \mid \mu_{\mathcal{B}}^+(x) \geq t \}\) and \(\mathcal{B}^-_x = \{ x \in \mathcal{X} \mid \mu_{\mathcal{B}}^-(x) \leq s \}\), which are called positive \(t\)-cut of \(B = \langle x, \mu_{\mathcal{B}}^+(x), \mu_{\mathcal{B}}^-(x) \rangle\) and the negative \(s\)-cut of \(\mathcal{B} = \langle x, \mu_{\mathcal{B}}^+(x), \mu_{\mathcal{B}}^-(x) \rangle\), respectively,

2. the set \(\geq \mathcal{B}^+_x = \{ x \in \mathcal{X} \mid \mu_{\mathcal{B}}^+(x) > t \}\) and \(\leq \mathcal{B}^-_x = \{ x \in \mathcal{X} \mid \mu_{\mathcal{B}}^-(x) < s \}\), which are called strong positive \(t\)-cut of \(\mathcal{B} = \langle x, \mu_{\mathcal{B}}^+(x), \mu_{\mathcal{B}}^-(x) \rangle\) and the strong negative \(s\)-cut of \(\mathcal{B} = \langle x, \mu_{\mathcal{B}}^+(x), \mu_{\mathcal{B}}^-(x) \rangle\), respectively,

3. the set \(X^{(s, t)}_\mathcal{B} = \{ x \in \mathcal{X} \mid \mu_{\mathcal{B}}^+(x) \geq t, \mu_{\mathcal{B}}^-(x) \leq s \}\) is called an \((s, t)\)-level subset of \(\mathcal{B}\),

4. the set \(SX^{(t, s)}_\mathcal{B} = \{ x \in \mathcal{X} \mid \mu_{\mathcal{B}}^+(x) > t, \mu_{\mathcal{B}}^-(x) < s \}\) is called a strong \((s, t)\)-level subset of \(\mathcal{B}\) for all \(x \in \mathcal{S}\).

3 Bipolar \((\lambda, \delta)\) Fuzzy Ideals in LA-semigroups

In this section, we will define the notion of bipolar \((\lambda, \delta)\)-fuzzy ideals in ternary semigroups and discuss some properties of these ideals. In what follows, let \(\lambda, \delta \in [0, 1]\) be such that \(0 \leq \lambda < \delta \leq 1\). Both \(\lambda\) and \(\delta\) are arbitrary but fixed.
Definition 3.1. A bipolar fuzzy subset \( \mathfrak{B} = \langle \mu_{\mathfrak{B}}^+, \mu_{\mathfrak{B}}^- \rangle \) of an LA-semigroup \( \mathfrak{S} \) is called a bipolar \((\lambda, \delta)\)-fuzzy LA-subsemigroup of \( \mathfrak{S} \) if

1. \( \max \{ \mu_{\mathfrak{B}}^+(xy), \lambda \} \geq \min \{ \mu_{\mathfrak{B}}^+(x), \mu_{\mathfrak{B}}^+(y), \delta \} \)
2. \( \min \{ \mu_{\mathfrak{B}}^-(xy), -\lambda \} \leq \max \{ \mu_{\mathfrak{B}}^-(x), \mu_{\mathfrak{B}}^-(y), -\delta \} \)

for all \( x, y \in \mathfrak{B} \).

Definition 3.2. A bipolar fuzzy subset \( \mathfrak{B} = \langle \mu_{\mathfrak{B}}^+, \mu_{\mathfrak{B}}^- \rangle \) of an LA-semigroup \( \mathfrak{S} \) is called a bipolar \((\lambda, \delta)\)-fuzzy left (right) ideal of \( \mathfrak{S} \) if

1. \( \max \{ \mu_{\mathfrak{B}}^+(xy), \lambda \} \geq \min \{ \mu_{\mathfrak{B}}^+(x), \mu_{\mathfrak{B}}^+(y), \delta \} \)
2. \( \min \{ \mu_{\mathfrak{B}}^-(xy), -\lambda \} \leq \max \{ \mu_{\mathfrak{B}}^-(x), \mu_{\mathfrak{B}}^-(y), -\delta \} \)

for all \( x, y \in \mathfrak{B} \). A bipolar fuzzy subset \( \mathfrak{B} = \langle \mu_{\mathfrak{B}}^+, \mu_{\mathfrak{B}}^- \rangle \) of a LA-semigroup \( \mathfrak{S} \) is called a bipolar \((\lambda, \delta)\)-fuzzy ideal of \( \mathfrak{S} \) if it is a bipolar \((\lambda, \delta)\)-fuzzy left ideal and bipolar \((\lambda, \delta)\)-fuzzy right ideal of \( \mathfrak{S} \).

Definition 3.3. A bipolar fuzzy subset \( \mathfrak{B} = \langle \mu_{\mathfrak{B}}^+, \mu_{\mathfrak{B}}^- \rangle \) of an LA-semigroup \( \mathfrak{S} \) is called a bipolar \((\lambda, \delta)\)-fuzzy generalized bi-ideal of \( \mathfrak{S} \) if

1. \( \max \{ \mu_{\mathfrak{B}}^+(xy)z, \lambda \} \geq \min \{ \mu_{\mathfrak{B}}^+(x), \mu_{\mathfrak{B}}^+(z), \delta \} \)
2. \( \min \{ \mu_{\mathfrak{B}}^-(xy)z, -\lambda \} \leq \max \{ \mu_{\mathfrak{B}}^-(x), \mu_{\mathfrak{B}}^-(z), -\delta \} \)

for all \( x, y, z \in \mathfrak{S} \).

Definition 3.4. A bipolar \((\lambda, \delta)\)-fuzzy subset \( \mathfrak{B} = \langle \mu_{\mathfrak{B}}^+, \mu_{\mathfrak{B}}^- \rangle \) of an LA-semigroup \( \mathfrak{S} \) is called a bipolar \((\lambda, \delta)\)-fuzzy bi-ideal of \( \mathfrak{S} \) if

1. \( \max \{ \mu_{\mathfrak{B}}^+(xy)z, \lambda \} \geq \min \{ \mu_{\mathfrak{B}}^+(x), \mu_{\mathfrak{B}}^+(z), \delta \} \)
2. \( \min \{ \mu_{\mathfrak{B}}^-(xy)z, -\lambda \} \leq \max \{ \mu_{\mathfrak{B}}^-(x), \mu_{\mathfrak{B}}^-(z), -\delta \} \)

for all \( x, y \in \mathfrak{S} \).

Theorem 3.1. A bipolar fuzzy subset \( \mathfrak{B} = \langle \mu_{\mathfrak{B}}^+, \mu_{\mathfrak{B}}^- \rangle \) of a LA-semigroup \( \mathfrak{S} \) is a bipolar \((\lambda, \delta)\)-fuzzy LA-subsemigroup, left (right generalized bi, bi-)ideal of \( \mathfrak{S} \) if and only if \( \mathfrak{S} \neq \mathfrak{S}^{[t,s]} \) is an LA-subsemigroup, left (right generalized bi, bi-)ideal of \( \mathfrak{S} \) for all \((s, t) \in [-\delta, -\lambda] \times (\lambda, \delta)\).

Proof. Suppose that \( B = \langle \mu_{\mathfrak{B}}^+, \mu_{\mathfrak{B}}^- \rangle \) be a bipolar \((\lambda, \delta)\)-fuzzy LA-subsemigroup of \( \mathfrak{S} \). Let \( x, y \in \mathfrak{S}, (s, t) \in [-\delta, -\lambda] \times (\lambda, \delta) \) and \( x, y \in \mathfrak{S}^{[t,s]} \). Then \( \mu_{\mathfrak{B}}^+(x) \geq t, \mu_{\mathfrak{B}}^+(y) \geq t \) also \( \mu^-(x) \leq s \) and \( \mu^-(y) \leq s \). Since \( \mathfrak{B} = \langle \mu_{\mathfrak{B}}^+, \mu_{\mathfrak{B}}^- \rangle \) is a bipolar \((\lambda, \delta)\)-fuzzy LA-subsemigroup of \( \mathfrak{S} \). Then

\[
\max \{ \mu_{\mathfrak{B}}^+(xy), \lambda \} \geq \min \{ \mu_{\mathfrak{B}}^+(x), \mu_{\mathfrak{B}}^+(y), \delta \} \geq \min \{ t, t, \delta \} = t.
\]

and

\[
\min \{ \mu_{\mathfrak{B}}^-(xy), -\lambda \} \leq \max \{ \mu_{\mathfrak{B}}^-(x), \mu_{\mathfrak{B}}^-(y), -\delta \} \geq \min \{ s, s, -\delta \} = s.
\]
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This implies that \(\mu_{S_2}^+(xy) \geq s\) and \(\mu_{S_2}(xy) \leq s\). Thus \(xy \in S_2^{(s,t)}\).

Hence \(S_2^{(s,t)}\) is an LA-subsemigroup of \(S\).

Conversely, suppose that \(S_2^{(s,t)}\) is an LA-subsemigroup of \(S\). Let \(x, y \in S\) such that

\[
\max \{\mu_{S_2}^+(xy), \lambda\} < \min \{\mu_{S_2}^+(x), \mu_{S_2}^+(y), \delta\}
\]

and

\[
\min \{\mu_{S_2}^-(xy), -\lambda\} > \max \{\mu_{S_2}^-(x), \mu_{S_2}^-(y), -\delta\}.
\]

Then there exists \((s,t) \in [-\delta, \lambda] \times (\lambda, \delta]\) such that

\[
\max \{\mu_{S_2}^+(xy), \lambda\} < t \leq \min \{\mu_{S_2}^+(x), \mu^- B(y), \delta\}
\]

and

\[
\min \{\mu_{S_2}^-(xy), -\lambda\} > s \geq \max \{\mu_{S_2}^-(x), \mu^- B(y), -\delta\}.
\]

This shows that \(\mu_{S_2}^+(x) \geq t, \mu_{S_2}^+(y) \geq t\) and \(\mu_{S_2}^+(xy) < t\), also \(\mu_{S_2}^-(x) \leq s\), \(\mu_{S_2}(y) \leq s\) and \(\mu_{S_2}^+(xy) > s\). Thus \(x, y \in S_2^{(t,s)}\), since \(S_2^{(s,t)}\) is an LA-subsemigroup of \(S\). Therefore \(xy \in S_2^{(s,t)}\), but this is a contradiction to \(\mu_{S_2}^+(xy) < t\) and \(\mu_{S_2}^-(xy) > s\).

Thus

\[
\max \{\mu_{S_2}^+(xy), \lambda\} \geq \min \{\mu^+ B(x), u^+(y), \delta\}
\]

and

\[
\min \{\mu_{S_2}^-(xy), -\lambda\} \leq \max \{\mu^+ B(x), u^+(y), -\delta\}.
\]

Hence \(B = \langle \mu_{S_2}^+, \mu_{S_2}^- \rangle\) is a bipolar \((\lambda, \delta)\)-fuzzy LA-subsemigroup of \(S\).

The other cases can be seen in a similar way.

\[\square\]

**Corollary 3.2.** Every bipolar fuzzy LA-subsemigroup, left (right generalized bi, bi-)ideal \(B = \langle \mu_{S_2}^+, \mu_{S_2}^- \rangle\) is a bipolar \((\lambda, \delta)\)-fuzzy LA-subsemigroup, left (right generalized bi, bi-)ideal of \(S\) with \(\lambda = 0\) and \(\delta = 1\).

**Theorem 3.3.** If a bipolar fuzzy subset \(B = \langle \mu_{S_2}^+, \mu_{S_2}^- \rangle\) is a bipolar \((\lambda, \delta)\)-fuzzy LA-subsemigroup, left (right generalized bi, bi-)ideal of \(S\), then the set \(B_\lambda = \langle B^\lambda_+ \cap B_-^\lambda \rangle\) is an LA-subsemigroup, left (right generalized bi, bi-)ideal of \(S\), where \(B_\lambda^+ = \{x \in S \mid \mu_{S_2}^+(x) = \lambda\}\) and \(B_\lambda^- = \{x \in S \mid \mu_{S_2}^-(x) = -\lambda\}\).

*Proof.* Suppose that \(B = \langle \mu_{S_2}^+, \mu_{S_2}^- \rangle\) is a bipolar \((\lambda, \delta)\)-fuzzy LA-subsemigroup of \(S\) and \(x, y \in S\) such that \(x, y \in B_\lambda\). Then \(\mu_{S_2}^+(x) > \lambda, \mu_{S_2}^+(y) > \lambda\) and \(\mu_{S_2}^-(x) > -\lambda, \mu_{S_2}^-(y) > -\lambda\). Since \(B = \langle \mu_{S_2}^+, \mu_{S_2}^- \rangle\) is a bipolar \((\lambda, \delta)\)-fuzzy LA-subsemigroup we have

\[
\max \{\mu_{S_2}^+(xy), \lambda\} \geq \min \{\mu_{S_2}^+(x), \mu_{S_2}^+(y), \delta\} \geq \min \{\lambda, \lambda, \delta\} = \lambda.
\]

and

\[
\min \{\mu_{S_2}^-(xy), -\lambda\} \leq \max \{\mu_{S_2}^-(x), \mu_{S_2}^-(y), -\delta\} \geq \min \{-\lambda, -\lambda, -\delta\} = -\lambda.
\]

Hence \(\mu_{S_2}^+(xy) > \lambda\) and \(\mu_{S_2}^-(xy) < -\lambda\) so \(xy \in B_\lambda\).

Therefore \(B_\lambda\) is an LA-subsemigroup of \(S\).

The other cases can be seen in a similar way.

\[\square\]
Theorem 3.4. A non-empty subset $A$ of an LA-semigroup $S$ is an LA-subsemigroup, left (right, generalized bi-, bi-) ideal of $S$ if and only if the bipolar fuzzy subset $B = (\mu_B^+, \mu_B^-)$ of $S$ defined as follows:

\[
\mu_B^+(x) = \begin{cases} 
\geq \delta & \text{if } x \in A \\
\lambda, & \text{if } x \notin A 
\end{cases} \quad \text{and} \quad \mu_B^-(x) = \begin{cases} 
\leq -\delta & \text{if } x \in A \\
-\lambda, & \text{if } x \notin A 
\end{cases}
\]

is a bipolar $(\lambda, \delta)$-fuzzy LA-subsemigroup, left (right, generalized bi-, bi-) ideal of $S$.

Proof. Suppose that $A$ is an LA-subsemigroup of $S$ and $x, y \in S$.

Case 1. If $x, y \in A$ then $xy \in A$. Thus $\mu_B^+(xy) \geq \delta$ and $\mu_B^-(xy) \leq -\delta$. Hence

\[
\max \{ \mu_B^+(xy), \lambda \} \geq \delta = \min \{ \mu_B^+(x), \mu_B^+(y), \delta \}
\]

and

\[
\min \{ \mu_B^-(xy), -\lambda \} \leq -\delta = \max \{ \mu_B^-(x), \mu_B^-(y), -\delta \}.
\]

Case 2. If $x \notin A$ or $y \notin A$ then $\mu_B^+(x) \geq \delta, \mu_B^+(y) \geq \delta$ and $\mu_B^-(x) \leq -\delta, \mu_B^-(y) \leq -\delta$.

Thus

\[
\max \{ \mu_B^+(xy), \lambda \} \geq \lambda = \min \{ \mu_B^+(x), \mu_B^+(y), \delta \}
\]

and

\[
\min \{ \mu_B^-(xy), -\lambda \} \leq -\lambda = \max \{ \mu_B^-(x), \mu_B^-(y), -\delta \}.
\]

Consequently $B = (\mu_B^+, \mu_B^-)$ is a bipolar $(\lambda, \delta)$-fuzzy LA-subsemigroup of $S$.

Conversely, let $B = (\mu_B^+, \mu_B^-)$ is a bipolar $(\lambda, \delta)$-fuzzy LA-subsemigroup of $S$ and $x, y \in A$. Then $\mu_B^+(x) \geq \delta, \mu_B^+(y) \geq \delta$ and $\mu_B^-(x) \leq -\delta, \mu_B^-(y) \leq -\delta$.

Since $B = (\mu_B^+, \mu_B^-)$ is a bipolar $(\lambda, \delta)$-fuzzy LA-subsemigroup of $S$, we have

\[
\max \{ \mu_B^+(xy), \lambda \} \geq \min \{ \mu_B^+(x), \mu_B^+(y), \delta \} \geq \min \{ \delta, \delta, \delta \} = \delta.
\]

and

\[
\min \{ \mu_B^-(xy), -\lambda \} \leq \max \{ \mu_B^-(x), \mu_B^-(y), -\delta \} \geq \min \{ -\delta, -\delta, -\delta \} = -\delta.
\]

This implies that $xy \in A$. Hence $A$ is an LA-subsemigroup of $S$.

The other cases can be seen in a similar way.

Theorem 3.5. A non-empty subset $A$ of an LA-semigroup $S$ is an LA-subsemigroup, left (right, generalized bi-, bi-) ideal of $B$ if and only if $B_A = (\mu_A^+, \mu_A^-)$ is a bipolar $(\lambda, \delta)$-fuzzy LA-subsemigroup, left (right, generalized bi-, bi-) ideal of $S$.

Proof. Let $A$ be an LA-subsemigroup of $S$. Then $B_A = (\mu_A^+, \mu_A^-)$ is a bipolar fuzzy LA-subsemigroup of $S$ and by Corollary 3.2, $B_A$ is a bipolar $(\lambda, \delta)$-fuzzy LA-subsemigroup of $S$.

Conversely, suppose that $B_A$ is a bipolar $(\lambda, \delta)$-fuzzy LA-subsemigroup of $S$ and $x, y \in A$. Let $x, y \in A$. Then $\mu_A^+(x) = \mu_A^+(y) = 1$ and $\mu_A^-(x) = \mu_A^-(y) = -1$.

Since $B_A$ is bipolar $(\lambda, \delta)$-fuzzy LA-subsemigroup of $S$ we have

\[
\max \{ \mu_A^+(xy), \lambda \} \geq \min \{ \mu_A^+(x), \mu_A^+(y), \delta \} \geq \min \{ 1, 1, \delta \} = \lambda.
\]
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and
\[
\min \{\mu_{\mathfrak{B}_A}(xy), -\lambda\} \leq \max \{\mu_{\mathfrak{B}_A}(x), \mu^\alpha_{\mathfrak{B}}(y), -\delta\} \\
\geq \min \{-1, -1, -\delta\} = -\delta.
\]

It implies that \(\mu^+_{\mathfrak{B}_A}(xy) \geq \delta\) and \(\mu^-_{\mathfrak{B}_A}(xy) \leq -\delta\). Thus \(xy \in \mathfrak{A}\).

Hence \(\mathfrak{B}_A = \langle \mu^+_{\mathfrak{B}_A} \rangle\) is a bipolar fuzzy LA-subsemigroup of \(\mathfrak{G}\).

The other cases can be seen in a similar way. \(\square\)

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