Electronic properties of
superconductor-semiconductor mesoscopic device

Attia.A. Awadalla

Abstract. In this paper we studied electronic properties through a semiconductor quantum dot which is coupled via tunnel barrier to two superconducting reservoirs. We derive an expression for the conductance and current density for a mesoscopic device. The tunneling probability is derived using the Bogoliubov-de Gennes equation (BdG) taking into consideration the effect of Coulomb blockade and the influence of magnetic field. The barrier height has been determined by Monte-Carlo simulation technique. The current density (current per unit energy) and conductance are calculated from the phase-coherent propagation of electron-like and hole-like excitation emitted by supercurrent reservoirs, together with electron and hole exciation from the semiconductor. The results show that an oscillatory behavior of both current density and conductance. These oscillations appears as random fluctuations in peak heights. This resonant behavior might be due to the multiple Andreev reflection at the interface and Coulomb oscillation due to Coulomb charging energy. Our results agree qualitatively with those in the literature.

Key words: Electronic stub tuner, Chaotic Dynamics, fluctuations, Andreev reflections.

Introduction

In recent years our understanding of electron transport in mesoscopic conductors has greatly improved. It has become clear that at low temperature electrons can maintain their phase coherence over considerable distances. So it has now become possible to study devices through which the electron can travel ballistically, with out being scattered by impurities [17, 16, 6, 11]. It was predicted [26, 15] that the supercurrent in such a device, measured as a function of the width of the channel, should exhibit steps each time an additional one-dimensional channel is opened. In this paper we describe the supercurrent flow through S-Sm-S device with the use of transmission and reflection formalism. This formalism has already been applied for the description of electron transport in normal metals and semiconductor [7]-[21], the device geometry
is illustrated in Fig.1. The length of the semiconductors is short compared to the elastic scattering length. The width may be larger or smaller than scattering length. At the interface between the semiconductor and superconductor elastic and Andreev scattering is present.

1 Method of Calculation

All elastic processes in the sample are represented by \(2N \times 2N\) transfer matrix \(S\) whose transmission \(T_{nm}, T'_{nm}\) and reflection \(R_{nm}, R'_{nm}\) coefficients can mix the left-hand side (LHS) and right-hand side (RHS) channels. \(T_{nm}, T'_{nm}\) is the probability of an electron traveling to the right (left) in the \(n^{th}\) channel on the LHS (RHS) to be transmitted into the \(n^{th}\) channel on the RHS, and \(R_{nm}, R'_{nm}\) the probability of that electron to be back scattered into the \(n^{th}\) channel on the LHS (RHS) [13].

We consider the device geometry is illustrated in Fig.1, mesoscopic semiconductor between superconducting contacts. The transmission and reflection of the electron waves at junction formed by lead 1, and 2 can be described by \(S\) matrix [24] which relates the amplitudes of outgoing waves to the amplitudes of the incoming waves at junction, where \(S\) is expressed as

\[
S = \begin{pmatrix}
-(a + b) & \varepsilon^{1/2} & \varepsilon^{1/2} \\
\varepsilon^{1/2} & a & b \\
\varepsilon^{1/2} & b & a
\end{pmatrix}
\]

(1.1)

with \(a = \frac{1}{2} \left[ \sqrt{(1 - 2\varepsilon)} - 1 \right] \) and \(b = \frac{1}{2} \left[ \sqrt{(1 - 2\varepsilon)} + 1 \right] \). The parameter \(\varepsilon\) describes the strength of the coupling between the 1D channel and the reservoirs.

We use a well-known model [22, 8] where the pair potential \(\Delta(x) = 0\) in semiconductor channel, and \(\Delta = \Delta_0 \exp(i\phi_{1,2})\) with \(\phi_{1,2}\) is the Cooper pair phases for the two leads. The wave functions can be found from the time-independent Bogoliubov de Gennes equation [10]:

\[
E \begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \begin{pmatrix} H - E_F & \Delta \\ \Delta^* & -(H^* - E_F) \end{pmatrix} \begin{pmatrix} u(x) \\ v(x) \end{pmatrix}
\]

(1.2)

In the previous equation, \(u(x)\) describes the electron wave function, and \(v(x)\) describes the hole wave function with an excitation energy \(E\) relative to the Fermi energy \(E_F\). Their expressions in the corresponding regions are:

\[
\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(\pm iq^+ x), \quad \text{with} \quad \hbar q^+ = \sqrt{2mE_F + E}
\]

(1.3)

with \(+(-)\) corresponding to electron excitation which move in positive (negative) \(x\) direction [29] and

\[
\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(\pm iq^- x), \quad \text{with} \quad \hbar q^- = \sqrt{2mE_F - E}
\]

(1.4)
and in the superconducting regions are [2]:

\[
\begin{pmatrix}
u(x) \\
v_o \exp(i\phi)
\end{pmatrix} = \begin{pmatrix}
u_o \\
v_o
\end{pmatrix} \exp(\pm ik^+_j x)
\]

where

\[
k^+_j = \left[ \frac{2m}{\hbar^2} \left( E_F + (E^2 - \Delta^2_0)^{0.5} + V_o + V_b + \frac{U_C N^2}{2} + e \eta V_g N + (0.5 \hbar \omega_c) \right) - \frac{j^2 \pi^2}{4 W^2} \right]^{0.5}
\]

with \(+\(-\)\) corresponding to electronlike excitations which travel in positive or (negative) \(x\) direction. \(V_o\) is the potential well depth of the semiconductor heterojunction quantum dot, \(E_F\) is the Fermi-energy, \(U_C\) is the charging energy and equals \(\frac{e^2}{2C}\) in which \(C\) is the quantum dot capacitance and \(e\) is electron charge, \(\omega_c\) is the cyclotron frequency and equals \(eB\) \(m^{-1}\), \(B\) is the magnetic field, \(N\) is the number of electrons in the quantum dot, \(\eta\) is the lever arm associated with the capacitance coupled to the gate, \(V_b\) is the Schottky barrier height, \(V_g\) is the gate voltage and \(m\) is the effective mass.

\[
\begin{pmatrix}
u(x) \\
v_o \exp(i\phi)
\end{pmatrix} = \begin{pmatrix}
u_o \\
v_o
\end{pmatrix} \exp(\pm ik^-_j x)
\]

\[
k^-_j = \left[ \frac{2m}{\hbar^2} \left( E_F - (E^2 - \Delta^2_0)^{0.5} + V_o + V_b + \frac{U_C N^2}{2} + e \eta V_g N + (0.5 \hbar \omega_c) \right) - \frac{j^2 \pi^2}{4 W^2} \right]^{0.5}
\]

with \(+\(-\)\) corresponding to electronlike excitations which travel in negative or (positive) \(x\) direction.

The coherence factors are given by

\[
u_o = \frac{1}{\sqrt{2}} \sqrt{1 + \left[ 1 - (\Delta_0/E)^2 \right]^{0.5}}
\]

\[
v_o = \frac{1}{\sqrt{2}} \sqrt{1 - \left[ 1 - (\Delta_0/E)^2 \right]^{0.5}}
\]

The formalism we use is based on Buttiker description of phase-coherent of electron transport in multiprobe normal conductor [5]. The reflection probability \(R_{nn}(\phi, E)\) is defined as current (in units of \(e\)) which flow back into the \(n^{th}\) reservoir as a result of the current which is emitted by the \(n^{th}\) channel at energy \(E\). Similarly the transmission probability \(T_{nn}(\phi, E)\) is defined as the ratio of the current which flow into the \(n^{th}\) channel as a result of particle at energy \(E\). Because the reservoirs can emit two types of particle, we add a superscript \(p\), with \(p = 1\) for electron (or electro-like) excitations and \(p = 2\) for hole (or hole-like) excitations. The current density (current per unit energy) \(J_{n}^{ex}(\phi, E)\) can be expressed as [19]:

\[
J_{n}^{ex}(\phi, E) = \frac{2e}{h} \sum_{p=1,2} C^p_n(E) - R_{nn}^p(\phi, E) - \sum_{m \neq n} T_{nm}^p(\phi, E)
\]
In equation (1.9), $C_{pn}(E)$ indicates the ratio between the current and particle current carried by the excitations which are emitted by the $n^{th}$ channel, and $R_{nn}(\phi, E)$, $T_{nm}(\phi, E)$ are the transmission and reflection probabilities from different channels. The conductance for considered device could be expressed in terms of the transmission and reflection probabilities of the various voltage probes [7, 4]:

\[
G = \frac{e^2}{\hbar} \left( \sum_{nm} T_{nm}(\phi, E) \right) \frac{2 \sum_{i} v_{i}^{-1}}{\sum_{i} (1 + \sum_{mn} R_{mn}(\phi, E) - \sum_{nm} T_{nm}(\phi, E)) v_{i}^{-1}}
\]

Where $v_{i}$ is the Fermi velocity in current direction in the $i^{th}$ channel, defined by $(\hbar K_{i}/m)$ ($K_{i}$ is the longitudinal Fermi wave vector of the $i^{th}$ channel), $\phi$ is phase angle and $T_{nm}, R_{nm}$ are the transmission and reflection probabilities [12].

## 2 Results and Discussions

We investigate the dependence of the conductance and current density on the magnetic field, gate voltage and phase angle (see Figs 2,3 and 4 and also Figs 5,6 and 7). So, numerical calculations have been performed as follow:

![Schematic representation of the model.](image)

**Fig. 1.** Schematic representation of the model.

![Graph of conductance versus magnetic field for different values of temperature $T$.](image)

**Fig. 2.** The conductance-magnetic field dependence for different values of temperature $T$. 

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The semiconductor heterostructure material was taken as GaAs/AlGaAs and superconducting leads are niobium (Nb). In our calculations we consider the values of both mean free path and coherence length of quasiparticles are larger than the dimensions of the present mesoscopic device, so the transport will be treated as ballistic one. The electron transport through the device is treated as stochastic process, so that the tunneled electron energy has been taken as random number relative to the superconducting energy gap, $\Delta_o$. Also, the Schottky barrier height $V_b$, is determined by using the Monte-Carlo-simulation technique and equals $0.47eV$. This value is in agreement with those found previously [2]-[4], [14]-[27].
Our results show the following features:

1. As shown from (Figs 2, 3 and 4), that the conductance $G$, oscillates as a functions of magnetic field $B$, gate voltage $V_g$, and phase angle $\phi$, and the oscillation peak, increases with increasing the above parameters ($B, V_g, \phi$). These oscillations are due
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to the Coulomb blockade effect [9] and the quantum interference of quasiparticles due to Andreev reflections processes at the semiconductor-superconductors interface [30].

2. Also from the Figs 5,6 and 7, the current density $J$, oscillates as functions of $B, V_g, \phi$, but the oscillations in this case is random oscillation. The behavior may be explained according to resonant tunneling through the bound state formed within the quantum dot belonging to different Landau levels [31]-[3].

References


Author’s address:

Attia.A.Awadalla
Physics Department, Faculty of Sciences, Beni-Suef University, Egypt.
email: atttiamd2005@yahoo.com