New classes of grill N-topological sets and functions

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Abstract. The prime aim of this paper is to introduce some new type of sets in grill N-topologies and subsequently establish that such collections of sets satisfy the topological axioms. We further develop the continuous and irresolute functions by using the weak forms of grill open sets. Finally, we derive the composition of such functions with suitable examples.

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1 Introduction

The supremacy of mathematics is upheld by the introduction of concept like grill N-topology. Choquet [5] was the first one to develop grill topology. He established that the study of grill topology leads to prove the aspects of proximity space, closure space and other related problems. Chattopadhyay and Thorn [4] extended this concept further that grill is always in union of ultra filters. Off late Roy and Mukherjee [14] constructed a topology for corresponding grill in a given topological space. Hatir et al. [7] and Noiri et al. [1] introduced a new type of set and discussed the properties of its continuous function in grill topological space. Many researchers [8, 11, 12, 13, 2, 6, 3] established weak forms of open sets and the of the related continuity. The concept of N-topological space was initiated by Lellis Thivagar et al. [9]. Lellis Thivagar et al. [10] extended the grill topology to the grill N-topology. In this paper we introduce and establish the properties of $\Lambda_K(A)$-sets for respective K in grill N-topological space and their relations are discussed. Further, we also establish the properties different types of functions such as continuous and irresolute functions with their relationship.

2 Preliminaries

In this preliminary section, we recall some familiar properties of grill N-topological spaces which are used in the following sections. Here the space X, we intend the grill N-topological space $(X, N\tau, G)$ without assuming separation axioms unless explicitly stated.

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Definition 2.1. [9] Let τ_1, τ_2, ... , τ_N be N-arbitrary topologies on a non-empty set X, then $N\tau = \{S \subseteq X : S = (\bigcup_{i=1}^{N} A_i) \cup (\bigcap_{i=1}^{N} B_i),\ A_i, B_i \in \tau_i\}$ is said to be a N-topology if the collection $N\tau$ satisfies the following axioms:

(i) $\varnothing, X \in N\tau$.

(ii) If $\{S_i\}_{i=1}^{\infty} \in N\tau$, then $\bigcup_{i=1}^{\infty} S_i \in N\tau$.

(iii) If $\{S_i\}_{i=1}^{n} \in N\tau$, then $\bigcap_{i=1}^{n} S_i \in N\tau$.

Then the ordered pair $(X, N\tau)$ is called an N-topological space. If $S \in N\tau$, then S is called $N\tau$-open sets and S^c is called $N\tau$-closed. We denote $N\tau$O(X) and $N\tau$C(X) respectively as the set of all $N\tau$-open and $N\tau$-closed sets on X.

Definition 2.2. [9] The interior and the closure operators of a subset A of $(X, N\tau)$ are respectively defined as

(i) $N\tau$-$int(A)$ is the union of all $N\tau$-open subsets of A.

(ii) $N\tau$-$cl(A)$ is the intersection of all $N\tau$-closed sets containing A.

Definition 2.3. [10] Grill G is a non empty collection of non empty subsets of $(X, N\tau)$ such that

(i) $A \in G$ and $A \subset B \Rightarrow B \in G$ and

(ii) $A, B \subset X$ and $A \cup B \in G \Rightarrow A \in G$ or $B \in G$.

Then a grill N-topological space is a N-topological space together with a grill G, and is denoted by $(X, N\tau, G)$.

Definition 2.4. [10] In a grill N-topological space $(X, N\tau, G)$, for each $A \subseteq X$, the local function is defined as $\Phi_G(A, N\tau) = \{x \in X : A \cap U \in G, \forall U \in N\tau(x)\}$. It is denoted as $\Phi_G(A)$ and where $N\tau(x)$ is the set of all $N\tau$-open sets containing x.

Definition 2.5. [10] For a corresponding grill G, the Kuratowski's closure operator $N\tau_G$-$cl(A) = A \cup \Phi_G(A)\ \forall A \subseteq X$, induces a finer topology $N\tau_G = \{U \subseteq X : N\tau_G$-$cl(U^c) = U^c\}$ of $N\tau$.

Definition 2.6. [10] Let a subset A of a grill N-topological space $(X, N\tau, G)$ is called

(i) ${}_GN\tau$-open if $A \subseteq N\tau$-$int(\Phi_G(A))$.

(ii) ${}_GN\tau$-α open if $A \subseteq N\tau$-$int(N\tau_G$-$cl(N\tau$-$int(A)))$.

(iii) ${}_GN\tau$-semi open if $A \subseteq N\tau_G$-$cl(N\tau$-$int(A))$.

(iv) ${}_GN\tau$-pre open if $A \subseteq N\tau$-$int(N\tau_G$-$cl(A))$.

(v) ${}_GN\tau$-β open if $A \subseteq N\tau$-$cl(N\tau$-$int(N\tau_G$-$cl(A)))$.

We denote ${}_GN\tau O(X)$ (resp. ${}_GN\tau\alpha O(X)$, ${}_GN\tau SO(X)$, ${}_GN\tau PO(X)$, ${}_GN\tau\beta O(X)$) by the set of all ${}_GN\tau$-open (resp. ${}_GN\tau$-αopen, ${}_GN\tau$-semi open, ${}_GN\tau$-pre open, ${}_GN\tau$-β open) sets. The complement of above grill N-topological open sets are called respective grill N-topological closed sets and we denote ${}_GN\tau C(X)$ (resp. ${}_GN\tau\alpha C(X)$, ${}_GN\tau SC(X)$, ${}_GN\tau PC(X)$, ${}_GN\tau\beta C(X)$) by the set of all ${}_GN\tau$-closed (resp. ${}_GN\tau$-α closed, ${}_GN\tau$-semi closed, ${}_GN\tau$-pre closed, ${}_GN\tau$-β closed) sets.

3 New classes of grill N-topological sets

Here we introduce and characterize some classes of grill N-topological sets namely $\Lambda_{GN\tau}(A)$, $\Lambda_{GN\tau\alpha}(A)$, $\Lambda_{GN\tau S}(A)$, $\Lambda_{GN\tau P}(A)$ and $\Lambda_{GN\tau\beta}(A)$. Through out this section, the notion $KO(X)$ means any one of the following collection: ${}_GN\tau O(X)$, ${}_GN\tau\alpha O(X)$, ${}_GN\tau SO(X)$, ${}_GN\tau PO(X)$, ${}_GN\tau\beta O(X)$.

Definition 3.1. Let A be a subset of agrill N-topological space $(X, N\tau, G)$. Then K-$int(A)$ and K-$cl(A)$ are respectively defined as K-$int(A) = \cup\{O \subseteq X : O \subseteq A, O \in KO(X)\}$ and K-$cl(A) = \cap\{F \subseteq X : A \subseteq F, F \in KC(X)\}$.

Theorem 3.1. *The following statements are true in a grill N-topological space $(X, N\tau, G)$ for the subsets A and B.*

(i) K-$cl(\varnothing) = \varnothing$.

(ii) $A \subseteq K$-$cl(A) \subseteq N\tau$-$cl(A)$.

(iii) *If $A \subseteq B$, then* K-$cl(A) \subseteq K$-$cl(B)$.

(iv) *For any $U \in KO(X)$ such that $x \in U$ and $A \cap U \neq \varnothing$ if and only if $x \in K$-$cl(A)$.*

Proof. (i) Obviously we have K-$cl(\varnothing) = \cap\{F : \varnothing \subseteq F, F \in KC(X)\} = \varnothing$.

(ii) Clearly, $A \subseteq K$-$cl(A)$. Suppose x is not an element of $N\tau$-$cl(A)$, then there is a $N\tau$-closed set F such that $x \notin F$ and $A \subseteq F$ implies $F \in KC(X)$ and x is not in K-$cl(A)$. Therefore, $N\tau$-$cl(A) \supseteq K$-$cl(A)$.

(iii) If $x \notin K$-$cl(B)$, then there is a $F \in KC(X)$ such that $x \notin F$ and $B \subseteq F$ implies $F \in KC(X)$ such that $x \notin F$ and $A \subseteq F$, then we get $x \notin K$-$cl(A)$. Therefore, K-$cl(B) \supseteq K$-$cl(A)$.

(iv) Assume that $x \in K$-$cl(A)$, $U \in KO(X)$ such that $x \in U$ and $A \cap U = \varnothing$, then $X - U \in KC(X)$ and $A \subseteq X - U$ implies K-$cl(A) \subseteq X - U$ and $x \in X - U$, which is a contradiction. Conversely, suppose $U \in KO(X)$ such that $x \in U$, $A \cap U \neq \varnothing$ and $x \notin K$-$cl(A)$, then there is a $G \in KO(X)$ such that $x \in G$ and $G \cap A = \varnothing$, which is contradicting the hypothesis. Therefore, $x \in K$-$cl(A)$. □

Definition 3.2. The subset $\Lambda_K(A)$ of a grill N-topological space $(X, N\tau, G)$ is defined for K as $\Lambda_K(A) = \cap\{U \subseteq X : A \subseteq U, U \in KO(X)\}$.

Theorem 3.2. *The following statements are true for the subsets $A, B, \{A_i\}_{i\in\Omega}$ of a grill N-topological space $(X, N\tau, G)$:*

(i) $A \subseteq \Lambda_K(A)$.

(ii) If $A \supseteq B$, then $\Lambda_K(A) \supseteq \Lambda_K(B)$.

(iii) If $A \in KO(X)$, then $A = \Lambda_K(A)$.

(iv) $\Lambda_K((\Lambda_K(A)) = \Lambda_K(A)$.

(v) $\Lambda_K(\bigcup_{i\in\Omega} A_i) = \bigcup_{i\in\Omega} \Lambda_K(A_i)$.

(vi) $\Lambda_K(\bigcap_{i\in\Omega} A_i) \subseteq \bigcap_{i\in\Omega} \Lambda_K(A_i)$.

Proof. Here we shall prove the claims (v) and (vi) only. The proof of the remaining claims similarly follows.

part(v) Since $\Lambda_K(A_i) \subseteq \Lambda_K(\bigcup A_i)$ for each $i \in \Omega$, then $\bigcup_{i\in\Omega} \Lambda_K(A_i) \subseteq \Lambda_K(\bigcup_{i\in\Omega} A_i)$. Conversely, if $x \notin \bigcup_{i\in\Omega} \Lambda_K(A_i)$, then $x \notin \Lambda_K(A_i)$ for each $i \in \Omega$. Then there exists a $U_i \in KO(X)$ such that $A_i \subseteq U_i$ and $x \notin U_i$ for each $i \in \Omega$ implies $\bigcup_{i\in\Omega} A_i \subseteq \bigcup_{i\in\Omega} U_i \in KO(X)$ and $x \notin \bigcup U_i$ for each $i \in \Omega$. Therefore, $x \notin \Lambda_K(\bigcup_{i\in\Omega} A_i)$ and $\Lambda_K(\bigcup_{i\in\Omega} A_i) \subseteq \bigcup_{i\in\Omega} \Lambda_K(A_i)$. Hence $\Lambda_K(\bigcup_{i\in\Omega} A_i) = \bigcup_{i\in\Omega} \Lambda_K(A_i)$.

part(vi) Since $\Lambda_K(A_i) \supseteq \Lambda_K(\bigcap A_i)$ for all $i \in \Omega$, then we have $\bigcap_{i\in\Omega} \Lambda_K(A_i) \supseteq \Lambda_K(\bigcap_{i\in\Omega} A_i)$. □

Definition 3.3. The subset $V_K(A)$ of a grill N-topological space $(X, N\tau, G)$ is defined for K as $V_K(A) = \cup\{F \subseteq X : F \subseteq A, F \in KC(X)\}$.

Theorem 3.3. *The following statements are true for any subsets $A, B, \{A_i\}_{i\in\Omega}$ of a grill N-topological space $(X, N\tau, G)$:*

(i) $A \supseteq V_K(A)$.

(ii) If $A \supseteq B$, then $V_K(A) \supseteq V_K(B)$.

(iii) If $A \in KC(X)$, then $A = V_K(A)$.

(iv) $V_K(V_K(A)) = V_K(A)$.

(v) $V_K(\bigcap_{i\in\Omega} A_i) = \bigcap_{i\in\Omega} V_K(A_i)$.

(vi) $V_K(\bigcup_{i\in\Omega} A_i) \supseteq \bigcup_{i\in\Omega} V_K(A_i)$.

Proof. The proof is similar to the one of Theorem 3.2. □

Definition 3.4. A subset A of a grill N-topological space $(X, N\tau, G)$ is said to be a Λ_K-set if $A = \Lambda_K(A)$.

Example 3.5. Let $N = 5$, $X = \{a, b, c\}$. Consider $\tau_1 O(X) = \{\varnothing, X, \{a, b\}\}$, $\tau_2 O(X) = \{\varnothing, X, \{b\}\}$, $\tau_3 O(X) = \{\varnothing, X, \{a\}\}$, $\tau_4 O(X) = \{\varnothing, X, \{b\}, \{a, b\}\}$ and $\tau_5 O(X) = \{\varnothing, X, \{a\}, \{a, b\}\}$. Then $5\tau O(X) = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}\}$ and let us consider the grill $G = P(X) - \{\varnothing\}$, then $(X, 5\tau, G)$ is a grill 5-topological space. Here $\Lambda_{G5\tau S}(X) = \Lambda_{G5\tau\beta}(X) = P(X)$, the power set of X and $\Lambda_{G5\tau}(X) = \Lambda_{G5\tau\alpha}(X) = \Lambda_{G5\tau P}(X) = \{\varnothing, \{a\}, \{b\}, \{a, b\}, X\}$.

Definition 3.6. A subset A of a grill N-topological space $(X, N\tau, G)$ is said to be a V_K-set if $A = V_K(A)$.

Lemma 3.4. *A subset A of a grill N-topological space $(X, N\tau, G)$ is Λ_K-set if and only if A^c is V_K-set.*

Proof. Suppose A is Λ_K-set, but A^c is not a V_K-set. Then either $A^c \subset V_K(A^c)$ and $A^c \not\supseteq V_K(A^c)$ or $A^c \supset V_K A^c)$ and $A^c \not\subseteq V_K(A^c)$.

Case I: If $A^c \subset V_K(A^c)$ and $A^c \not\supseteq V_K(A^c)$, then there is a $x \in V_K(A^c)$ such that x is not an element of A^c implies $x \in \cup\{F \subseteq X : F \subseteq A^c, F \in KC(X)\}$. Then for every $F \in KC(X)$ which contains x such that $F \subseteq A^c$, which is a contradiction.

Case II: If $A^c \supset V_K(A^c)$ and $A^c \not\subseteq V_K(A^c)$, then there is a $x \in A^c$ such that x is not an element of $V_K(A^c)$ implies x is not in $\cup\{F \subseteq X : F \subseteq A^c, F \in KC(X)\}$. Then for every $F \in KC(X)$ which does not contain x such that $F \subseteq A^c$, which is a contradiction.

Thus, from both cases, we have A^c is a V_K-set. We can prove the other hand side in the similar manner. □

Theorem 3.5. *The following statements are true for the subset A of the grill N-topological space $(X, N\tau, G)$:*

(i) Every set $\Lambda_K(A)$ is a Λ_K-set.

(ii) If $A \in KO(X)$, then A is a Λ_K-set.

(iii) Arbitrary union of Λ_K-sets is a Λ_K-set.

(iv) Arbitrary intersection of Λ_K-sets is a Λ_K-set.

Proof. The proof follows from Theorem 3.2 and Definition 3.4. □

Theorem 3.6. *The following statements are true in a grill N-topological space:*

(i) Every $\Lambda_{GN\tau}$-set is $\Lambda_{GN\tau P}$-set.

(ii) A subset $A \subseteq X$ is $\Lambda_{GN\tau\alpha}$-set if and only if it is $\Lambda_{GN\tau S}$-set and $\Lambda_{GN\tau P}$-set.

(iii) Every $\Lambda_{GN\tau S}$-set is $\Lambda_{GN\tau\beta}$-set.

(iv) Every $\Lambda_{GN\tau P}$-set is $\Lambda_{GN\tau\beta}$-set.

Proof. The proof immediately derives from Theorem 4.2 of [10]. □

Example 3.7. Let $N = 5$, $X = \{a, b, c\}$. Consider $\tau_1 O(X) = \{\varnothing, X, \{a, b\}\}$, $\tau_2 O(X) = \{\varnothing, X, \{a\}\}$, $\tau_3 O(X) = \{\varnothing, X, \{b\}\}$, $\tau_4 O(X) = \{\varnothing, X, \{b\}, \{a, b\}\}$ and $\tau_5 O(X) = \{\varnothing, X, \{a\}, \{a, b\}\}$. Then $5\tau O(X) = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}\}$ and let us consider the grill $G = \{X, \{a, b\}, \{a, c\}, \{a\}\}$. Then $(X, 5\tau, G)$ is a grill 5-topological space. Clearly, $\{a, b\}$ is $\Lambda_{G5\tau P}$-set but it is not $\Lambda_{G5\tau}$-set and $\{a, c\}$ is $\Lambda_{G5\tau S}$-set but it is not $\Lambda_{G5\tau P}$-set and it is not $\Lambda_{G5\tau\alpha}$-set. Also, $\{b, c\}$ is $\Lambda_{G5\tau\beta}$-set but it is not $\Lambda_{G5\tau S}$-set, not $\Lambda_{G5\tau P}$-set and not $\Lambda_{G5\tau\alpha}$-set.

Example 3.8. Let $N = 2$, $X = \{a, b, c\}$. Consider $\tau_1 O(X) = \{\varnothing, X\}$ and $\tau_2 O(X) = \{\varnothing, X, \{a\}\}$. Then $2\tau O(X) = \{\varnothing, X, \{a\}\}$ and let us take the grill $G = \{X, \{a, b\}\}$. Then $(X, 2\tau, G)$ is a grill bi-topological space. Clearly, $\{a, b\}$ is $\Lambda_{G2\tau P}$-set but it is not $\Lambda_{G2\tau}$-set, not $\Lambda_{G2\tau S}$-set and not $\Lambda_{G2\tau\alpha}$-set.

Theorem 3.7. *The collection of all $\Lambda_{GN\tau\alpha}$-sets (resp. $\Lambda_{GN\tau S}$-sets, $\Lambda_{GN\tau P}$-sets, $\Lambda_{GN\tau\beta}$-sets) forms a topology in a grill N-topological space.*

Proof. Clearly, $\varnothing$ and X are $\Lambda_{GN\tau\alpha}$-sets (resp. $\Lambda_{GN\tau S}$-sets, $\Lambda_{GN\tau P}$-sets, $\Lambda_{GN\tau\beta}$-sets). The remaining axioms derive from Theorem 3.5. □

Remark 3.9. The collection of all $\Lambda_{GN\tau}$-sets, need not form a topology in a grill N-topological space $(X, N\tau, G)$. For example, we can observe from example 3.8 that $\varnothing$ is a $\Lambda_{G2\tau}$-set, since $\Phi_G(\varnothing) = \varnothing$, but X is not.

Theorem 3.8. *If the grill N-topological open sets $KO(X)$ form a topology, then the collection of all Λ_K-sets is equal to $KO(X)$.*

Proof. From Theorem 3.5(ii), we observe that every K-open set is Λ_K-set. Conversely, assume that A is Λ_K-set, then $A = \Lambda_K(A) = \cap\{U : A \subseteq U, U \in KO(X)\}$ implies $A \in KO(X)$, since $KO(X)$ is a topology. □

4 New classes of grill N-topological continuous functions

This section introduces and establishes the properties of some grill N-topological continuous and irresolute functions.

Definition 4.1. A function f from a grill N-topological space $(X, N\tau, G)$ to a N-topological space $(Y, N\sigma)$ is said to be

(i) ${}_GN^*$-α continuous if $f^{-1}(F) \in_G N\tau\alpha C(X)$ for every $F \in N\sigma C(X)$.

(ii) ${}_GN^*$-semi continuous if $f^{-1}(F) \in_G N\tau SC(X)$ for every $F \in N\sigma C(X)$.

(iii) ${}_GN^*$-pre continuous if $f^{-1}(F) \in_G N\tau PC(X)$ for every $F \in N\sigma C(X)$.

(iv) ${}_GN^*$-β continuous if $f^{-1}(F) \in_G N\tau\beta C(X)$ for every $F \in N\sigma C(X)$.

The relation between these grill N-topological continuous functions as in the following theorem is obvious.

Theorem 4.1. *The following statements are true for a function f from a grill N-topological space $(X, N\tau, G)$ to a N-topological space $(Y, N\sigma)$:*

(i) Every N^*-continuous function is ${}_GN^*$-α continuous.

(ii) Every ${}_GN^*$-α continuous function is both ${}_GN^*$-semi continuous and ${}_GN^*$-pre continuous and vice versa.

(iii) Every ${}_GN^*$-semi continuous function is ${}_GN^*$-β continuous.

(iv) *Every ${}_GN^*$-pre continuous function is ${}_GN^*$-β continuous.*

Remark 4.2. The following examples show that the converse of above theorem need not be true.

Example 4.3. Let $N = 2$, $X = \{x, y, z, t\}$, consider $\tau_1O(X) = \{\varnothing, X, \{y\}\}$ and $\tau_2O(X) = \{\varnothing, \{x\}, X\}$. Then $2\tau O(X) = \{\varnothing, \{x\}, \{y\}, \{x, y\}, X\}$ and let us take the grill $G = \{X, \{x, z, t\}, \{x, y, t\}, \{x, y, z\}, \{x, t\}, \{x, z\}, \{x, y\}, \{x\}\}$, then ${}_G2\tau\alpha O(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}, \{x, y, z\}, \{x, y, t\}\}$. Let $Y = \{a, b, c, d\}$, $\sigma_1O(Y) = \{\varnothing, \{a\}, Y\}$, and $\sigma_2O(Y) = \{\varnothing, \{a, b\}, Y\}$. Then $2\sigma O(Y) = \{\varnothing, Y\{a\}, \{a, b\}\}$ is a bi-topology on Y and define a function f from a grill bi-topological space $(X, 2\tau, G)$ to a bi-topological space $(Y, 2\sigma)$ by $f(x) = a, f(y) = a,$ $f(z) = b$ and $f(t) = c$. Clearly the function f is ${}_G2^*$-α continuous function but it is not 2^*-continuous.

Example 4.4. Let $N = 2$, $X = \{x, y, z\}$, consider $\tau_1O(X) = \{\varnothing, X, \{y\}\}$ and $\tau_2O(X) = \{\varnothing, \{x\}, X\}$. Then $2\tau O(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}\}$ and let us consider the grill $G = \{X, \{x\}, \{x, y\}, \{x, z\}\}$, then ${}_G2\tau\alpha O(X) = {}_G2\tau PO(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}\}$, ${}_G2\tau SO(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}, \{x, z\}\}$ and ${}_G2\tau\beta O(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}, \{x, z\}, \{y, z\}\}$. Let $Y = \{a, b, c\}$, $\sigma_1O(Y) = \{\varnothing, Y, \{a, b\}\}$, and $\sigma_2O(Y) = \{\varnothing, Y\}$. Then $2\sigma O(Y) = \{\varnothing, \{a, b\}, Y\}$ is a bi-topology on Y. Define $g : (X, 2\tau, G) \to (Y, 2\sigma)$ by $g(x) = c, g(y) = a$ and $g(z) = b$. Here g is ${}_G2^*$-β continuous function but not ${}_G2^*$-α continuous, not ${}_G2^*$-semi continuous and not ${}_G2^*$-precontinuous.

Notation 1. The notion K^*-continuous means any the following continuity for functions: ${}_GN^*$-α continuous, ${}_GN^*$-semi continuous, ${}_GN^*$-pre continuous, ${}_GN^*$-β continuous. Thus a function f from a grill N-topological space $(X, N\tau, G)$ to a N-topological space $(Y, N\sigma)$ is said to be K^*-continuous if $f^{-1}(V) \in KC(X)$, for every $V \in N\sigma C(Y)$.

Theorem 4.2. *The following statements are equivalent for a function f from a grill N-topological space $(X, N\tau, G)$ to a N-topological space $(Y, N\sigma)$:*

(i) *f is K^*-continuous.*

(ii) *$f^{-1}(V) \in KO(X)$, for every $V \in N\sigma O(Y)$.*

(iii) *For each $x \in X$ and each $V \in N\sigma O(Y)$ containing $f(x)$, there is a $U \in KO(X)$ and U containing x such that $f(U) \subseteq V$.*

(iv) *$N\sigma$-$cl(f(A)) \supseteq f(K$-$cl(A))$ for any $A \subseteq X$.*

(v) *$f^{-1}(N\sigma$-$cl(B)) \supseteq K$-$cl(f^{-1}(B))$ for any $B \subseteq Y$.*

(vi) *$f^{-1}(N\sigma$-$int(B)) \subseteq K$-$int(f^{-1}(B))$ for any $B \subseteq Y$.*

Proof. $(i) \Rightarrow (ii)$: The proof follows from the fact that $f^{-1}(Y - V) = X - f^{-1}(V)$, for any $V \subseteq Y$.

$(ii) \Rightarrow (iii)$: Assume $x \in X$ and $V \in N\sigma O(Y)$ containing $f(x)$, then $f^{-1}(V) \in KO(X)$ and $x \in f^{-1}(V)$. Take $U = f^{-1}(V)$, $U \in KO(X)$ and $x \in U$ such that $V \supseteq f(U)$.

$(iii) \Rightarrow (iv)$: Let $x \in K\text{-}cl(A)$ and $V \in N\sigma O(Y)$ containing $f(x)$, then by hypothesis, there is a $U \in KO(X)$ and $x \in U$ such that $V \supseteq f(U)$. By Theorem 3.1, $A \cap U \neq \varnothing$ and $\varnothing \neq f(A \cap U) \subseteq f(A) \cap f(U) \subseteq f(A) \cap V$ implies $f(x) \in N\sigma\text{-}cl(f(A))$. Therefore, $N\sigma\text{-}cl(f(A)) \supseteq f(K\text{-}cl(A))$.

$(iv) \Rightarrow (v)$: Let $B \subseteq Y$, then by hypothesis, $N\sigma\text{-}cl(B) \supseteq N\sigma\text{-}cl(f(f^{-1}(B))) \subseteq f(K\text{-}cl(f^{-1}(B)))$. Thus $f^{-1}(N\sigma\text{-}cl(B)) \supseteq K\text{-}cl(f^{-1}(B))$.

$(v) \Rightarrow (vi)$: Let $B \subseteq Y$, then $f^{-1}(N\sigma\text{-}cl(Y-B)) \supseteq K\text{-}cl(f^{-1}(Y-B))$ implies $X - f^{-1}(N\sigma\text{-}int(B)) \supseteq X - K\text{-}int(f^{-1}(B))$. Therefore, $f^{-1}(N\sigma\text{-}int(B)) \subseteq K\text{-}int(f^{-1}(B))$.

$(vi) \Rightarrow (i)$: Let $F \in N\sigma C(Y)$, then by hypothesis, $X - K\text{-}cl(f^{-1}(F)) = K\text{-}int(f^{-1}(Y-F)) \supseteq f^{-1}(N\sigma\text{-}int(Y-F)) = f^{-1}(Y-F)$ implies $K\text{-}cl(f^{-1}(F)) \subseteq f^{-1}(F) \in KC(X)$. Thus f is K^*-continuous. □

Definition 4.5. A function f from a grill N-topological space $(X, N\tau, G_1)$ to a grill N-topological space $(Y, N\sigma, G_2)$ is said to be

(i) ${}_{G_1^2}N^*$-α irresolute if $f^{-1}(G) \in_{G_1} N\tau\alpha O(X)$ for every $G \in_{G_2} N\sigma\alpha O(Y)$.

(ii) ${}_{G_1^2}N^*$-semi irresolute if $f^{-1}(G) \in_{G_1} N\tau SO(X)$ for every $G \in_{G_2} N\sigma SO(Y)$.

(iii) ${}_{G_1^2}N^*$-pre irresolute if $f^{-1}(G) \in_{G_1} N\tau PO(X)$ for every $G \in_{G_2} N\sigma PO(Y)$.

(iv) ${}_{G_1^2}N^*$-β irresolute if $f^{-1}(G) \in_{G_1} N\tau\beta O(X)$ for every $G \in_{G_2} N\sigma\beta O(Y)$.

Notation 2. The notion ${}_1^2K^*$-irresolute refers to any one of the functions as being irresolute of the following types: ${}_{G_1^2}N^*$-α irresolute, ${}_{G_1^2}N^*$-semi irresolute, ${}_{G_1^2}N^*$-pre irresolute, ${}_{G_1^2}N^*$-β irresolute. Thus a function f from a grill N-topological space $(X, N\tau, G_1)$ to a grill N-topological space $(Y, N\sigma, G_2)$ is a ${}_1^2K^*$-irresolute if $f^{-1}(V) \in KO(X)$, for every $V \in KO(Y)$.

Theorem 4.3. *A function f from a grill N-topological space $(X, N\tau, G_1)$ to a grill N-topological space $(Y, N\sigma, G_2)$ is a ${}_1^2K^*$-irresolute if and only if $f^{-1}(V) \in KC(X)$, for every $V \in KC(Y)$.*

Proof. Assume that $V \in KC(Y)$, then $Y - V \in KO(Y)$ and so $X - f^{-1}(V) \in KO(X)$. Thus $f^{-1}(V) \in KC(X)$ for every $V \in KC(Y)$. On the other hand, let $V \in KO(Y)$, then $Y - V \in KC(Y)$ implies $f^{-1}(Y-V) \in KC(X)$ and $f^{-1}(V) \in KO(X)$. Therefore, f is ${}_1^2K^*$-irresolute. □

Theorem 4.4. *A function f from a grill N-topological space $(X, N\tau, G_1)$ to a grill N-topological space $(Y, N\sigma, G_2)$ is a ${}_1^2K^*$-irresolute, then f is K^*-continuous.*

Proof. The proof is a straightforward consequence of the fact that every $N\sigma$-open set in Y is in $KO(Y)$. □

Remark 4.6. The converse of the above theorem needs not be true, as shown in the following examples.

Example 4.7. Let $N = 2$, $X = \{x, y, z, t\}$, $\tau_1 O(X) = \{\varnothing, X, \{x\}\}$ and $\tau_2 O(X) = \{\varnothing, X, \{y\}\}$. Then $2\tau O(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}\}$ and let us consider the grill $G_1 = \{X, \{x, z, t\}, \{x, y, t\}, \{x, y, z\}, \{x, t\}, \{x, z\}, \{x, y\}, \{x\}\}$. Then ${}_{G_1}2\tau\alpha O(X) = {}_{G_1}2\tau PO(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}, \{x, y, z\}, \{x, y, t\}\}$, ${}_{G_1}2\tau SO(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}, \{x, z\}, \{x, t\}, \{x, y, z\}, \{x, y, t\}, \{x, z, t\}\}$ and ${}_{G_1}2\tau\beta O(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}, \{x, z\}, \{x, t\}, \{y, z\}, \{y, t\}, \{x, y, z\}, \{x, y, t\}, \{x, z, t\}, \{y, z, t\}\}$.
Let $Y = \{a, b, c, d\}$ and $\sigma_1 O(Y) = \{\varnothing, Y, \{a\}\}$ and $\sigma_2 O(Y) = \{\varnothing, Y, \{a, b\}\}$. Then $2\sigma O(Y) = \{\varnothing, Y, \{a\}, \{a, b\}\}$ and let us consider the grill $G_2 = \{Y, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{a, d\}, \{a, c\}, \{a, b\}, \{a\}\}$. Then ${}_{G_2}2\sigma\alpha O(Y) = {}_{G_2}2\sigma SO(Y) = {}_{G_2}2\sigma PO(Y) = {}_{G_2}2\sigma\beta O(Y) = \{\varnothing, Y, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{a, d\}, \{a, c\}, \{a, b\}, \{a\}\}$. Define $f : (X, 2\tau, G_1) \to (Y, 2\sigma, G_2)$ by $f(x) = b, f(y) = a$, $f(z) = d$ and $f(t) = d$. Here f is ${}_{G_1}2^*$-α continuous, ${}_{G_1}2^*$-semi function and ${}_{G_1}2^*$-pre continuous but not ${}_{G_1^2}2^*$-α irresolute, not ${}_{G_1^2}2^*$-semi irresolute and not ${}_{G_1^2}2^*$-pre irresolute.

Example 4.8. Let $N = 2$, $X = \{x, y, z\}$, consider $\tau_1 O(X) = \{\varnothing, X, \{x\}\}$ and $\tau_2 O(X) = \{\varnothing, X, \{y\}\}$. Then $2\tau O(X) = \{\varnothing, X, \{x, y\}, \{y\}, \{x\}\}$ and let us take the grill $G_1 = \{X, \{x, z\}, \{x, y\}, \{x\}\}$, then ${}_{G_1}2\tau\alpha O(X) = {}_{G_1}2\tau PO(X) = \{\varnothing, X, \{x, y\}, \{y\}, \{x\}\}$, ${}_{G_1}2\tau SO(X) = \{\varnothing, X, \{x, z\}, \{x, y\}, \{y\}, \{x\}\}$ and ${}_{G_1}2\tau\beta O(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}, \{x, z\}, \{y, z\}\}$. Let $Y = \{a, b, c\}$, $\sigma_1 O(Y) = \{\varnothing, Y, \{a, b\}\}$ and $\sigma_2 O(Y) = \{\varnothing, Y\}$. Then $2\sigma O(Y) = \{\varnothing, Y, \{a, b\}\}$ and let us take the grill $G_2 = \{Y, \{b, c\}, \{a, c\}, \{a, b\}, \{b\}, \{a\}\}$, then ${}_{G_2}2\sigma PO(Y) = {}_{G_2}2\sigma\beta O(Y) = \{\varnothing, Y, \{b, c\}, \{a, c\}, \{a, b\}, \{b\}, \{a\}\}$ and ${}_{G_2}2\sigma\alpha O(Y) = {}_{G_2}2\sigma SO(Y) = \{\varnothing, Y, \{a, b\}\}$. Define $g : (X, 2\tau, G_1) \to (Y, 2\sigma, G_2)$ by $g(x) = c, g(y) = a$, $g(z) = b$. Here g is ${}_{G_1}2^*$-β continuous function but not ${}_{G_1^2}2^*$-β irresolute.

Theorem 4.5. *The following statements are true for a function f from a grill N-topological space $(X, N\tau, G_1)$ to a grill N-topological space $(Y, N\sigma, G_2)$:*

(i) f is ${}_{G_1^2}N^*$-α irresolute if and only if it is ${}_{G_1^2}N^*$-semi irresolute and ${}_{G_1^2}N^*$-pre irresolute.

(iii) Every ${}_{G_1^2}N^*$-semi irresolute function is ${}_{G_1^2}N^*$-β irresolute.

(iv) Every ${}_{G_1^2}N^*$-pre irresolute function is ${}_{G_1^2}N^*$-β irresolute.

Proof. The proof is trivially following from Theorem 4.1. □

Remark 4.9. The following figure illustrates the relationship between the grill N-topologicalfunctions, where the reverse implication is not valid.

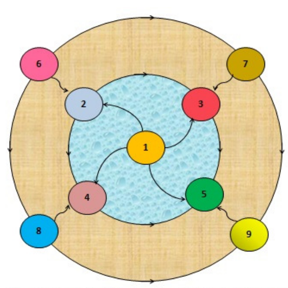

Figure 4.1 The relationship between the grill N-topological functions.

(1) N^*-continuous	(2) ${}_GN^*$-α continuous	(3) ${}_GN^*$-semi continuous
(4) ${}_GN^*$-pre continuous	(5) ${}_GN^*$-β continuous	(6) ${}_{G_1^2}N^*$-α irresolute
(7) ${}_{G_1^2}N^*$-semi irresolute	(8) ${}_{G_1^2}N^*$-pre irresolute	(9) ${}_{G_1^2}N^*$-β irresolute

5 Applications of grill N-topological functions

This section discuss the composition of continuous functions and irresolute functions in grill N-topology.

Theorem 5.1. *Let $f : (X, N\tau, G_1) \to (Y, N\sigma, G_2)$ be a ${}_1^2K^*$-irresolute function and $g : (Y, N\sigma, G_2) \to (Z, N\eta, G_3)$ be a ${}_2^3K^*$-irresolute function. Then $g \circ f : (X, N\tau, G_1) \to (Z, N\eta, G_3)$ is ${}_1^3K^*$-irresolute.*

Proof. Assume f is a ${}_1^2K^*$-irresolute function and g is a ${}_2^3K^*$-irresolute function. Define $gof : (X, N\tau, G_1) \to (Z, N\eta, G_3)$ by $(gof)(x) = g(f(x))$, for all $x \in X$ and assume $V \in KO(Z)$, $g^{-1}(V) \in KO(Y)$. This implies $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) \in KO(X)$. Therefore, $g \circ f$ is ${}_1^3K^*$-irresolute. □

Remark 5.1. The compositions of two K^*-continuous functions not necessarily be a K^*-continuous function as shown in the following examples.

Example 5.2. Let $N = 2$, $X = \{t, x, y, z\}$. Consider $\tau_1O(X) = \{\varnothing, X, \{x\}\}$ and $\tau_2O(X) = \{\varnothing, X, \{y\}\}$. Then $2\tau O(X) = \{\varnothing, X, \{x, y\}, \{y\}, \{x\}\}$ and let grill $G_1 = \{X, \{x, z, t\}, \{x, y, t\}, \{x, y, z\}, \{x, t\}, \{x, z\}, \{x, y\}, \{x\}\}$, then ${}_{G_1}2\tau\alpha O(X) = {}_{G_1}2\tau PO(X) = \{\varnothing, X, \{x, y, t\}, \{x, y, z\}, \{x, y\}, \{y\}, \{x\}\}$, ${}_{G_1}2\tau SO(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}, \{x, z\}, \{x, t\}, \{x, y, z\}, \{x, y, t\}, \{x, z, t\}\}$ and ${}_{G_1}2\tau\beta O(X) = \{\varnothing, X, \{x\}, \{y\}, \{x, y\}, \{x, z\}, \{x, t\}, \{y, z\}, \{y, t\}, \{x, y, z\}, \{x, y, t\}, \{x, z, t\}, \{y, z, t\}\}$.
Let $Y = \{a, b, c, d\}$, $\sigma_1O(Y) = \{\varnothing, Y, \{a\}\}$ and $\sigma_2O(Y) = \{\varnothing, Y, \{a, b\}\}$, then $2\sigma O(Y) = \{\varnothing, Y, \{a\}, \{a, b\}\}$. Let us consider the grill $G_2 = \{Y, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{a, d\}, \{a, c\}, \{a, b\}, \{a\}\}$, then ${}_{G_2}2\sigma\alpha O(Y) = {}_{G_2}2\sigma SO(Y) = {}_{G_2}2\sigma PO(Y) = {}_{G_2}2\sigma\beta O(Y) = \{\varnothing, Y, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{a, d\}, \{a, c\}, \{a, b\}, \{a\}\}$. Let $Z = \{p, q, r, s\}$, $\eta_1O(Z) = \{\varnothing, \{p\}, Z\}$ and $\eta_2O(Z) = \{\varnothing, \{q\}, Z\}$, then $2\eta O(Z) = \{\varnothing, Z, \{p, q\}, \{q\}, \{p\}\}$. Then a function $f : (X, 2\tau, G_1) \to (Y, 2\sigma, G_2)$ defined by $f(x) = b$, $f(y) = a$, $f(z) = d$ and $f(t) = d$. Clearly, f is ${}_{G_1}2^*$-α continuous, ${}_{G_1}2^*$-semi function and ${}_{G_1}2^*$-pre continuous. Also a function $g : (Y, 2\sigma, G_2) \to (Z, 2\eta)$ defined by $g(a) = p, g(b) = r$, $g(c) = r$ and $g(d) = p$. Clearly, g is ${}_{G_2}2^*$-α continuous, ${}_{G_2}2^*$-semi function and ${}_{G_2}2^*$-pre continuous. Then $g \circ f : (X, 2\tau, G_1) \to (Z, 2\eta)$ is not ${}_{G_1}2^*$-α continuous, not ${}_{G_1}2^*$-semi function and not ${}_{G_1}2^*$-pre continuous.

Example 5.3. Let $N = 2$, $X = \{x, y, z\}$, consider $\tau_1O(X) = \{\varnothing, X, \{x\}\}$ and $\tau_2O(X) = \{\varnothing, X, \{y\}\}$. Then $2\tau O(X) = \{\varnothing, X, \{x, y\}, \{y\}, \{x\}\}$ and let us consider the grill $G_1 = \{X, \{x, z\}, \{x, y\}, \{x\}\}$, then ${}_{G_1}2\tau\beta O(X) = \{\varnothing, X, \{x, z\}, \{y, z\}, \{x, y\}, \{y\}, \{x\}\}$. Let $Y = \{a, b, c\}$, $\sigma_1O(Y) = \{\varnothing, Y, \{a\}\}$ and $\sigma_2O(Y) = \{\varnothing, Y, \{a, b\}\}$. Then $2\sigma O(Y) = \{\varnothing, Y, \{a, b\}, \{a\}\}$ and let us consider the grill $G_2 = \{Y, \{a, c\}, \{b, c\}, \{a, b\}, \{b\}, \{a\}\}$, then ${}_{G_2}2\sigma\beta O(Y) = \{\varnothing, Y, \{a, c\}, \{b, c\}, \{a, b\}, \{b\}, \{a\}\}$. Let $Z = \{p, q, r\}$, $\eta_1O(Z) = \{\varnothing, Z, \{p\}\}$ and $\eta_2O(Z) = \{\varnothing, Z, \{q\}\}$. Then $2\eta O(Z) = \{\varnothing, Z, \{p, q\}, \{q\}, \{p\}\}$. Define $f : (X, 2\tau, G_1) \to (Y, 2\sigma, G_2)$ by $f(x) =$

$a, f(y) = c$, $f(z) = b$, then it is ${}_{G_1}2^*$-β continuous. Define $g : (Y, 2\sigma, G_2) \to (Z, 2\eta)$ by $g(a) = q, g(b) = p$, $g(c) = r$. Here g is ${}_{G_2}2^*$-β continuous. Clearly, $g \circ f : (X, 2\tau, G_1) \to (Z, 2\eta)$ is not ${}_{G_1}2^*$-β continuous.

Theorem 5.2. *If a function f from a grill N-topological space $(X, N\tau, G_1)$ to a grill N-topological space $(Y, N\sigma, G_2)$ is ${}^2_1K^*$-irresolute and a function g from a grill N-topological space $(Y, N\sigma, G_2)$ to a N-topological space $(Z, N\eta)$ is K^*-continuous, then the function $g \circ f$ from a grill N-topological space $(X, N\tau, G_1)$ to a N-topological space $(Z, N\eta)$ is K^*-continuous.*

Proof. Assume that V is a $N\eta$-open set in Z, then by hypothesis, $g^{-1}(V) \in KO(Y)$. Since f is ${}^2_1K^*$-irresolute, then $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) \in KO(X)$. Thus $g \circ f$ is K^*-continuous. □

Theorem 5.3. *A function f from a grill N-topological space $(X, N\tau, G_1)$ to a grill N-topological space $(Y, N\sigma, G_2)$ is ${}^2_1K^*$-irresolute if and only if for each $x \in X$ and each $V \in KO(Y)$ containing $f(x)$, there is a $U \in KO(X)$ and U containing x such that $V \supseteq f(U)$.*

Proof. Necessity: Assume that f is ${}^2_1K^*$-irresolute, $x \in X$ and $V \in KO(Y)$ containing $f(x)$, then $f^{-1}(V) \in KO(X)$. Take $f^{-1}(V) = U$, then $U \in KO(X)$ containing x such that $V \supseteq f(f^{-1}(V)) = f(U)$.
Sufficiency: Assume that $V \in KO(Y)$ and x is an element of $f^{-1}(V)$, then $f(x)$ is in V and by hypothesis there is a $U_x \in KO(X)$ such that x is in U_x and $V \supseteq f(U_x)$. Then $x \in U_x \subseteq f^{-1}(V)$, for every $x \in f^{-1}(V)$ implies $f^{-1}(V) = \cup\{U_x : x \in f^{-1}(V)\}$. Then by our assumption, $f^{-1}(V) \in KO(X)$ and so f is ${}^2_1K^*$-irresolute. □

The proof of the following theorems are similar to the one of the above theorem.

Theorem 5.4. *If f is a function from a grill N-topological space $(X, N\tau, G)$ to a N-topological space $(Y, N\sigma)$ is K^*-continuous and g is a function from N-topological space $(Y, N\sigma)$ to a N-topological space $(Z, N\eta)$ is N^*-continuous, then the function $g \circ f$ from a grill N-topological space $(X, N\tau, G)$ to a N-topological space $(Z, N\eta)$ is K^*-continuous.*

Theorem 5.5. *The composition of two N^*-continuous functions is K^*-continuous.*

6 Conclusions

The systematic development of classical grill concept not only in bi-topological spaces but also in tri-topological spaces and in general N-topological spaces was done by us. In this paper we made an attempt to present a new type of sets in grill N-topological space with suitable examples which explains our theory vividly. Having established the grill N-topological space we have extended this concept to new type of continuous and irresolute functions in grill N-topology with some properties. We have discussed about the composition of such functions. These findings can be execute further to the other research areas of General topology such as Supra topology, Rough topology, Fuzzy topology and so on.

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